STATUS OF LOW ENERGY SUPERSYMMETRY* **

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We review 1) constraints on low energy supersymmetry from the search for Higgs boson and from precision data, 2) dependence of coupling unification on the superpartner spectrum, 3) naturalness and fine tuning in the minimal and non- minimal scenarios.

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1. Constraints from the search for Higgs boson and from precision data

The most appropriate starting point for reviewing the status of low energy supersymmetry is the status of the Standard Model itself. Its success in describing all the available experimental data becomes more and more pronounced, with all potential deviations disappearing with the increasing precision of data. At present, the Standard Model is successfully tested at 1 permille accuracy up to the LEP2 and TEVATRON energies.

One of the most important results following from the precision tests of the Standard Model is the strong indirect indication for a light Higgs boson. Although the sensitivity of electroweak observables to the Higgs boson mass is only logarithmic, the precision of both data and calculations is high enough for obtaining from the fits the upper bound on the Higgs boson mass M_h of about 250 GeV at 95% C.L. The best value of M_h in the fits is in the region of the present direct experimental lower bound, $M_h \gtrsim 90$ GeV.

One should stress that these results are obtained strictly in the Standard Model. The best fitted value of M_h can be changed if we admit new physics in the $\Delta\rho$ parameter. There is the well known (see, for instance, [1]) "flat

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direction" in χ^2, which correlates almost linearly $\ln M_h$ with $(\Delta\rho)^{\rm NEW}$ (previously known as $\ln M_h$–m_t correlation), as the two effects compensate each other in the electroweak observables like ρ, $\sin^2\Theta_{\rm eff}$ and M_W. In general, however, it is very difficult to find a self-consistent extension of the Standard Model that would use this freedom[1].

The result for M_h from the Standard Model fits to precision data raises strong hopes for experimental discovery of the Higgs boson in a relatively near future. Secondly, it is in agreement with the most robust prediction of supersymmetric extensions of the Standard Model, which is the existence of a light Higgs boson. This prediction is generic for low energy effective supersymmetric models [2, 3] and becomes particularly quantitative in the Minimal Supersymmetric Standard Model (MSSM), defined by three assumptions: a) minimal particle content consistent with the observed particle spectrum and with supersymmetry, b) R-parity conservation, c) most general soft supersymmetry breaking terms consistent with the SM gauge group[2].

In the MSSM the lightest Higgs boson mass is predicted [4, 5] (now at two- loop level [6, 7]) in terms of free parameters of the model:

$$M_h = M_h(M_Z, G_\mu, \alpha_{EM}, m_t, \tan\beta, M_A, \text{superpartner masses})\,. \qquad (1)$$

In practise, only the third generation sfermions are important in Eq. (1) and M_h depends logarithmically on their masses. It is worth recalling the dependence of M_h on $\tan\beta$ and M_A (see, for instance [5]): for fixed $\tan\beta$ and superpartner masses, M_h reaches its maximal value for $M_A \approx 250$ GeV and for larger values of M_A it stays then approximately constant. As a function of $\tan\beta$, these maximal values rise with $\tan\beta$ and remain almost constant for $\tan\beta \gtrsim 4$, with $M_h \approx 130$ GeV for superpartner masses lighter than 1 TeV. There is also a rather strong dependence on the left-right mixing in the stop sector, with the $\tan\beta$ dependent upper bounds for M_h reached for large mixing [7].

Clearly, the upper bound on M_h from precision fits in the Standard Model is encouraging for supersymmetry. However, one can also ask how *constraining* for the MSSM is the present direct experimental lower limit $M_h > 90$ GeV[3]. This question has been studied in Ref. [8] and the reader

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[1] Note also that $M_h \lesssim \mathcal{O}(500)$ GeV by unitarity and "triviality" arguments. So the corrections to $\Delta\rho$ must be just right (not too big), to explore only relatively small variation of $\ln M_h$.

[2] There is often some confusion about the terminology "MSSM". We always understand the MSSM as an effective low energy model with parameters unconstrained by any further high scale model assumptions.

[3] Strictly speaking, this limit is valid only in the Standard Model. In the MSSM, in some small regions of parameter space, the limit is actually lower, but we ignore this effect here.

is invited to consult it for details. The main conclusion is that, indeed, the low $\tan\beta$ region (interesting in the context of the quasi-infrared fixed point scenario) is strongly constrained. It can be realized in Nature only if at least one stop is heavy, $\mathcal{O}(1)$ TeV, and with large left-right mixing. This follows from the fact that for low $\tan\beta$ the tree level lightest Higgs boson mass is small and large radiative corrections have to account for the experimental bound. The infrared fixed point scenario with low $\tan\beta$ will be totally ruled out if a Higgs boson is not found at LEP2 operating at 200 GeV. For intermediate and large values of $\tan\beta$, those constraints, of course, disappear.

