PRECISION TESTS OF THE STANDARD MODEL*

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The status of the Standard Model is reviewed on the basis of precise calculations for the electroweak observables associated with the W and Z bosons together with the recent experimental high precision data. A brief discussion of the status of precision observables in the MSSM is also included.

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1. Introduction

The generation of high-precision experiments imposes stringent tests on the standard model of particle physics. The e^+e^- colliders LEP and the SLC have collected an enormous amount of electroweak precision data on Z and W bosons [1,2]. The W boson properties have also been determined at the $p\bar{p}$ collider Tevatron with a constant increase in accuracy [2,3], and the top quark mass has been measured [4] to 173.8 ± 5.0 GeV, a value that agrees with the mass range obtained indirectly, through the radiative corrections. Nowadays, with the top mass as an additional precise experimental data point one can fully exploit the virtual sensitivity to the Higgs mass.

The experimental sensitivity in the electroweak observables, at the level of the quantum effects, requires the highest standards on the theoretical side as well. A sizeable amount of work has contributed, over the recent years, to a steadily rising improvement of the standard model predictions, pinning down the theoretical uncertainties to the level required for the current interpretation of the precision data. The availability of both highly accurate measurements and theoretical predictions, at the level of 0.1% precision and better, provides tests of the quantum structure of the standard model, thereby probing the still untested scalar sector, and simultaneously accesses alternative scenarios such as the supersymmetric extension of the standard model.

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2. Theoretical basis for precision tests

2.1. Calculation of radiative corrections

The possibility of performing precision tests is based on the formulation of the standard model as a renormalizable quantum field theory preserving its predictive power beyond tree-level calculations. With the experimental accuracy being sensitive to the loop-induced quantum effects, also the Higgs sector of the standard model is being probed. The higher-order terms induce the sensitivity of electroweak observables to the top and Higgs mass m_t, M_H and to the strong coupling constant α_s.

Before predictions can be made from the theory, a set of independent parameters has to be taken from experiment. For practical calculations the physical input quantities α, G_μ, M_Z, m_f, M_H, α_s are commonly used to fix the free parameters of the standard model. Differences between various schemes are formally of higher order than the one under consideration. The study of the scheme dependence of the perturbative results, after improvement by resummation of the leading terms, allows us to estimate the missing higher-order contributions (see *e.g.* [5] for a comprehensive study).

Related to charge and mass renormalization, there occur two sizeable effects in the electroweak loops that deserve a special discussion:

(i) Charge renormalization and light fermion contribution:

Charge renormalization introduces the concept of electric charge for real photons ($q^2 = 0$) to be used for the calculation of observables at the electroweak scale set by M_Z. Hence the difference

$$\mathrm{Re}\,\hat{\Pi}^\gamma(M_Z^2)) = \mathrm{Re}\,\Pi^\gamma(M_Z^2) - \Pi^\gamma(0) \tag{1}$$

of the photon vacuum polarization is a basic entry in the predictions for electroweak precision observables. The contribution from the leptons and the 5 light hadronic flavors

$$\begin{aligned} \Delta\alpha &= \Delta\alpha_{\mathrm{lept}} + \Delta\alpha_{\mathrm{had}} \\ &= -\,\mathrm{Re}\,\hat{\Pi}^\gamma_{\mathrm{lept}}(M_Z^2) - \mathrm{Re}\,\hat{\Pi}^\gamma_{\mathrm{had}}(M_Z^2) \end{aligned} \tag{2}$$

corresponds to a QED-induced shift in the electromagnetic fine structure constant

$$\alpha \to \alpha(1 + \Delta\alpha)\,, \tag{3}$$

which can be resummed in accordance with the renormalization group. The result can be interpreted as an effective fine structure constant at the Z mass scale:

$$\alpha(M_Z^2) = \frac{\alpha}{1 - \Delta\alpha}\,. \tag{4}$$

$\Delta\alpha$ is an input of crucial importance because of its universality and of its remarkable size of $\sim 6\%$. The leptonic content can be directly evaluated in terms of the known lepton masses; the 2-loop correction has been known already for a long time [6], and also the 3-loop contribution is now available [7], yielding altogether

$$\Delta\alpha_{\rm lept} \;=\; 314.97687\cdot 10^{-4}\,. \tag{5}$$

For the light hadronic part, perturbative QCD is not applicable. Instead, the 5-flavour contribution to $\hat{\Pi}^{\gamma}_{\rm had}$ can be derived with the help of a dispersion relation

$$\Delta\alpha_{\rm had} = -\,\frac{\alpha}{3\pi}\,M_Z^2\,{\rm Re}\int\limits_{4m_\pi^2}^{\infty} {\rm d}s'\,\frac{R^{\gamma}(s')}{s'(s'-M_Z^2-i\varepsilon)} \tag{6}$$

with

$$R^{\gamma}(s) = \frac{\sigma(e^+e^- \to \gamma^* \to {\rm hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

as an experimental input quantity in the problematic low energy range.

