NEW RESULTS ON THE PRECISION OF THE LEP LUMINOSITY * **

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We present recent progress on the theoretical precision limits of the LEP luminosity process, as calculated by the Monte Carlo event generator BHLUMI4.04. We include exact results for all two-photon radiative corrections to the process $e^+e^- \to e^+e^-$ at small angles and LEP energies. These results reduce the precision estimate for the $\mathcal{O}(\alpha^2)$ photonic radiative correction from 0.1% to 0.027%, leading to an overall precision of 0.061% for the currently published version of BHLUMI4.04. This precision level is important for the final precision Z physics measurements at LEP1. We also present precision estimates for LEP2.

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1. Introduction

A precise measurement of the luminosity is required for all experiments at LEP measuring normalized cross sections. Any errors become a limiting factor in high-precision measurements of the electroweak parameters in the standard model. Luminosity is measured via the low-angle Bhabha scattering process $e^+e^- \to e^+e^- + n\gamma$. This process was chosen because it has a clean, strong signal and is dominated by pure QED, with weak interactions entering below 3%, so that it can be calculated cleanly as well.

It is important to maintain a parity between the experimental and theoretical precision in the luminosity process. Recent progress with the new luminosity monitors at LEP has reduced the experimental uncertainty to below 0.05%. The best theoretical uncertainty has been obtained using the Monte Carlo program BHLUMI4.04 [1], which cites a precision of 0.11% for an acceptance matching the SICAL luminometer [2] at ALEPH. In light of the experimental progress, it is important to re-examine the theoretical precision to obtain a more accurate bound on the uncertainty.

We have found [3] that a careful analysis of the two-photon radiative corrections leads to a revised precision of 0.061% for BHLUMI with LEP1 parameters. We also report results for LEP2 parameters. We obtained these results using exact small-angle matrix elements for all of the two photon (real and virtual) processes contributing to low-angle Bhabha scattering. These can be used to further reduce the uncertainty in the theoretical results as the need arises. We also discuss the technical precision of the implementation of these new results in the context of the BHLUMI Monte Carlo generator.

TABLE I

Theoretical uncertainty for an ALEPH SICAL-type calorimetric detector. L is the logarithm (1) in the leading log expansion. For LEP1, the CMS energy is the Z mass, and the angular range is 1°-3°, and for LEP2, the CMS energy may be up to 176 GeV, and angular range within 1°-3° and 3°-6°. "Past" results are from Refs. [1,10,11].

	LEP1		LEP2	
Type of correction	Past	Present	Past	Present
Missing photonic $\mathcal{O}(\alpha^2)$ [3]	.10%	.027%	0.20%	0.04%
Missing photonic $\mathcal{O}(\alpha^3 L^2)$ [4]	.015%	.015%	0.03%	0.03%
Vacuum polarization [5,6]	.04%	.04%	0.10%	0.10%
Light pairs [7,8]	.03%	.03%	0.05%	0.05%
Z exchange [9]	.015%	.015%	0.0%	0.0%
Total	.11%	.061%	0.25%	0.122%

The currently published uncertainty for BHLUMI may be broken down as shown in the "Past" columns of Table I, following Ref. [1,10]. The largest contribution comes from the missing $\mathcal{O}(\alpha^2)$ photonic correction, which alone contributes 0.1% to the total precision for LEP1 parameters.

To obtain the results in the "Present" column, we have re-examined the two photon bremsstrahlung contributions. These have been incorporated in BHLUMI in a leading log expansion in terms of the logarithm

$$L = 2\ln\left\{\frac{E_{\rm cms}}{m_e}\sin(\theta/2)\right\}, \quad (1)$$

which is on the order of $15 - 20$ for the LEP1 and LEP2 parameters. The leading contribution at order α^2 has a factor of L^2, and terms with lower powers of L may be added systematically as needed. BHLUMI4.04 includes the $\mathcal{O}(\alpha^2 L^2)$ leading log matrix element together with Yennie–Frautschi–Suura (YFS) exponentiation [12]. We have used the exact results in Refs. [13–15] and the exact result in Ref. [16] to make a more realistic estimate of the true size of this dominant error [1,10].

It is important to reexamine the technical precision of the Monte Carlo program's generation of two hard real photons together with the implementation of the new exact matrix element. This can be done by implementing the same matrix element in both BHLUMI4.04 and an independent Monte Carlo program. We will present the results of this test, and show that the technical precision remains very high compared to the physical precision.

The missing part of the $\mathcal{O}(\alpha^2)$ correction due to one hard and one virtual photon can be found by implementing the exact (one loop) expression of Ref. [13] in BHLUMI4.04, and comparing it to the leading log expression already in use. In Fig. 1, we show the electron-line emission cross-section obtained by running BHLUMI for 10^6 events with ALEPH SICAL-type acceptance, for both LEP1 and LEP2 parameters. We display the differences between the BHLUMI leading log expression and two more precise expressions: the exact one from Ref. [13] and a semi-collinear expression from Ref. [17].

The BHLUMI results are within .02% of the exact result in units of the respective Born cross section throughout the experimentally interesting regime $0.2 \le 1 - z_{\rm min} \le 1.0$. This is the main reason we have been able to reduce the estimated precision of the BHLUMI4.04 prediction in comparison to Ref. [1,10].