Superpartner masses that appear in radiative corrections to the Higgs boson mass also appear in the calculation of $\Delta\rho$ and related observables such as $\sin^2\theta_{\rm eff}$, M_W *etc.* It is well known (see, for instance, [10, 11]) that the main new contribution to $\Delta\rho$ comes from the third generation left-handed sfermions. The custodial $\mathrm{SU}_V(2)$ breaking in other sectors of the MSSM is very weak. Thus, the quality of description of precision electroweak data in the MSSM depends on those superpartner masses. Instead of attempting an overall fit, it is more instructive to focus on very well measured $\sin^2\theta_{\rm eff}$ and on soon very well measured M_W. Calculating *e.g.* $\sin^2\theta_{\rm eff}$ in terms of M_Z, G_μ, α_{EM} and the superpartner masses and comparing with the experimental value we expect to get bounds on the left-handed third generation sfermions. We said earlier that for low $\tan\beta$ strong lower bounds on the stop mass follow from the experimental lower limit on M_h. For such low values of $\tan\beta$ the bound from precision data is somewhat weaker but, contrary to the other bound, it remains very significant for all values of $\tan\beta$. The absolute lower bound on the left-handed stop and slepton masses is obtained for intermediate and large $\tan\beta$ since the data can then accommodate larger correction to $\Delta\rho$ due to a heavier Higgs boson -see the earlier discussion. In Fig. 1 we show [11] the dependence of $\sin^2\Theta_{\rm eff}$ and of M_W on the stop and slepton masses. We conclude that for 2σ precision in these observables one needs $m_{\tilde{q}_{\rm L}} > \mathcal{O}(400)$ GeV and $m_{\tilde{l}_{\rm L}} > \mathcal{O}(150)$ GeV, with stronger bounds in low $\tan\beta$ region. Precision data (and the lower limit on M_h for low $\tan\beta$) indicate that at least some superpartners are well above the electroweak scale!

On the other hand, the right-handed sfermions of the third generation and all sfermions of the first two generations as well as the gaugino/ higgsino sectors are essentially unconstrained by the LEP precision data. They decouple from those observables even if their masses are $\mathcal{O}(M_Z)$.

Are there other observables that are more sensitive to the rest of the spectrum, that is in which its decoupling in virtual effects is slower? Indeed, there are such examples. The decay $b\to s\gamma$, the $K-\bar{K}$ and $B-\bar{B}$ mixing are sensitive to the right-handed third generation sfermions and to the higgsino-

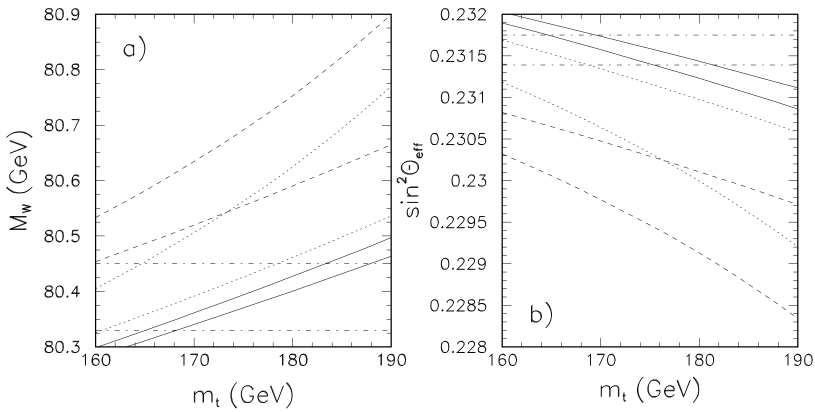

Fig. 1. (a) $W^{\pm}$ mass, and (b) $\sin^2\theta^{\rm eff}_{\rm lept}$ as a function of the top quark mass, calculated for all but the heavier stop and the heavier slepton superpartner masses equal to 90 GeV. Top-down in (a) and bottom-up curves in (b): dashed — the heavier slepton (degenerate) masses at 90 GeV and the heavier stop masses at 200 and 400 GeV, respectively; dotted — slepton masses at 150 GeV and stop masses as before; solid — slepton masses at 250 GeV, stop mass at 500 GeV and at 1 TeV, respectively; dashed-dotted horizontal curves — experimental 1σ bands.

like chargino/neutralino. This is because the relevant coupling is the top quark Yukawa coupling. For a review see [12]. (Even the gaugino and the first and the second generation sfermion contribution decouples quite slowly in the $b \to s\gamma$ decay.) Those processes have still good prospects to reveal indirect effects of supersymmetry once the precision of data is improved.