Integrating by means of the trapezoidal rule (averaging data in bins) over e^+e^- data for the energy range below 40 GeV and applying perturbative QCD for the high-energy region above, the expression (6) yields the value [8, 9]

$$\Delta\alpha_{\rm had} = -0.0280 \pm 0.0007\,, \tag{7}$$

which agrees with another independent analysis [10] with a different error treatment. Because of the lack of precision in the experimental data a large uncertainty is associated with the value of $\Delta\alpha_{\rm had}$, which propagates into the theoretical error of the predictions of electroweak precision observables. Including additional data from τ-decays [11] yields about the same result with a slightly improved uncertainty. Recently, other attempts have been made to increase the precision of $\Delta\alpha$ [12–15] by "theory-driven" analyses of the dispersion integral (6). The common basis is the application of perturbative QCD down to the energy scale given by the τ mass for the calculation of the quantity $R^{\gamma}(s)$ outside the resonances. Those calculations were made possible by the recent availability of the quark-mass-dependent $O(\alpha_s^2)$ QCD corrections [16] for the cross section down to close to the thresholds for b and c production. [A first step in this direction was done in [17] in the massless approximation.] The results obtained for $\Delta\alpha_{\rm had}$ are very similar:

$$\begin{array}{ll} 0.02763 \pm 0.0016 & \text{Ref. [12]}\,, \\ 0.02777 \pm 0.0017 & \text{Ref. [13]}\,. \end{array}$$

In [15] the $\overline{MS}$ quantity $\hat{\alpha}(M_Z)$ has been derived with the help of an unsubtracted dispersion relation in the $\overline{MS}$-scheme, yielding a comparable error.

Although the error in the QCD-based evaluation of $\Delta\alpha_{\rm had}$ is considerably reduced, it should be kept in mind that the conservative estimate in Eq. (7) is independent of theoretical assumptions on QCD at lower energies and thus less sensitive to potential systematic effects not under consideration now [19].

(ii) Mixing angle renormalization and the ρ-parameter:

The ρ-parameter, originally defined as the ratio of the neutral to the charged current strength in neutrino scattering [20], is unity in the standard model at the tree level, but gets a deviation $\Delta\rho$ from 1 by radiative corrections. The relation between the gauge boson masses and the electroweak mixing angle is modified in higher orders according to

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} + \frac{M_W^2}{M_Z^2}\Delta\rho + \cdots\,, \tag{8}$$

where the main contribution to the ρ-parameter is from the (t,b) doublet [21], at the present level calculated as follows:

$$\Delta\rho = 3x_t\cdot[1 + x_t\,\rho^{(2)} + \delta\rho_{\rm QCD}] \quad \text{with} \quad x_t = \frac{G_\mu m_t^2}{8\pi^2\sqrt{2}}\,. \tag{9}$$

The electroweak 2-loop part [22,23] is described by the function $\rho^{(2)}(M_H/m_t)$, and $\delta\rho_{\rm QCD}$ is the QCD correction to the leading $G_\mu m_t^2$ term [24, 25]

$$\delta\rho_{\rm QCD} = -\,2.86\,\frac{\alpha_s(m_t)}{\pi} \,+\, 14.6\left(\frac{\alpha_s(m_t)}{\pi}\right)^2 \tag{10}$$

with the on-shell top mass m_t and 6 flavors. This reduces the scale dependence of ρ significantly and hence is an important entry to decrease the theoretical uncertainty of the standard model predictions for precision observables.

