SUPERSYMMETRY SEARCHES AT e^+e^- LINEAR COLLIDERS*

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The physics potential of discovering and exploring supersymmetry at future e^+e^- linear colliders is reviewed. Such colliders are planned to start to operate at a center-of-mass energy of 500 GeV to 800 GeV, with a final energy of about 2 TeV expected. They are ideal facilities for the discovery of supersymmetric particles. High precision measurements of their properties and interactions will help to uncover the mechanism of supersymmetry breaking and will allow for tests of grand unification scenarios.

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1. Introduction

The Standard Model is exceedingly successful in describing leptons, quarks and their interactions. Nevertheless, the Standard Model (SM) is not considered as the ultimate theory since neither the fundamental parameters, masses and couplings, nor the symmetry pattern are predicted. These elements are merely built into the model. Likewise, the spontaneous electroweak symmetry breaking is simply parametrized by a single Higgs doublet field.

Even though many aspects of the Standard Model are experimentally supported to a very high accuracy [1], the embedding of the model into a more general framework is to be expected. The argument is closely related to the mechanism of the electroweak symmetry breaking. If the Higgs boson is light, the Standard Model can naturally be embedded in a grand unified theory. The large energy gap between the low electroweak scale and the high grand unification scale can be stabilized by supersymmetry. Supersymmetry [2] actually provides the link between the experimentally explored

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interactions at electroweak energy scales and physics at scales close to the Planck scale where the gravity is important. If the Higgs boson is very heavy, or if no fundamental Higgs boson exists, new strong interactions between the massive electroweak gauge bosons are predicted by unitarity at the TeV scale. With possibly many more new layers of matter before the Planck scale is reached, no direct link between electroweak and Planck scales in such a scenario is expected at present. In either case, the next generation of accelerators which will operate in the TeV energy range can uncover the structure of physics beyond the Standard Model.

Despite the lack of direct experimental evidence[1] for supersymmetry (SUSY), the concept of symmetry between bosons and fermions has so many attractive features that the supersymmetric extension of the Standard Model is widely considered as a most natural scenario. SUSY ensures the cancellation of quadratically divergent quantum corrections from scalar and fermion loops and thus the Higgs boson mass can be kept in the desired range of order 10^2 GeV, which is preferred by precision tests of the SM. The prediction of the renormalized electroweak mixing angle $\sin^2\theta_W$, based on the spectrum of the Minimal Supersymmetric Standard Model (MSSM), is in striking agreement with the measured value. Last but not least, supersymmetry provides the opportunity to generate the electroweak symmetry breaking radiatively.

In the next section the silent features of supersymmetric models are briefly summarized. We will stress the importance of determining experimentally all SUSY parameters in a model independent way. For this purpose the e^+e^- linear colliders (LC) [4] are indispensable tools. It is illustrated in the next chapter where some of the recently developed strategies to "measure" SUSY parameters in the gaugino and sfermion sectors are discussed. For a discussion of the SUSY Higgs sector we refer to [5].

2. Low-energy MSSM

Supersymmetry predicts the quarks and leptons to have scalar partners, called squarks and sleptons, the gauge bosons to have fermionic partners, called gauginos. In the MSSM [6] two Higgs doublets with opposite hypercharges, and with their superpartners — higgsinos – are required to give masses to the up and down type fermions and to ensure anomaly cancellation. Thus the particle content of the MSSM is given by

$(l_a,\nu_a)_L$	l^c_{aL}	$(u_a,d_a)_L$	u^c_{aL}	d^c_{aL}	γ	$W^\pm$	Z^0	g_i	H_1	H_2
$(\tilde{l}_a,\tilde{\nu}_a)_L$	$\tilde{l}^c_{aL}$	$(\tilde{u}_a,\tilde{d}_a)_L$	$\tilde{u}^c_{aL}$	$\tilde{d}^c_{aL}$	$\tilde{\gamma}$	$\tilde{W}^\pm$	$\tilde{Z}^0$	$\tilde{g}_i$	$\tilde{H}_1$	$\tilde{H}_2$