The superpartner mass spectrum is the low energy window to the mechanism of supersymmetry breaking and to the theory of soft supersymmetry breaking terms. For instance, with the lower bound on the left-handed stop from the precision data and with the still open possibility of a much lighter, say $\mathcal{O}(100)$ GeV, right-handed stop one can envisage the case of a strongly split spectrum. This case is discussed in Ref. [13] as an illustration of the bottom-up approach to the problem of supersymmetry breaking. It is shown that for low $\tan\beta$ it needs, at the GUT scale, scalar masses much larger than the gluino mass and the strong non-universality pattern, $m^2_Q : m^2_U : m^2_{H_2} = 1:2:3$. This is related to the fact that the hypothetical spectrum considered in this example departs from the well known sum rules valid in the infrared fixed point limit and with the scalar and gaugino masses of the same order of magnitude. In such a scenario, the lighter chargino is generically also light, $\mathcal{O}(100)$ GeV, and it can be gaugino- or higgsino-like.

2. Dependence of coupling unification on the superpartner spectrum

The gauge coupling unification [14,15] within the MSSM has been widely publicized as the most important piece of indirect evidence for supersymmetry at accessible energies. The unification idea is predictive if physics at the GUT scale is described in terms of only two parameters: α_U and M_U. Then we can predict, for instance, $\alpha_s(M_Z)$ in terms of $\alpha_{EM}(M_Z)$ and $\sin^2\theta_W(M_Z)$. Here we mean the running coupling constants defined in the $\bar{M}S$ renormalization scheme in the SM which, we assume, is the correct renormalizable theory at the electroweak scale. The value of $\alpha_{EM}(M_Z)$ is obtained from the on-shell α_{EM}^{OS}=1/137.0359895(61) via the RG running in the SM, with .01% uncertainty due to the continuous hadronic contribution to the photon propagator. The most precise value of $\sin^2\theta_W(M_Z)$ in the SM is at present obtained from its calculation in terms of G_μ, M_Z, α_{EM} and the top quark mass (with some dependence on the Higgs boson mass). The unification prediction for $\alpha_s(M_Z)$ is obtained by using two loop RG equations in the MSSM, for the running from M_Z up to the GUT scale defined by the crossing of the electroweak couplings. For the two–loop consistency (and precision), one must include the supersymmetric threshold corrections in the leading logarithmic approximation. (For a spectrum that is above the present experimental lower limits on the superpartner masses, the finite threshold effects $\mathcal{O}(\frac{M_Z}{m_{\rm SUSY}})$ are already small enough to be neglected. [15]) In this approximation the dependence of $\alpha_s(M_Z)$ on the supersymmetric spectrum can to a very good approximation be described by a single parameter $T_{\rm SUSY}$ [16]. We get

$$\alpha_s(M_Z) = f(G_\mu, M_Z, \alpha_{EM}, m_t, M_h, T_{\rm SUSY})\,, \tag{2}$$

where

$$T_{\rm SUSY} = |\mu| \left(\frac{m_{\tilde{W}}}{m_{\tilde{g}}})\right)^{3/2} \left(\frac{M_{\tilde{l}}}{M_{\tilde{q}}}\right)^{3/16} \left(\frac{M_{A^\circ}}{|\mu|}\right)^{3/19} \left(\frac{m_{\tilde{W}}}{|\mu|}\right)^{4/19} . \tag{3}$$

We observe that the effective scale $T_{\rm SUSY}$ depends strongly on the values of μ and of the ratio $m_{\tilde{W}}$ to $m_{\tilde{g}}$ but very weakly on the values of the squark and slepton masses. It is also clear that the scale $T_{\rm SUSY}$ can be much smaller than M_Z even if all superpartner masses are heavier than the Z boson. Only for a fully degenerate spectrum $T_{\rm SUSY}$ is the common superpartner mass. Moreover we note that the supersymmetric threshold effects are absent for $T_{\rm SUSY} = M_Z$. The unification prediction for the strong coupling constant as a function of $T_{\rm SUSY}$ is shown in Fig. 2(a). We see that the variation of $\alpha_s(M_Z)$ with $T_{\rm SUSY}$ is substantial. The central experimental value