2.2. The vector boson mass correlation

The interdependence between the gauge boson masses is established through the accurately measured muon lifetime or, equivalently, the Fermi coupling constant G_μ. Beyond the well-known 1-loop QED corrections [26], the 2-loop QED corrections in the Fermi model have been calculated quite recently [27], yielding the expression (the error in the 2-loop term is from the hadronic uncertainty)

$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left(1 - \frac{8m_e^2}{m_\mu^2}\right)\left[1 + 1.810\,\frac{\alpha}{\pi} + (6.701\,\pm 0.002)\left(\frac{\alpha}{\pi}\right)^2\right]\,. \tag{11}$$

leading to the value [27]

$$G_\mu = (1.16637 \pm 0.00001) 10^{-5}\ \mathrm{GeV}^{-2}\,. \tag{12}$$

In the standard model, G_μ can be calculated in terms of the basic standard model parameters,yielding the correlation between the masses M_W, M_Z of the vector bosons; in 1-loop order it is given by [28]:

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha}{2 s_W^2 M_W^2}[1 + \Delta r(\alpha, M_W, M_Z, M_H, m_t)]\,. \tag{13}$$

with $\quad s_W^2 = 1 - M_W^2/M_Z^2$.

The presence of large terms in Δr requires the consideration of effects higher than 1-loop. The modification of Eq. (13) according to

$$1 + \Delta r \;\rightarrow\; \frac{1}{(1-\Delta\alpha)\cdot(1+\frac{c_W^2}{s_W^2}\Delta\rho)\;-\;(\Delta r)_{\mathrm{rem}}} \equiv \frac{1}{1-\Delta r} \tag{14}$$

accommodates the following higher-order terms (Δr in the denominator is an effective correction including higher orders):

(i) the leading log resummation [29] of $\Delta\alpha$: $1 + \Delta\alpha \;\rightarrow\; (1-\Delta\alpha)^{-1}$;

(ii) the resummation of the leading m_t^2 contribution [30] in terms of $\Delta\rho$ in Eq. (9). Beyond the QCD higher-order contributions through the ρ-parameter, the complete $O(\alpha\alpha_s)$ corrections to the self energies are available [31, 32]. All these higher-order terms contribute with the same positive sign to Δr. Non-leading QCD corrections to Δr of are also available [33].

(iii) With the quantity $(\Delta r)_{\mathrm{rem}}$ in the denominator, non-leading higher-order terms containing mass singularities of the type $\alpha^2 \log(M_Z/m_f)$ from light fermions are incorporated [34].

(iv) The subleading $G_\mu^2 m_t^2 M_Z^2$ contribution of the electroweak 2-loop order [35] in an expansion in terms of the top mass. This subleading term turned out to be sizeable, about as large as the formally leading term of $O(m_t^4)$ via the ρ-parameter. In view of the present and future experimental accuracy it constitutes a non-negligible shift in the W mass.

Meanwhile exact results have been derived for the Higgs-dependence of the fermionic 2-loop corrections in Δr [36], and comparisons were performed with those obtained via the top mass expansion [37]. Differences in the values of M_W of several MeV (up to 8 MeV) are observed when M_H is varied over the range from 65 GeV to 1 TeV.

Pure fermion-loop contributions (n fermion loops at n-loop order) have also been investigated [37, 38]. In the on-shell scheme, explicit results have been worked out up to 4-loop order, which allows an investigation of the validity of the resummation (14) for the non-leading 2-loop and higher-order terms. It was found that numerically the resummation (14) works remarkably well, within 2 MeV in M_W.

2.3. Z boson widths and asymmetries

With M_Z used as a precise input parameter, together with α and G_μ, the predictions for the width, partial widths and asymmetries can conveniently be calculated in terms of effective neutral current coupling constants for the various fermions: (see *e.g.* [39]):

$$\begin{aligned} J_\nu^{\mathrm{NC}} &= \left(\sqrt{2}G_\mu M_Z^2\right)^{1/2} \left(g_V^f\,\gamma_\nu - g_A^f\,\gamma_\nu\gamma_5\right) \\ &= \left(\sqrt{2}G_\mu M_Z^2\rho_f\right)^{1/2} \left((I_3^f - 2Q_f s_f^2)\gamma_\nu - I_3^f\gamma_\nu\gamma_5\right). \end{aligned} \tag{15}$$

The subleading 2-loop corrections $\sim G_\mu^2 m_t^2 M_Z^2$ for the leptonic mixing angle [35] s_ℓ^2 have also been obtained in the meantime, as well as for ρ_ℓ [40].

Meanwhile exact results have been derived for the Higgs-dependence of the fermionic 2-loop corrections in s_ℓ^2 [37, 38], and comparisons were performed with those obtained via the top mass expansion [37]. Differences in the values of s_ℓ^2 can amount to $0.8 \cdot 10^{-4}$ when M_H is varied over the range from 100 GeV to 1 TeV.

