

$F_2^\gamma$  AT LOW  $Q^2$  AND  $\sigma_{\gamma\gamma}$  AT HIGH ENERGIES\*

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The parametrisation of the photon structure function in the low  $Q^2$  region is formulated. It includes the VMD contribution and the QCD improved parton model component suitably extrapolated to the low  $Q^2$  region. The parametrisation describes reasonably well existing experimental data on  $\sigma_{\gamma\gamma}$  for real photons and the low  $Q^2$  data on  $\sigma_{\gamma^*\gamma}$ . Predictions for  $\sigma_{\gamma\gamma}$  and for  $\sigma_{\gamma^*\gamma}$  for energies which may be accessible in future linear colliders are also given.

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The structure function of the photon is described at large scales  $Q^2$  by the QCD improved parton model [1-3]. It is expected however that in the low  $Q^2$  region the Vector Meson Dominance (VMD) contribution [4] may also become important. Here, as usual,  $Q^2 = -q^2$  where  $q$  denotes the four momentum of the virtual photon probing the real photon with four momentum  $p$ . The CM energy squared  $W^2$  of the  $\gamma^*\gamma$  system is  $W^2 = (q + p)^2$ .

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In this talk we wish to present the representation of the photon structure function which includes both the VMD contribution together with the QCD improved parton model term suitably extrapolated to the low  $Q^2$  region. This representation of the photon structure function is based on the extension of similar representation of the nucleon structure function to the case of the photon “target” [5,6]. Possible parametrization of the photon structure function which extends to the low  $Q^2$  region has also been discussed in Ref. [7]. There do also exist several microscopic models describing the energy dependence of the total  $\gamma\gamma$  cross-section [8].

Our representation of the structure function  $F_2(W^2, Q^2)$  is based on the following decomposition:

$$F_2(W^2, Q^2) = F_2^{\text{VMD}}(W^2, Q^2) + F_2^{\text{partons}}(W^2, Q^2), \quad (1)$$

where in what follows we shall consider the structure function of the photon, *i.e.*  $F_2 \equiv F_2^\gamma$ . The terms  $F_2^{\text{VMD}}(W^2, Q^2)$  and  $F_2^{\text{partons}}(W^2, Q^2)$  denote the VMD and partonic contributions respectively. The VMD part is given by the following formula:

$$F_2^{\text{VMD}}(W^2, Q^2) = \frac{Q^2}{4\pi} \sum_{v=\rho,\omega,\phi} \frac{M_v^4 \sigma_{\gamma v}(W^2)}{\gamma_v^2 (Q^2 + M_v^2)^2}, \quad (2)$$

where  $M_v$  is the mass of the vector meson  $v$  and  $\sigma_{\gamma v}(W^2)$  denotes the  $\gamma v$  total cross-section. The parameters  $\gamma_v$  can be determined from the leptonic widths  $\Gamma_{e^+e^-}^v$  [4,5]:

$$\frac{\gamma_v^2}{\pi} = \frac{\alpha^2 M_v}{3\Gamma_{e^+e^-}^v}. \quad (3)$$

The partonic contribution is expressed in terms of the structure function  $F_2^{\text{QCD}}$  obtained from the QCD improved parton model analysis of the photon structure function in the large  $Q^2$  region [6]:

$$F_2^{\text{partons}}(W^2, Q^2) = \frac{Q^2}{Q^2 + Q_0^2} F_2^{\text{QCD}}(\bar{x}, Q^2 + Q_0^2), \quad (4)$$

where

$$\bar{x} = x \left( 1 + \frac{Q_0^2}{Q^2} \right) = \frac{Q^2 + Q_0^2}{W^2 + Q^2} \quad (5)$$

with  $x$  denoting the Bjorken variable, *i.e.*  $x = Q^2/(2pq)$ . The parameter  $Q_0^2$  should have its magnitude greater than the mass squared of the heaviest vector meson included in the VMD part and its value will be taken to be the same as in Ref. [6], *i.e.*  $Q_0^2 = 1.2 \text{ GeV}^2$ .