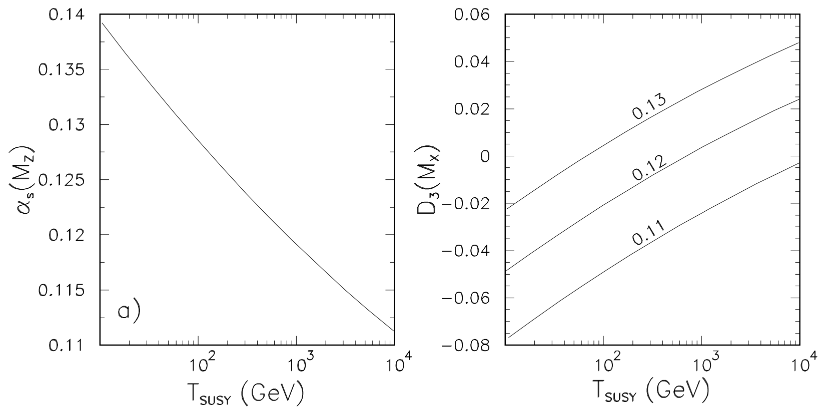

Fig. 2. Unification prediction for $\alpha_3(M_Z)$ (a) and D_3 defined by Eq. (5) (b), as a function of $T_{\rm SUSY}$ ($m_t = 175$ GeV, $\tan\beta = 2$).

$\alpha_s(M_Z) = 0.118$ is obtained for $T_{\rm SUSY} = 1$ TeV. The values $T_{\rm SUSY} = M_Z$ and $T_{\rm SUSY} = 10$ TeV change the prediction by $\delta\alpha \approx \mp 0.01$ which is 3σ away from the central value. It is interesting to see that the value as large as $T_{\rm SUSY} = 10$ TeV is equally acceptable (or unacceptable) as $T_{\rm SUSY} = M_Z$. As we already mentioned, a given value of $T_{\rm SUSY}$ not necessarily implies similar scale for the superpartner masses. It should be stressed that in models with universal gaugino masses at the GUT scale we have the relation

$$\frac{m_{\tilde{W}}}{m_{\tilde{g}}} \approx \frac{\alpha_2(M_Z)}{\alpha_3(M_Z)} \quad \text{and} \qquad T_{\rm SUSY} \approx |\mu| \left(\frac{\alpha_2(M_Z)}{\alpha_3(M_Z)}\right)^{3/2} \approx \frac{1}{7}|\mu|\,. \tag{4}$$

Moreover, radiative electroweak symmetry breaking correlates the μ parameter with the soft parameters that determine the sfermion masses, and large μ implies squark masses of the same order of magnitude (but not vice versa!). Of course, it is also conceivable to have large $T_{\rm SUSY}$ with small μ. This requires $M_2/M_3 \gg 1$ [17] and, therefore, a gauge dependent transmission to the visible sector of the supersymmetry breaking mechanism.

The minimal unification may be too restrictive as it is generally expected that there are some GUT/stringy threshold correction or higher dimension operator effects. Therefore is also interesting to reverse the question and to study the convergence in the MSSM of all the three couplings in the bottom-up approach, starting with their experimental values at the scale M_Z.

It is convenient to define the "mismatch" parameter [18]

$$D_3 = \frac{\alpha_3(M_{\rm GUT}) - \alpha_2(M_{\rm GUT})}{\alpha_2(M_{\rm GUT})}\,, \tag{5}$$

where $M_{\rm GUT}$ is again defined by the crossing of the electroweak couplings. The results for D_3 as a function of $T_{\rm SUSY}$ are shown in Fig. 2(b) for three values of $\alpha_3(M_Z)$. We recall that the experimental value is $\alpha_3(M_Z) = 0.118 \pm 0.003$. We see that for $T_{\rm SUSY}$ changing from M_Z up to 10 TeV all the three couplings unify within 2% accuracy! On the one hand, this is certainly an impressive success of the MSSM, but on the other hand we conclude that the gauge coupling unification does not put any significant upper bounds on the superpartner spectrum.