[1] The status of low-energy supersummetry is discussed by Pokorski [3].

where the first row lists the (left-handed) fermion fields of one generation (a=1–3), the gauge fields (for gluons i=1–8) and two Higgs doublets, and the second row — their superpartners. The higgsinos and electroweak gauginos mix; the mass eigenstates are called charginos and neutralinos for electrically charged and neutral states, respectively. The MSSM is defined by the superpotential

$$W = Y^e_{ab}\hat{L}_a\hat{H}_1\hat{E}^c_b + Y^d_{ab}\hat{Q}_a\hat{H}_1\hat{D}^c_b + Y^u_{ab}\hat{Q}_a\hat{H}_2\hat{U}^c_b - \mu\hat{H}_1\hat{H}_2\,, \qquad (1)$$

where standard notation is used for the superfields of left-handed doublets of (s)leptons ($\hat{L}_a$) and (s)quarks ($\hat{Q}_a$), the right-handed singlets of charged (s)leptons ($\hat{E}_a$), up- ($\hat{U}_a$) and down-type (s)quarks ($\hat{D}_a$), and for the Higgs doublet superfields which couple to the down ($\hat{H}_1$) and up quarks ($\hat{H}_2$); the indices a, b denote the generations and a summation is understood, Y^f_{ab} are Yukawa couplings and μ is the Higgs mixing mass parameter. The W respects a discrete multiplicative symmetry under R-parity, defined as $R_\mathrm{p} = (-1)^{3B+L+2S}$, where B, L and S denote the baryon and lepton number, and the spin of the particle. The R_p conservation implies that the lightest supersymmetric particle (LSP — preferably the lightest neutralino) is stable and superpartners can be produced only in pairs in collisions and decays of particles.

If realized in Nature, supersymmetry must be broken at low energy since no superpartners of ordinary particles have been observed so far. It is technically achieved [7] by introducing the soft–supersymmetry breaking

(i) gaugino mass terms for bino $\tilde{B}$, wino $\tilde{W}^i$ [i=1-3] and gluino $\tilde{g}^i$ [i=1-8]

$$\tfrac{1}{2}M_1\,\overline{\tilde{B}}\,\tilde{B} \;+\; \tfrac{1}{2}M_2\,\overline{\tilde{W}}^i\,\tilde{W}^i \;+\; \tfrac{1}{2}M_3\,\overline{\tilde{g}}^i\,\tilde{g}^i\,, \qquad (2)$$

(ii) trilinear couplings (generation indices are understood)

$$A^u H_2\tilde{Q}\tilde{u}^c + A^d H_1\tilde{Q}\tilde{d}^c + A^l H_1\tilde{L}\tilde{l}^c - \mu B H_1 H_2\,, \qquad (3)$$

(iii) and squark and slepton mass terms

$$m^2_{\tilde{Q}}[\tilde{u}^*_\mathrm{L}\tilde{u}_\mathrm{L} + \tilde{d}^*_\mathrm{L}\tilde{d}_\mathrm{L}] + m^2_{\tilde{u}}\tilde{u}^*_\mathrm{R}\tilde{u}_\mathrm{R} + m^2_{\tilde{d}}\tilde{d}^*_\mathrm{R}\tilde{d}_\mathrm{R} + \;\cdots\,, \qquad (4)$$

where the ellipses stand for the soft mass terms for sleptons and Higgs bosons.