The  $\gamma^*\gamma$  total cross-section  $\sigma_{\gamma^*\gamma}$  is related in the following way to the photon structure function:

$$\sigma_{\gamma^*\gamma}(W^2, Q^2) = \frac{4\pi^2\alpha}{Q^2} F_2(W^2, Q^2). \tag{6}$$

After taking in equation (6) the limit  $Q^2 = 0$  (for fixed  $W$ ) we obtain the total cross-section  $\sigma_{\gamma\gamma}(W^2)$  corresponding to the interaction of two real photons. The representation (1) and equations (2) and (4) give the following expression for this cross-section:

$$\sigma_{\gamma\gamma}(W^2) = \alpha\pi \sum_{v=\rho,\omega,\phi} \frac{\sigma_{\gamma v}(W^2)}{\gamma_v^2} + \frac{4\pi^2\alpha}{Q_0^2} F_2^{\text{QCD}}(Q_0^2/W^2, Q_0^2). \tag{7}$$

In the large  $Q^2$  region the structure function given by Eq. (1) becomes equal to the QCD improved parton model contribution  $F_2^{\text{QCD}}(x, Q^2)$ . The VMD component gives the power correction term which vanishes as  $(1/Q^2)$  for large  $Q^2$ . The modifications of the QCD parton model contribution (*i.e.* replacement of the parameter  $x$  by  $\bar{x}$  defined by equation (5), the shift of the scale  $Q^2 \rightarrow Q^2 + Q_0^2$  and the factor  $Q^2/(Q^2 + Q_0^2)$  instead of 1) are also negligible at large  $Q^2$  and introduce the power corrections which vanish as  $1/Q^2$ .

In the quantitative analysis of  $\sigma_{\gamma^*\gamma}$  and of  $\sigma_{\gamma\gamma}(W^2)$  we have taken the  $F_2^{\text{QCD}}$  from the LO analysis presented in Ref. [9].

The VMD part was estimated using the following assumptions:

1. The numerical values of the couplings  $\gamma_v^2$  are the same as those used in Ref. [5]. They were estimated from relation (3) which gives the following values :

$$\frac{\gamma_\rho^2}{\pi} = 1.98 \quad \frac{\gamma_\omega^2}{\pi} = 21.07 \quad \frac{\gamma_\phi^2}{\pi} = 13.83. \tag{8}$$

2. The cross-sections  $\sigma_{\gamma v}$  are represented as the sum of the Reggeon and Pomeron contributions:

$$\sigma_{\gamma v}(W^2) = R_{\gamma v}(W^2) + P_{\gamma v}(W^2), \tag{9}$$

where

$$R_{\gamma v}(W^2) = a_{\gamma v}^R \left( \frac{W^2}{W_0^2} \right)^{\lambda_R}, \tag{10}$$

$$P_{\gamma v}(W^2) = a_{\gamma v}^P \left( \frac{W^2}{W_0^2} \right)^{\lambda_P} \tag{11}$$

with

$$\lambda_R = -0.4525, \quad \lambda_P = 0.0808 \quad (12)$$

and  $W_0^2 = 1\text{GeV}^2$  [10].

3. The pomeron couplings  $a_{\gamma v}^P$  are related to the corresponding couplings  $a_{\gamma p}^P$  controlling the pomeron contributions to the total  $\gamma p$  cross-sections assuming the additive quark model and reducing the total cross-sections for the interaction of strange quarks by a factor equal 2. This gives:

$$\begin{aligned} a_{\gamma\rho}^P &= a_{\gamma\omega}^P = \frac{2}{3}a_{\gamma p}^P, \\ a_{\gamma\phi}^P &= \frac{1}{2}a_{\gamma\rho}^P. \end{aligned} \quad (13)$$

4. The reggeon couplings  $a_{\gamma v}^R$  are estimated assuming additive quark model and duality (*i.e.* dominance of planar quark diagrams). We also assume that the quark couplings to a photon are proportional to the quark charge with the flavour independent proportionality factor. This gives:

$$\begin{aligned} a_{\gamma\rho}^R &= a_{\gamma\omega}^R = \frac{5}{9}a_{\gamma p}^R, \\ a_{\gamma\phi}^R &= 0. \end{aligned} \quad (14)$$

5. The couplings  $a_{\gamma p}^P$  and  $a_{\gamma p}^R$  are taken from the fit discussed in Ref. [10] which gave:

$$a_{\gamma p}^R = 0.129\text{mb}, \quad a_{\gamma p}^P = 0.0677\text{mb}. \quad (15)$$