The effective mixing angles are of particular interest, since they determine the on-resonance asymmetries via the combinations

$$A_f = \frac{2g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}\,, \tag{16}$$

namely

$$A_{\mathrm{FB}} = \frac{3}{4}\,A_e A_f, \quad A_\tau^{\mathrm{pol}} = A_\tau, \quad A_{\mathrm{LR}} = A_e\,. \tag{17}$$

Measurements of the asymmetries hence are measurements of the ratios

$$g_V^f/g_A^f = 1 - 2Q_f s_f^2 \tag{18}$$

or the effective mixing angles, respectively.

The total Z width Γ_Z can be calculated essentially as the sum over the fermionic partial decay widths. Expressed in terms of the effective coupling constants, they read up to second order in the fermion masses:

$$\Gamma_f = \Gamma_0 \left[(g_V^f)^2 + (g_A^f)^2 \left(1 - \frac{6m_f^2}{M_Z^2} \right) \right] \left(1 + Q_f^2 \, \frac{3\alpha}{4\pi} \right) + \Delta\Gamma_{\rm QCD}^f \qquad (19)$$

with

$$\Gamma_0 \; = \; N_C^f \, \frac{\sqrt{2} G_\mu M_Z^3}{12\pi}, \qquad N_C^f = 1 \text{ (leptons)} \quad = 3 \text{ (quarks)}.$$

The QCD corrections, also for the massive case, are calculated up to third order in α_s, except for the m_b-dependent singlet terms, which are known to $O(\alpha_s^2)$. For a review of the QCD corrections to the Z width, see [41]. Also the mixed $O(\alpha\alpha_s)$ 2-loop contributions have been completed by now [42–46].

2.4. Accuracy of the calculations

For a discussion of the theoretical reliability of the standard model predictions, one has to consider the various sources contributing to their uncertainties:

Parametric uncertainties result from the limited precision in the experimental values of the input parameters, essentially $\alpha_s = 0.119 \pm 0.002$ [45], $m_t = 173.8 \pm 5.0$ GeV [4], $m_b = 4.7 \pm 0.2$ GeV, and the hadronic vacuum polarization as discussed in section 2.1. The conservative estimate of the error in Eq. (7) leads to $\delta M_W = 13$ MeV in the W-mass prediction, and $\delta \sin^2\theta = 0.00023$ common to all of the mixing angles.

The uncertainties from the QCD contributions can essentially be traced back to those in the top quark loops in the vector boson self-energies. The knowledge of the $O(\alpha_s^2)$ corrections to the ρ-parameter and Δr yields a significant reduction; they are small, although not negligible (*e.g.* $\sim 3\times 10^{-5}$ in s_ℓ^2).

The size of unknown higher-order contributions can be estimated by different treatments of non-leading terms of higher order in the implementation of radiative corrections in electroweak observables ('options') and by investigations of the scheme dependence. Explicit comparisons between the results of 5 different computer codes based on on-shell and $\overline{MS}$ calculations for the Z-resonance observables are documented in the "Electroweak Working Group Report" [39] in Ref. [5]. The inclusion of the non-leading 2-loop corrections $\sim G_\mu^2 m_t^2 M_Z^2$ reduce the uncertainty in M_W below 10 MeV and in s_ℓ^2 below 10^{-4}, typically to $\pm 4 \times 10^{-5}$ [47].

3. Status of Standard Model precision tests

We now confront the standard model predictions for the discussed set of precision observables with the most recent sample of experimental data [1,2]. In Table I the standard model predictions for Z-pole observables and the W mass are put together for the best fit input data set, given in (22). The experimental results on the Z observables are from LEP and the SLC, the W mass is from combined LEP and $p\bar{p}$ data. The leptonic mixing angle determined via $A_{\rm LR}$ by the SLD experiment [48] and the s_ℓ^2 average from LEP:

$$s_e^2(A_{\rm LR}) = 0.23109 \pm 0.00029, \quad s_\ell^2(\text{LEP}) = 0.23189 \pm 0.00024\,, \tag{20}$$

have come closer to each other in their central value; owing to their smaller errors, however, they still differ by 2.8 standard deviations.