The missing part of the $\mathcal{O}(\alpha^2)$ correction due to a pair of hard photons can be found by implementing the exact (tree level) expression of Ref. [15] in BHLUMI4.04, and comparing it to the leading log expression already in use. In Fig. 2, we show the electron-line emission cross-section obtained by running BHLUMI for 10^6 events with ALEPH SICAL-type acceptance,

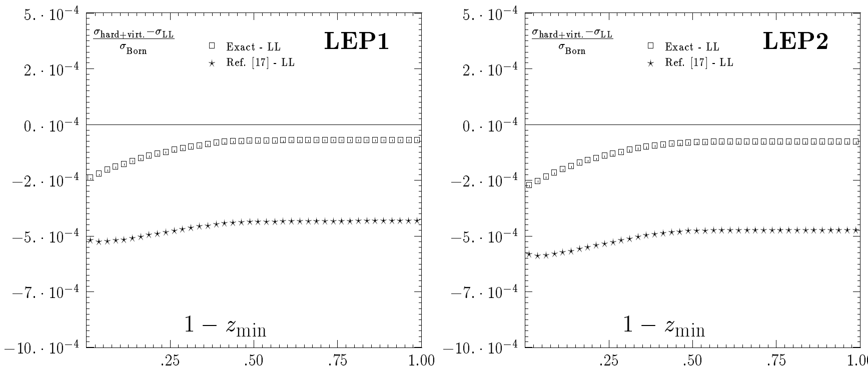

Fig. 1. Monte Carlo results (10^6 events) for the $\mathcal{O}(\alpha^2)$ cross section for single photon emission from the electron line. Differences between three matrix elements are shown for the SICAL Wide-Narrow trigger, divided by the Narrow-Narrow Born cross section, with z_{min} defined as in Fig. 2 of Ref. [9]. The two graphs display results for LEP1 and LEP2 parameters, respectively.

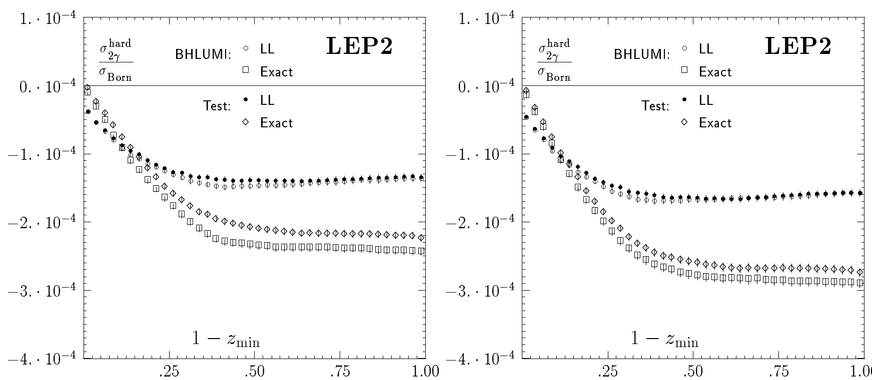

Fig. 2. Monte Carlo results (10^6 events) for the $\mathcal{O}(\alpha^2)$ cross section for two photon emission from the electron line. Exact and leading log hard photon cross sections are shown for the SICAL Wide-Narrow trigger, divided by the Narrow-Narrow Born cross section, with z_{min} defined as in Fig. 2 of Ref. [9]. The two graphs display results for LEP1 and LEP2 parameters, respectively.

for both LEP1 and LEP2 parameters. The hard photon part of the double bremsstrahlung cross section (technically, the averaged YFS residual $\bar{\beta}_2$ [12]) is displayed for both matrix elements.

As a check on the technical precision, we also implement both of these matrix elements in an independent "test" Monte Carlo program optimized to generate exactly two hard photons. The same two-photon cross sections are calculated, and compared to the BHLUMI results.

We find that the error introduced by using the leading log approximation is 0.012% for the relevant range of parameters, in agreement with the estimate in Ref. [1]. The difference between the two Monte Carlo generators is below 0.003% of the Born cross section. This shows that the technical precision is still good on the scale of the improved physical precision.

Finally, we turn to the exact result for two virtual photons, and compare it to the exact result in BHLUMI4.04. This has been obtained by analytically continuing the result of Ref. [16] for the $\mathcal{O}(\alpha^2)$ (two-loop) QED charge form factor from the s-channel to the t-channel. For the ALEPH SICAL type acceptance at the Z^0 peak, this was found [3] to yield a 0.014% contribution to the cross section.

Adding the three errors above in quadrature, find that the current calculation of the $\mathcal{O}(\alpha^2)$ photonic corrections in BHLUMI4.04 are accurate to 0.027%. Using this result in Table I for Ref. [1] we arrive at the precision tag 0.061% for in BHLUMI4.04 at the Z^0 peak. Repeating this analysis for LEP2 parameters, we find that the corresponding precision of BHLUMI4.04, for both the SICAL and LCAL type acceptances, is now reduced to 0.122% compared to the estimate in Ref. [1] of 0.25%. This new LEP2 result applies up to cms energies of 200GeV.

Our new estimate for the missing $\mathcal{O}(\alpha^2)$ bremsstrahlung contribution in BHLUMI4.04 agrees with the estimate of 0.03% made by Montagna *et al.* [18] using an approximation with one hard collinear external photon and an acollinear internal photon. Our exact result confirms that their approximation actually gives the bulk of the $\mathcal{O}(\alpha^2)$ correction.

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