In Fig.1 we compare our predictions with the data on  $\sigma_{\gamma\gamma}(W^2)$  [11–14]. We show experimental points corresponding to the “low” energy region ( $W < 10$  GeV) [11–13] and the recent preliminary high energy data obtained by the L3 and OPAL collaborations at LEP [14]. We can see that the representation (7) for the total  $\gamma\gamma$  cross-section describes the data reasonably well. It should be stressed that our prediction is essentially parameter free. The magnitude of the cross-section is dominated by the VMD component yet the partonic part is also non-negligible. The latter term is in particular responsible for generating steeper increase of the total cross-section with increasing  $W$  than that embodied in the VMD part which is described by the soft pomeron

contribution. The decrease of the total cross-section with increasing energy in the low  $W$  region is controlled by the Reggeon component of the VMD part (see Eqs. (9), (11) and (12)) and by the valence part of the partonic contribution.

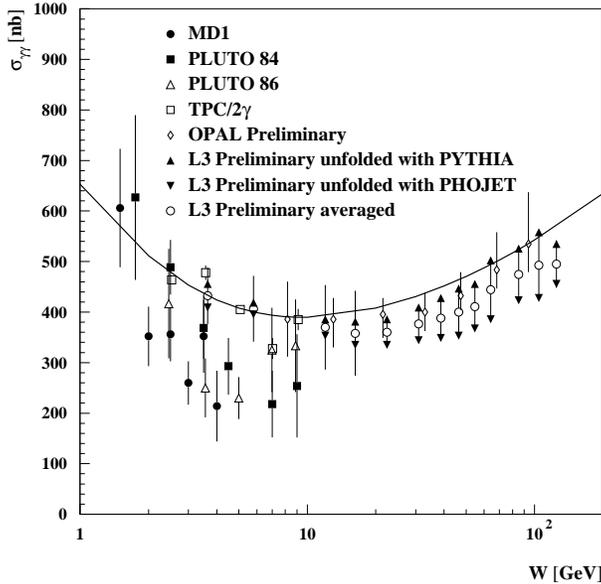


Fig. 1. Comparison of our predictions for  $\sigma_{\gamma\gamma}(W^2)$  based on equation (7) with experimental results [11–14].

In Fig. 2 we show predictions for the total  $\gamma\gamma$  cross-section as the function of the total CM energy  $W$  in the wide energy range which includes the energies that might be accessible in future linear colliders. We also show in this Figure the decomposition of  $\sigma_{\gamma\gamma}(W^2)$  into its VMD and partonic components. We see that at very high energies these two terms exhibit different energy dependence. The VMD part is described by the soft pomeron contribution which gives the  $W^{2\lambda}$  behaviour with  $\lambda = 0.0808$  (12). The partonic component increases faster with energy since its energy dependence reflects increase of  $F_2^{\text{QCD}}(\bar{x}, Q_0^2)$  with decreasing  $\bar{x}$  generated by the QCD evolution [9].

This increase is stronger than that implied by the soft pomeron exchange. As the result the total  $\gamma\gamma$  cross-section, which is the sum of the VMD and partonic components does also exhibit stronger increase with the increasing energy than that of the VMD component. It is however milder than the increase generated by the partonic component alone, at least for  $W < 10^3$  GeV. This follows from the fact that in this energy range the magnitude