TABLE I

Precision observables: experimental results from combined LEP and SLD data for Z observables and combined $p\bar{p}$ and LEP data for M_W, together with the standard model predictions for the best fit, *i.e.* for the parameter values given in Eq. (22). ρ_ℓ and s_ℓ^2 are derived from the experimental values of $g_{V,A}^\ell$ according to Eq. (15), averaged under the assumption of lepton universality.

Observable	Exp.	SM best fit
M_Z (GeV)	91.1867 ± 0.0019	91.1865
Γ_Z (GeV)	2.4939 ± 0.0024	2.4956
$\sigma_0^{\rm had}$ (nb)	41.491 ± 0.058	41.476
$R_{\rm had}$	20.765 ± 0.026	20.745
R_b	0.21656 ± 0.00074	0.2159
R_c	0.1732 ± 0.0048	0.1722
$A_{\rm FB}^\ell$	0.01683 ± 0.00096	0.0162
$A_{\rm FB}^b$	0.0990 ± 0.0021	0.1029
$A_{\rm FB}^c$	0.0709 ± 0.0044	0.0735
A_b	0.867 ± 0.035	0.9347
A_c	0.647 ± 0.040	0.6678
ρ_ℓ	1.0041 ± 0.0012	1.0051
s_ℓ^2	0.23157 ± 0.00018	0.23155
M_W (GeV)	80.39 ± 0.06	80.372

Table I contains the combined LEP/SLD value. ρ_ℓ and s_ℓ^2 are the leptonic neutral current couplings in Eq. (15), derived from partial widths and asymmetries under the assumption of lepton universality.

Note that the experimental value for ρ_ℓ points at the presence of genuine electroweak corrections by 3.5 standard deviations. In s_ℓ^2 the presence of purely bosonic radiative corrections is clearly established when the experimental result is compared with a theoretical value containing only the fermion loop corrections, an observation that has been persisting already for several years [49]. The deviation from the standard model prediction in the quantity R_b has been reduced below one standard deviation by now. Other small deviations are observed in the asymmetries: the purely leptonic $A_{\rm FB}$ is slightly higher than the standard model predictions, and $A_{\rm FB}$ for b quarks is lower. Whereas the leptonic $A_{\rm FB}$ favours a very light Higgs boson, the b quark asymmetry needs a heavy Higgs.

The effective mixing angle is an observable most sensitive to the mass M_H of the Higgs boson. Since a light Higgs boson corresponds to a low value of s_ℓ^2, the strongest upper bound on M_H is from $A_{\rm LR}$ at the SLC [48]. The inclusion of the two-loop electroweak corrections $\sim m_t^2$ from [35] yields a sizeable positive contribution to s_ℓ^2. The inclusion of this term hence strengthens the upper bound on M_H.

The W mass prediction in Table I is obtained from Eq. (13) (including the higher-order terms) from M_Z, G_μ, α and M_H, m_t. The present experimental value for the W mass from the combined LEP 2, UA2, CDF and D0 results is in best agreement with the standard model prediction.

The quantity s_W^2 resp. the ratio M_W/M_Z can indirectly be measured in deep-inelastic neutrino–nucleon scattering. The average from the experiments CCFR, CDHS and CHARM with the recent NUTEV result [50],

$$s_W^2 = 1 - M_W^2/M_Z^2 = 0.2255 \pm 0.0021 \tag{21}$$

for $m_t = 175$ GeV and $M_H = 150$ GeV, corresponds to $M_W = 80.25 \pm 0.11$ GeV and is hence fully consistent with the direct vector boson mass measurements and with the standard theory.

Global fits:

The FORTRAN codes ZFITTER [51] and TOPAZ0 [52] have been updated by incorporating all the recent precision calculation results that were discussed in the previous section. Comparisons have shown good agreement between the predictions from the two independent programs [47,53]. Global fits of the standard model parameters to the electroweak precision data done by the Electroweak Working Group [1] are based on these recent versions. Including m_t and M_W from the direct measurements in the experimental data set, together with s_W^2 from neutrino scattering, the standard model parameters for the best fit result are:

$$\begin{aligned} m_t &= 171.1 \pm 4.9\,\text{GeV}\,, \\ M_H &= 76^{+85}_{-47}\,\text{GeV}\,, \\ \alpha_s &= 0.119 \pm 0.003\,. \end{aligned} \tag{22}$$