Yukawa coupling unification is a much more model dependent issue. It strongly relies on GUT models and has no generic backing in string theory. Nevertheless, it happens that in the bottom-up approach the b and τ Yukawa couplings approximately unify at the same scale as the gauge couplings. On a more quantitative level, it is well known that exact b–τ Yukawa coupling unification, at the level of two-loop renormalization group equations for the running from the GUT scale down to M_Z, supplemented by three-loop QCD running down to the scale M_b of the pole mass and finite two-loop QCD corrections at this scale, is possible only for very small or very large values of $\tan\beta$. This is due to the fact that renormalization of the b-quark mass by strong interactions is too strong, and has to be partly compensated by a large t-quark Yukawa coupling. This result is shown in Fig. 3(a). We compare there the running mass $m_b(M_Z)$ obtained by the running down from $M_{\rm GUT}$, where we take $Y_b = Y_\tau$, with the range of $m_b(M_Z)$ obtained from the pole mass $M_b = (4.8 \pm 0.2)$ GeV [20], taking into account the above-mentioned low-energy corrections. These translate the range of the pole mass: $4.6 < M_b < 5.0$ GeV into the following range of the running mass $m_b(M_Z)$: $2.72 < m_b(M_Z) < 3.16$ GeV. To remain conservative, we use $\alpha_s(M_Z) = 0.115(0.121)$ to obtain an upper (lower) limit on $m_b(M_Z)$.

It is also well known [21, 22] that, at least for large values of $\tan\beta$, supersymmetric finite one-loop corrections (neglected in Fig. 3(a)) are very important. These corrections are usually not considered for intermediate values of $\tan\beta$, but they are also very important there [28] and make b–τ unification viable in much larger range of $\tan\beta$ than generally believed (see also [23]).

One-loop diagrams with bottom squark-gluino and top squark-chargino loops make a contribution to the bottom-quark mass which is proportional to $\tan\beta$ [21, 22]. We recall that, to a good approximation, the one-loop correction to the bottom quark mass is given by the expression:

$$\frac{\Delta m_b}{m_b} \approx \frac{\tan\beta}{4\pi}\mu\left[\frac{8}{3}\alpha_s m_{\tilde g} I(m_{\tilde g}^2, M_{\tilde b_1}^2, M_{\tilde b_2}^2) + Y_t A_t I(\mu^2, M_{\tilde t_1}^2, M_{\tilde t_2}^2)\right] , \qquad (6)$$

where

$$I(a,b,c) = -\frac{ab\log(a/b) + bc\log(b/c) + ca\log(c/a)}{(a-b)(b-c)(c-a)}$$

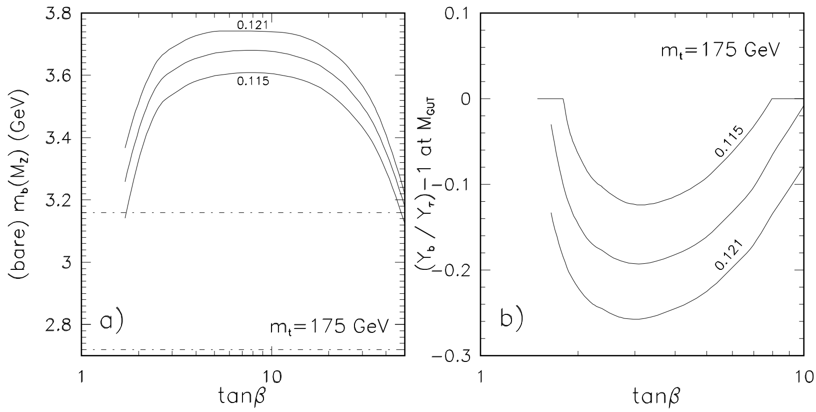

Fig. 3. a) The running mass $m_b(M_Z)$ obtained from strict b–τ Yukawa coupling unification at $M_{\rm GUT} = 2\times 10^{16}$ GeV for different values of $\alpha_s(M_Z)$, before inclusion of one-loop supersymmetric corrections. b) The minimal departure from $Y_b = Y_\tau$ at $M_{\rm GUT}$ measured by the ratio $Y_b/Y_\tau - 1$, which is necessary for obtaining the correct b mass in the minimal supergravity model with one-loop supersymmetric corrections included.

and the function $I(a,b,c)$ is always positive and approximately inversely proportional to its largest argument. This is the correction to the running $m_b(M_Z)$. It is clear from Fig. 3(a) that for b–τ unification in the intermediate $\tan\beta$ region we need a negative correction of order (15–20)% for $3 \lesssim \tan\beta \lesssim 20$, and about a 10% correction for $\tan\beta = 30$. According to Eq. (6), such corrections require $\mu < 0$.