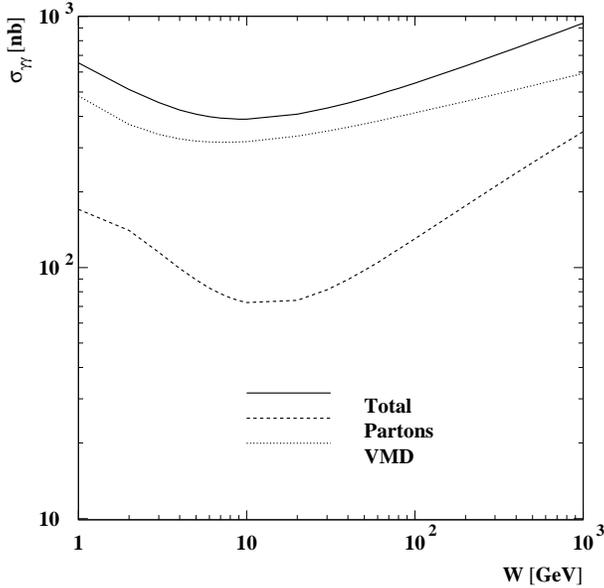


Fig. 2. The total  $\gamma\gamma$  cross-sections  $\sigma_{\gamma\gamma}(W^2)$  (continuous line) calculated from equation (7) and plotted as the function of the CM energy  $W$  in the wide energy range which includes the region that will be accessible in future linear colliders. We also show separately the VMD (dotted line) and partonic (dashed line) components of  $\sigma_{\gamma\gamma}(W^2)$ . They correspond to the first and the second term on the r.h.s. of equation (7), respectively.

of the cross-section is still dominated by its VMD component. We found that for sufficiently high energies  $W$  the total  $\gamma\gamma$  cross-section  $\sigma_{\gamma\gamma}(W^2)$  described by Eq. (7) can be parametrized by the effective power law dependence  $\sigma_{\gamma\gamma}(W^2) \sim (W^2)^{\lambda_{\text{eff}}}$  with  $\lambda_{\text{eff}}$  slowly increasing with energy within the range  $\lambda_{\text{eff}} \sim 0.1 - 0.12$  for  $30 \text{ GeV} < W < 10^3 \text{ GeV}$ .

In Fig. 3 we compare our predictions for  $\sigma_{\gamma^*\gamma}(W^2, Q^2)$  based on equations (1), (2), (4) and (6) with the experimental data in the low  $Q^2$  region [12]. We can see that in this case the model is also able to give a good description of the data.

Finally in Fig. 4 we show our results for  $\sigma_{\gamma^*\gamma}(W^2, Q^2)$  plotted as the function of  $Q^2$  for different values of the total CM energy  $W$ . We notice that for low values  $Q^2$  the cross-section does only weakly depend upon  $Q^2$ . In the large  $Q^2$  region it follows the  $1/Q^2$  scaling behaviour modulated by the logarithmic scaling violations implied by perturbative QCD.

To sum up we have presented an extension of the representation developed in Refs. [5,6] for the nucleon structure function  $F_2$  for arbitrary values

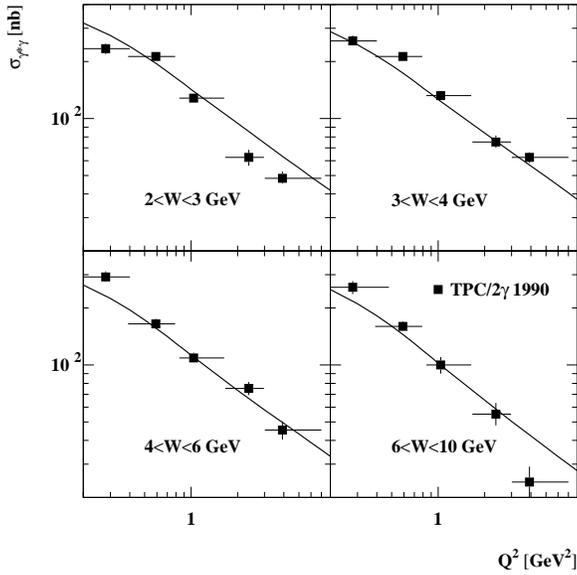


Fig. 3. Comparison of predictions for  $\sigma_{\gamma^*\gamma}(W^2, Q^2)$  in the low  $Q^2$  region based on equations (1), (2), (4) and (6) with experimental results [12].