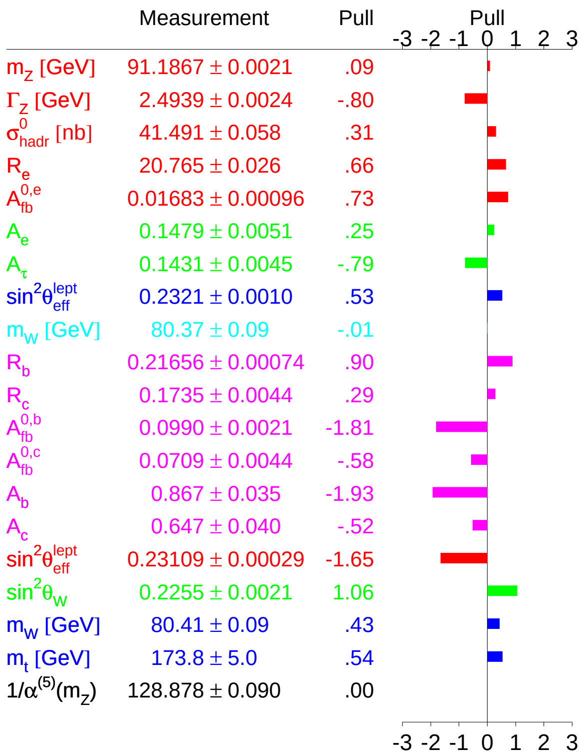

Fig. 1. Experimental results and pulls from a standard model fit (from Ref. [1,2]). pull = obs(exp)-obs(SM)/(exp.error).

The upper limit to the Higgs mass at the 95% C.L. is $M_H < 262$ GeV, where the theoretical uncertainty is included. Thereby the hadronic vacuum polarization in Eq. (7) has been used (solid line in figure 2). With the theory-driven result on $\Delta\alpha_{\rm had}$ of Ref. [12] one obtains [1] $M_H = 92^{+64}_{-41}$ (dashed line). The 1σ upper bound on M_H is influenced only marginally. The reason is that simultaneously with the error reduction the central value of M_H is shifted upwards (see Fig. 2). Another recent analysis [54] (for earlier studies see [55, 56]) based on the data set of summer 1998 yields a Higgs mass $M_H = 107^{+67}_{-45}$ GeV. About one half of the difference with (22) can be ascribed to the use of $\alpha(M_Z)$ of Ref. [15], which is very close to the value in Ref. [12, 13]; the residual shift might be interpreted as due to different renormalization schemes and different treatments of α_s.

With an overall $\chi^2/\text{d.o.f.} = 15/15$ the quality of the fit is remarkably high. As can be seen from figure 1, the deviation of the individual quantities from the standard model best-fit values are below 2 standard deviations.

The remaining theoretical uncertainty associated with the Higgs mass bounds should be taken very seriously. The effect of the inclusion of the next-to-leading term in the m_t-expansion of the electroweak 2-loop corrections in the precision observables has shown to be sizeable, at the upper margin of the estimate given in [39]. It is thus not guaranteed that the subsequent subleading terms in the m_t-expansion are indeed smaller in size. Also the variation of the M_H-dependence at different stages of the calculation, as discussed in sections 2.2 and 2.3, indicate the necessity of more complete results at two-loop order. Having in mind also the variation of the Higgs mass bounds under the fluctuations of the experimental data [2], the limits for M_H derived from the analysis of electroweak data in the frame of the standard model still carry a noticeable uncertainty. Nevertheless, as a central message, it can be concluded that the indirect determination of the Higgs mass range has shown that the Higgs is light, with its mass well below the non-perturbative regime.

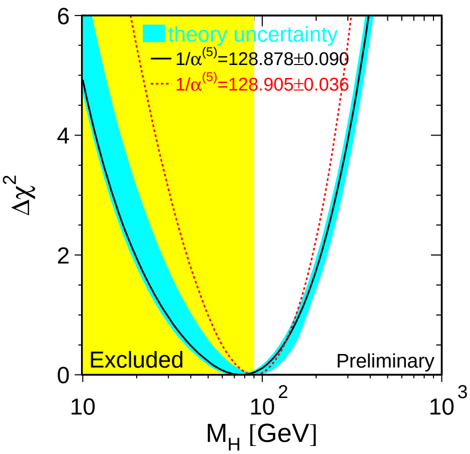

Fig. 2. Higgs mass dependence of χ^2 in the global fit to precision data (from Ref. [1, 2]). The shaded band displays the error from the theoretical uncertainties obtained from various options in the codes ZFITTER and TOPAZ0.