We notice that, as expected from (6), b–τ unification is easier for $\tan\beta = 30$ than for $\tan\beta \approx 10$. In the latter case it requires $A_t \gtrsim 0$, in order to obtain an enhancement in (6) or at least to avoid any cancellation between the two terms in (6). This is a strong constraint on the parameter space. Since A_t is given by [19]:

$$A_t \approx (1-y)A_0 - \mathcal{O}(1-2)M_{1/2}\,, \tag{7}$$

where $y = Y_t/Y_t^{\rm FP}$ is the ratio of the top Yukawa coupling to its quasi-infrared fixed point value, b–τ unification requires large positive A_0 and not too large a $M_{\tilde g}$ (*i.e.*, $M_{1/2}$ for universal gaugino masses). In addition, the low-energy value of A_t is then always relatively small and this implies a stronger upper bound on M_h (for a similar conclusion, see [23]). We see in Fig. 3(b) that, for $\tan\beta \lesssim 10$, the possibility of exact b–τ unification evaporates quite quickly, with a non-unification window for $2 \lesssim \tan\beta \lesssim 8$–10, depending on the value of α_s. However, we also see that supersymmetric one-loop corrections are large enough to assure unification within 10% in almost the whole range of small and intermediate $\tan\beta$.

For $\tan\beta > 10$, the qualitative picture changes gradually. The overall factor of $\tan\beta$, on the one hand, and the need for smaller corrections, on the other hand, lead to the situation where a partial cancellation of the two terms in (6) is necessary, or both corrections must be suppressed by sufficiently heavy squark masses. Therefore, b–τ unification for $\tan\beta = 30$ typically requires a negative value of A_t, and is only marginally possible for positive A_t, for heavy enough squarks. A similar but more extreme situation occurs for very large $\tan\beta$ values. It is worth recalling that the second term in (6) is typically at most of order of (20–30)% of the first term [22], due to Eq. (7). Thus, cancellation of the two terms is limited, and for very large $\tan\beta$ the contribution of (6) must be anyway suppressed by requiring heavy squarks. This trend is visible in Fig. 4(a) already for $\tan\beta = 30$. The Higgs-boson mass is not constrained by b–τ unification, since A_t can be negative and large.

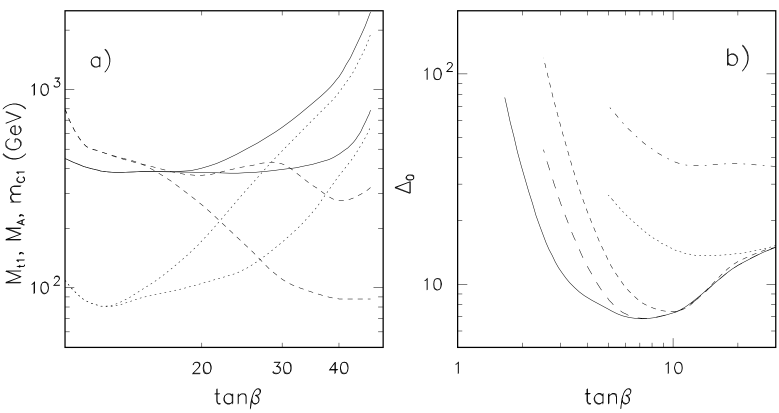

Fig. 4. (a) Lower limits on the lighter (dotted lines) and heavier (solid lines) stop and on the CP-odd Higgs boson A° (dashed lines) in the minimal supergravity scenario with b–τ Yukawa coupling unification, as functions of $\tan\beta$. Upper (lower) lines refer to the case with the $b \to s\gamma$ constraint imposed (not imposed). (b) Fine-tuning measures as functions of $\tan\beta$. Lower limits on the Higgs boson mass of 90 GeV (solid), 100 GeV (long-dashed), 105 GeV (dashed) 110 GeV (dotted) and 115 GeV (dot-dashed) have been assumed.

We turn our attention now to a deeper understanding of the $b \to s\gamma$ constraint and its interplay with b–τ unification. The first point we would like to make is that $b \to s\gamma$ decay is a rigid constraint in the minimal supergravity model, but is only an optional one for the general low-energy effective MSSM. Its inclusion depends on the strong assumption that the stop-chargino-strange quark mixing angle is the same as the CKM element V_{ts}. This is the case only if squark mass matrices are diagonal in the super-KM basis, which is realized, for instance, in the minimal supergravity model. However, for the right-handed up-squark sector such an assumption is not imposed upon us by FCNC processes [24]. Indeed, aligning the squark flavour basis with that of the quarks, the up-type squark right-handed flavour off-diagonal mass squared matrix elements $(m^2_{\tilde{U}})^{13}_{\rm RR}$ and $(m^2_{\tilde{U}})^{23}_{\rm RR}$ are unconstrained by other FCNC processes.