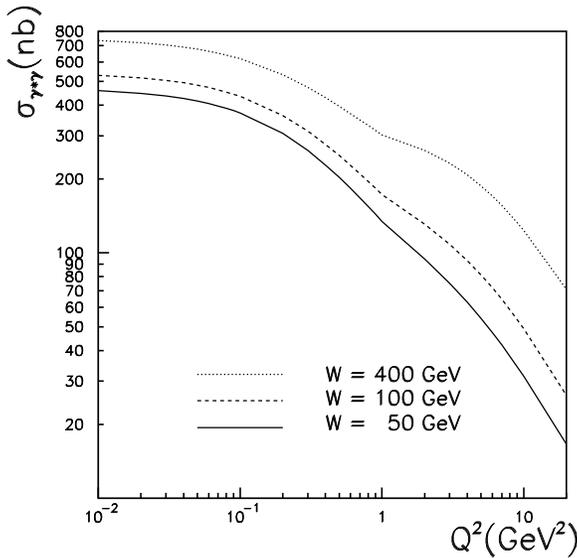


Fig. 4. Plot of  $\sigma_{\gamma^*\gamma}(W^2, Q^2)$  as the function of  $Q^2$  for different values of the CM energy  $W$ .

of  $Q^2$ , onto the structure function of the real photon. This representation includes both the VMD contribution and the QCD improved parton model component suitably extrapolated to the region of low  $Q^2$ . We showed that it is fairly successful in describing the experimental data on  $\sigma_{\gamma\gamma}(W^2)$  and on  $\sigma_{\gamma^*\gamma}(W^2, Q^2)$  at low  $Q^2$ . We also showed that one can naturally explain the fact that the increase of the total  $\gamma\gamma$  cross-section with increasing CM energy  $W$  is stronger than that implied by soft pomeron exchange. The calculated total  $\gamma\gamma$  cross-section was found to exhibit approximate power-law increase with increasing energy  $W$ , *i.e.*  $\sigma_{\gamma\gamma}(W^2) \sim (W^2)^{\lambda_{\text{eff}}}$  with  $\lambda_{\text{eff}}$  slowly increasing with energy within the range  $\lambda_{\text{eff}} \sim 0.1 - 0.12$  for  $30 \text{ GeV} < W < 10^3 \text{ GeV}$ .

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## REFERENCES

- [1] H. Abramowicz, M. Krawczyk, K. Charchuła, A. Levy, U. Maor, *Int. J. Mod. Phys. A* **8**, 1005 (1993) and references therein.
- [2] Report of the Working Group on  $\gamma\gamma$  Physics, P. Aurenche, G.A. Schuler (conveners), in: G. Altarelli, T. Sjöstrand, P. Zwirner (Eds.), Proceedings of the Workshop on Physics at LEP2, CERN yellow preprint 96-01, hep-ph/9601317.
- [3] M. Krawczyk, *Acta Phys. Pol.* **B28**, 2659 (1997); M. Krawczyk, A. Zembrzusi, M. Staszal, DESY-98-013, hep-ph/9806291.
- [4] T.H. Bauer *et al.*, *Rev. Mod. Phys.* **50**, 261 (1978) and references therein.
- [5] J. Kwieciński, B. Badełek, *Z. Phys.* **C43**, 251 (1989).
- [6] B. Badełek, J. Kwieciński, *Phys. Lett.* **B295**, 263 (1992).
- [7] E. Gotsman, A. Levy, U. Maor, *Z. Phys.* **C40**, 117 (1988).
- [8] A. Corsetti, R.M. Godbole, G. Pancheri, *Phys. Lett.* **B435**, 441 (1998) and references therein.
- [9] M. Glück, E.Reya, A.Vogt, *Phys. Rev.* **D46**, 1973 (1992).
- [10] A. Donnachie, P.V. Landshoff, *Phys. Lett.* **B296**, 227 (1992).
- [11] Ch. Berger *et al.*, PLUTO collaboration, *Phys. Lett.* **B149**, 421 (1984); *Z. Phys.* **C26**, 353 (1984).

- [12] H. Aihara *et al.*, TPC/2 $\gamma$  Collaboration, *Phys. Rev.* **D41**, 2667 (1990).
- [13] S.E. Baru *et al.*, MD-1 Collaboration, *Z. Phys.* **C53**, 219 (1992).
- [14] Stefan Soldner-Rembold, FREIBURG-EHEP-98-06, Talk given at 29th International Conference on High-Energy Physics (ICHEP 98), Vancouver, Canada, 23-29 Jul 1998, hep-ex/9810011; M. Erdmann, Plenary talk given at 29th International Conference on High-Energy Physics (ICHEP 98), Vancouver, Canada, 23-29 July 1998, DESY-98-209B, hep-ex/9812030, and references therein.