4. Implication for the Higgs sector

The minimal model with a single scalar doublet is the simplest way to implement the electroweak symmetry breaking. The experimental result that the ρ-parameter is very close to unity is a natural feature of models with doublets and singlets. In the standard model, the mass M_H of the Higgs boson appears as the only additional parameter beyond the vector boson and fermion masses. M_H cannot be predicted but has to be taken from experiment. The present lower limit (95% C.L.) from the search at LEP [57] is 90 GeV; indirect determinations of M_H from precision data have already been discussed in Section 3.

There are also theoretical constraints on the Higgs mass from vacuum stability and absence of a Landau pole [58, 59], and from lattice calculations [60]. Explicit perturbative calculations of the decay width for $H \to W^+W^-, ZZ$ in the large-M_H limit in 2-loop order [61] have shown that the 2-loop contribution exceeds the 1-loop term in size (same sign) for $M_H > 930$ GeV. This result is confirmed by the calculation of the next-to-leading order correction in the $1/N$ expansion, where the Higgs sector is treated as an $O(N)$ symmetric σ-model [62]. A similar increase of the 2-loop perturbative contribution with M_H is observed for the fermionic decay [63] $H \to f\bar{f}$, but with opposite sign leading to a cancellation of the one-loop correction for $M_H \simeq 1100$ GeV. The requirement of applicability of perturbation theory therefore puts a stringent upper limit on the Higgs mass. The indirect Higgs mass bounds obtained from the precision analysis show, however, that the Higgs boson is well below the mass range where the Higgs sector becomes non-perturbative.

The behaviour of the quartic Higgs self-coupling λ, as a function of a rising energy scale μ, follows from the renormalization group equation with the β-function dominated by λ and the top quark Yukawa coupling g_t contributions:

$$\beta_\lambda = 24\,\lambda^2 + 12\,\lambda\, g_t^2 - 6\, g_t^4 + \cdots \tag{23}$$

In order to avoid unphysical negative quartic couplings from the negative top quark contribution, a lower bound on the Higgs mass is derived. The

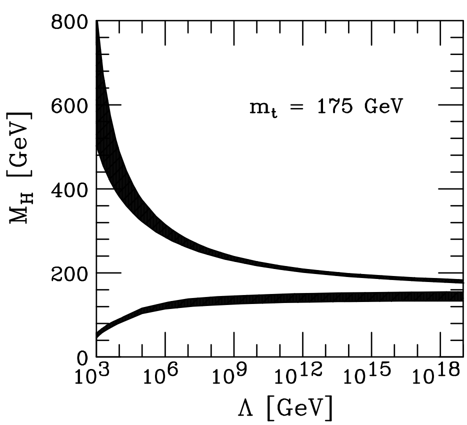

Fig. 3. Theoretical limits on the Higgs boson mass from the absence of a Landau pole and from vacuum stability (from Ref. [59]).

requirement that the Higgs coupling remains finite and positive up to a scale Λ yields constraints on the Higgs mass M_H, which have been evaluated at the 2-loop level [58, 59]. These bounds on M_H are shown in figure 3 as a function of the cut-off scale Λ up to which the standard Higgs sector can be extrapolated, for $m_t = 175$ GeV and $\alpha_s(M_Z) = 0.118$. The allowed region is the area between the lower and the upper curves. The bands indicate the theoretical uncertainties associated with the solution of the renormalization group equations [59]. It is interesting to note that the indirect determination of the Higgs mass range from electroweak precision data via radiative corrections is compatible with a value of M_H where Λ can extend up to the Planck scale.

5. Precision tests of the MSSM

Among the extensions of the standard model, the minimal supersymmetric standard model (MSSM) is the theoretically favoured scenario as the most predictive framework beyond the standard model. A definite prediction of the MSSM is the existence of a light Higgs boson with mass below ~ 135 GeV [64]. The detection of a light Higgs boson at LEP could be a significant hint for supersymmetry.

The structure of the MSSM as a renormalizable quantum field theory allows a similarly complete calculation of the electroweak precision observables as in the standard model in terms of one Higgs mass (usually taken as the CP-odd 'pseudoscalar' mass M_A) and $\tan\beta = v_2/v_1$, together with the set of SUSY soft-breaking parameters fixing the chargino/neutralino and scalar fermion sectors. It has been known for quite some time [65] that light non-standard Higgs bosons as well as light stop and charginos predict larger values for the ratio R_b [66, 68]. Complete 1-loop calculations are available for Δr [67] and for the Z boson observables [68].