In the minimal supergravity model the dominant contributions to $b \to s\gamma$ decay come from the chargino-stop and charged Higgs-boson/top-quark loops. For intermediate and large $\tan\beta$, one can estimate these using the formulae of [25] in the approximation of no mixing between the gaugino and higgsinos, *i.e.*, for $M_W \ll \max(M_2, |\mu|)$. We get [26]

$$\mathcal{A}_W \approx \mathcal{A}_0^\gamma \frac{3}{2}\frac{m_t^2}{M_W^2} f^{(1)}\left(\frac{m_t^2}{M_W^2}\right), \tag{8}$$

$$\mathcal{A}_{H^+} \approx \mathcal{A}_0^\gamma \frac{1}{2}\frac{m_t^2}{M_{H^+}^2} f^{(2)}\left(\frac{m_t^2}{M_{H^+}^2}\right), \tag{9}$$

$$\begin{aligned}\mathcal{A}_C \approx -\mathcal{A}_0^\gamma \Bigg\{ & \left(\frac{M_W}{M_2}\right)^2 \left[\cos^2\theta_{\tilde{t}} f^{(1)}\left(\frac{M^2_{\tilde{t}_2}}{M_2^2}\right) + \sin^2\theta_{\tilde{t}} f^{(1)}\left(\frac{M^2_{\tilde{t}_1}}{M_2^2}\right)\right] \\ & - \left(\frac{m_t}{2\mu}\right)^2 \left[\sin^2\theta_{\tilde{t}} f^{(1)}\left(\frac{M^2_{\tilde{t}_2}}{\mu^2}\right) + \cos^2\theta_{\tilde{t}} f^{(1)}\left(\frac{M^2_{\tilde{t}_1}}{\mu^2}\right)\right] \\ & - \frac{\tan\beta}{2}\frac{m_t}{\mu}\frac{m_t A_t}{M^2_{\tilde{t}_2} - M^2_{\tilde{t}_1}} \left[f^{(3)}\left(\frac{M^2_{\tilde{t}_2}}{\mu^2}\right) - f^{(3)}\left(\frac{M^2_{\tilde{t}_1}}{\mu^2}\right)\right]\Bigg\},\end{aligned} \tag{10}$$

where $\tilde{t}_1(\tilde{t}_2)$ denotes the lighter (heavier) stop,

$$\cos^2\theta_{\tilde{t}} = \frac{1}{2}\left(1+\sqrt{1-a^2}\right), \quad a \equiv \frac{2m_t A_t}{M^2_{\tilde{t}_2} - M^2_{\tilde{t}_1}}, \quad \mathcal{A}_0^\gamma \equiv G_F\sqrt{\alpha/(2\pi)^3}\; V_{ts}^\star V_{tb} \tag{11}$$

and the functions $f^{(k)}(x)$ given in [25] are negative. The contribution $\mathcal{A}_C$ is effectively proportional to the stop mixing parameter A_t, and the sign of $\mathcal{A}_C$ relative to $\mathcal{A}_W$ and $\mathcal{A}_{H^+}$ is negative for $A_t\mu < 0$.

We can discuss now the interplay of the b–τ unification and $b \to s\gamma$ constraints. The chargino-loop contribution (10) has to be small or positive, since the Standard Model contribution and the charged Higgs-boson exchange (both negative) leave little room for additional constructive contributions. Hence, one generically needs $A_t\mu < 0$. Since $\mu < 0$ for b–τ unification, both constraints together require $A_t > 0$. This is in line with our earlier results for the proper correction to the b mass for $\tan\beta \lesssim 10$, [4] but typically in conflict with such corrections for larger values of $\tan\beta$. In the latter case, both constraints can be satisfied only at the expense of heavy squarks (to suppress a positive A_t correction to the b-quark mass or a negative A_t correction to $b \to s\gamma$) and a heavy pseudoscalar A°, as seen in Fig. 4(a).

3. Naturalness and fine tuning

The main theoretical motivation for the appearance of sparticles at the accessible energies is in order to alleviate the fine tuning required to maintain the electroweak hierarchy, and sparticles become less effective in this task the heavier their masses. This is a widely accepted qualitative argument and a common sense expectation is that sparticles are lighter than, for instance, 100 TeV or may be even 10 TeV. On the other hand it is very difficult, if not impossible, to quantify this argument in a convincing and fully objective way. One particular measure often used in such discussions is $\Delta_{a_i} = \frac{a_i}{M_Z}\frac{\partial M_Z}{\partial a_i}$, where a_i's are input parameters of the MSSM, but other measures can be as well considered. In any case it is unclear what is quantitatively acceptable as the fine tuning and what is not. Moreover, the fine tuning can be discussed only in concrete models for the soft supersymmetry breaking terms, and any conclusion refers to the particular model under consideration. It is clear that, in spite of being uncontested qualitative notion, the naturalness and fine-tuning criteria cannot be used for setting any absolute upper bounds on the sparticle spectra. Instead, however, the idea which is promoted in Ref. [28] is that, any sensible measure of the amount of fine-tuning becomes an interesting criterion for at least comparing the relative naturalness of various theoretical models for the soft mass terms in the MSSM lagrangian, that are consistent with the stronger and stronger experimental constraints.

In the first place, it has been shown [27, 28, 30, 31] that, comparing the situation before and after LEP, the fine-tuning price in the minimal supergravity model (that is, with universal soft terms at the GUT scale) has significantly increased, largely as a result of the unsuccessful Higgs boson search. Comparing different values of $\tan\beta$, we find that in this model naturalness favours an intermediate range. Fine tuning increases for small values because of the lower limit on the Higgs boson mass and increases for large values because of the difficulty in assuring correct electroweak symmetry breaking. This is shown in Fig. 4(b). In the intermediate $\tan\beta$ region the

[4] This does not constrain the parameter space more than b–τ unification itself. Note also that, if we do not insist on b–τ unification, the $b \to s\gamma$ constraint is easily satisfied since $\mu > 0$ is possible.

fine tuning price still remains moderate but would strongly increase with higher limits on the chargino mass.

In view of the above results and in the spirit of using the fine-tuning considerations as a message for theory rather than experiment, it is interesting to discuss the departures from the minimal supergravity model that would ease the present (particularly in the low and large $\tan\beta$ region, where the price is high) and possibly future fine-tuning problem. The first step in this direction is to identify the parameters that are really relevant for the Higgs potential. It has been emphasized in Ref. [29] that scalar masses that enter into the Higgs potential at one-loop level are only the soft Higgs mass parameters and the third generation sfermion masses. Thus, breaking the universality between the first two generation sfermion masses and the remaining scalars, with the former much heavier, does not cost much of the fine-tuning, but unfortunately it is not useful now (the present bounds on the first two generation sfermions are still low enough not to be the source of the fine tuning in the minimal model)[5].

Furthermore, the attention has recently been drawn [32] to the fact that, at one-loop, the Higgs potential depends on the gluino mass but not on the wino and bino masses. This is interesting as it means that in models with $M_3 \neq M_{1,2}$ the fine tuning price is in fact weakly dependent on the limits on the chargino mass (but not totally independent because of the constraints on the μ parameter, which is present in the Higgs potential at the tree level) and the Tevatron direct bound on the gluino mass is weaker than the indirect bound obtained from LEP2 assuming gaugino mass universality. Allowing for $M_3 < M_{1,2}$[6], the after-LEP fine-tuning price is reduced mainly in the intermediate $\tan\beta$ region, where it was still quite modest even in the universal model, but this possibility may be particularly interesting for intermediate $\tan\beta$ region when the lower limit on the chargino mass is pushed higher.

After identifying the parameters which are relevant for the Higgs potential at one-loop level, that is the Higgs, stop and gluino soft masses and the μ parameter, it is clear that the fine tuning price does not increase much even if other superparticles are substantially heavier. The question remains what sort of pattern for soft terms would reduce the fine tuning caused by the present and future limits on the relevant parameters. One obvious possibility are non-universal soft Higgs boson masses [34], which can significantly reduce the fine-tuning price [27,32] and particularly in the large $\tan\beta$ region [28]. Another alternative is that a model with independent mass parameters is in fact simply inadequate to address the naturalness problem.

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[5] This possibility has been discussed as a way to ameliorate the FCNC problem in the MSSM. [29,33] One should stress, however, that it cannot *solve* the FCNC problem.

[6] As discussed earlier, such a scenario may also be interesting for the gauge coupling unification [17].

A relevant example is related to the so-called μ-problem. One hopes that in the ultimate theory the μ parameter will be calculated in terms of the soft supersymmetry breaking masses. A model which correlates μ and the gluino mass may have dramatic effects on the fine tuning price [28], as they should not be any more considered as independent parameters. Thus the fine tuning price will depend on the actual solution to the μ-problem.

4. Conclusions

Direct searches for superpartners have so far come up empty-handed. Nevertheless, we get from experiment a handful of important information on supersymmetric models, which makes the whole concept of low energy supersymmetry much more constrained than a decade ago. Simplest ideas like the minimal supergravity model with universal but otherwise independent mass terms may soon become inadequate.

For a more complete list of earlier references see, for instance, [9–11, 35].

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