A possible mass splitting between $\tilde{b}_L$ and $\tilde{t}_L$ yields a contribution to the ρ-parameter of the same sign as the standard top term. As a universal loop contribution, it enters the quantity Δr and the Z boson couplings and is thus significantly constrained by the data on M_W and the leptonic widths. Recently the 2-loop α_s corrections have been computed [69], which can amount to 30% of the 1-loop $\Delta\rho_{\tilde{b}\tilde{t}}$.

Figure 4 displays the range of predictions for M_W in the minimal model and in the MSSM. It is thereby assumed that no direct discovery has been made at LEP 2. As can be seen, precise determinations of M_W and m_t can become decisive for the separation between the models.

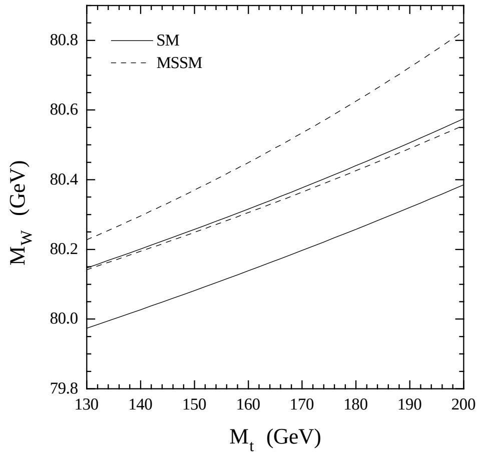

Fig. 4. The W mass range in the standard model (——) and in the MSSM (- - -). Bounds are from the non-observation of Higgs bosons and SUSY particles at LEP2.

As the standard model, the MSSM yields a good description of the precision data. A global fit [56] to all electroweak precision data, including the top mass measurement, shows that the χ^2 of the fit is slightly better than in the standard model; but, owing to the larger numbers of parameters, the probability is about the same as for the standard model (figure 5).

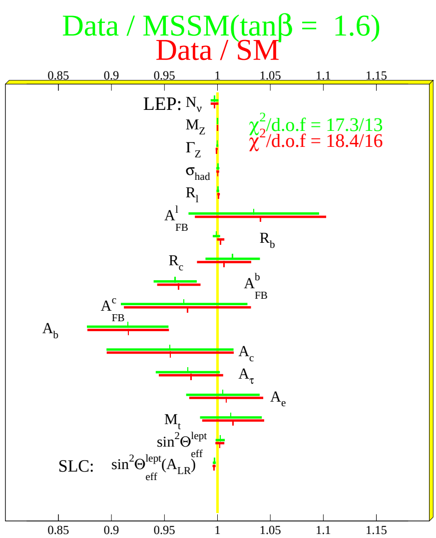

Fig. 5. Best fits in the SM and in the MSSM, normalized to the data. Error bars are those from data. (Updated from Ref. [56].)

The virtual presence of SUSY particles in the precision observables can be exploited also in the other way of constraining the allowed range of the MSSM parameters. Since the quality of the standard model description can be achieved only for those parameter sets where the standard model with a light Higgs boson is approximated, deviations from this scenario result in a rapid decrease of the fit quality. An analysis of the precision data in this spirit can be found in Ref. [70].

6. Conclusions

The experimental data for tests of the standard model have achieved an impressive accuracy. In the meantime, many theoretical contributions have become available to improve and stabilize the standard model predictions and to reach a theoretical accuracy clearly better than 0.1%.

The overall agreement between theory and experiment for the entire set of the precision observables is remarkable and instructively confirms the validity of the standard model. Fluctuations of data around the predictions are within two standard deviations, with no compelling evidence for deviations. Direct and indirect determinations of the top mass are compatible, and a light Higgs boson is clearly favoured by the analysis of precision data in the standard model context, which is far below the mass range where the standard Higgs sector becomes non-perturbative.

As a consequence of the high quality performance of the standard model, any kind of New Physics can only provoke small effects, at most of the size that is set by the radiative corrections. The MSSM, mainly theoretically advocated, is competitive to the standard model in describing the data with about the same quality in global fits. Since the MSSM predicts the existence of a light Higgs boson, the detection of a Higgs at LEP could be an indication of supersymmetry. The standard model can also accommodate such a light Higgs, but with the consequence that its validity cannot be extrapolated to energies much higher than the TeV scale.

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