The more than doubling the spectrum of states in the MSSM together with the necessity of including the SUSY breaking terms gives rise to a large

number of parameters. Even with the R-parity conserving and CP-invariant SUSY sector, which we will assume in what follows, in total more than 100 new parameters are introduced! This number of parameters can be reduced by additional physical assumptions. The most radical reduction is achieved in the so called mSUGRA, by embedding the low–energy supersymmetric theory into a grand unified (SUSY-GUT) framework by requiring at the GUT scale M_G:

(*i*) the unification of the U(1), SU(2) and SU(3) coupling constants

$$\alpha_3(M_G) = \alpha_2(M_G) = \alpha_1(M_G) = \alpha_G, \tag{5}$$

(*ii*) a common gaugino mass $m_{1/2}$. The gaugino masses M_i at the electroweak scale are then related through renormalization group equations (RGEs) to the gauge couplings

$$M_i = \frac{\alpha_i(M_Z)}{\alpha_G} m_{1/2}, \tag{6}$$

(*iii*) a universal trilinear coupling A_G

$$A_G = A^u(M_G) = A^d(M_G) = A^l(M_G), \tag{7}$$

(*iv*) a universal scalar mass m_0

$$m_0 \;=\; m_{\tilde{Q}} = m_{\tilde{u}} = m_{\tilde{d}} = \cdots, \tag{8}$$

(*v*) radiative breaking of the electroweak symmetry.

The last requirement allows to solve for B and μ (to within a sign) once the values of the GUT parameters $m_{1/2}$, m_0, A_G as well as the ratio of the vacuum expectation values of the fields H_2^0 and H_1^0, $\tan\beta = v_2/v_1$, are fixed. As a result, the mSUGRA is fully specified by $m_{1/2}$, m_0, A_G, $\tan\beta$ and sign(μ) — the couplings, masses and mixings at the electroweak scale are determined by the RGEs [8].

From the experimental point of view, however, all low-energy parameters should be measured independently of any theoretical assumptions. Therefore the experimental program to search for and explore SUSY at present and future colliders should include the following points:

(a) discover supersymmetric particles and measure their quantum numbers to prove that they are *the* superpartners of standard particles,

(b) determine the low-energy Lagrangian parameters,

(c) verify the relations among them in order to distinguish between various SUSY models.

If SUSY is at work it will be a matter of days for the LC to discover the kinematically accessible supersymmetric particles. Once they are discovered, the priority will be to measure the low-energy SUSY parameters independently of theoretical prejudices and then check whether the correlations among parameters, if any, support a given theoretical framework, like SUSY-GUT relations. A clear strategy is needed to deal with so many a prori arbitrary parameters. One should realize, that the low-energy parameters are of two distinct categories. The first one includes all the gauge and Yukawa couplings and the Higgs mixing mass parameter μ. They are related by exact supersymmetry which is crucial for the cancellation of quadratic divergencies. For example, at tree-level the qqZ, $\tilde{q}\tilde{q}Z$ gauge and $q\tilde{q}\tilde{Z}$ Yukawa couplings have to be equal. The relations among these parameters (with calculable radiative corrections) have to be confirmed experimentally; if not — the supersymmetry is excluded. The second category encompasses all soft supersymmetry breaking parameters: Higgs, gaugino and sfermion masses and mixings, and trilinear couplings. They are soft in the sense that they do not reintroduce dangerous quadratic divergencies. Each one should be measured independently by experiment to shed light on the mechanism of supersymmetry breaking.

Particularly in this respect (points (b) and (c) above) the e^+e^- linear colliders are invaluable. An intense activity during last decade in Europe, the USA and Japan on physics at a linear e^+e^- collider [9] has convincingly demonstrated the advantages and benefits of such a machine and its complementarity to the Large Hadron Collider (LHC). Many studies have shown that the LHC can cover a mass range for SUSY particles up to ~ 2 TeV, in particular for squarks and gluinos [10]. The problem however is that many different sparticles will be accessed at once with the heavier ones cascading into the lighter which will in turn cascade further leading to a complicated picture. Simulations for the extraction of parameters have been attempted for the LHC [10] and demonstrated that some of them can be extracted with a good precision. However it must be stressed that these checks were done with the assumption of an underlying model, like mSUGRA and it has not been demonstrated so far that the same can be achieved in a model independent way.

From the practical point of view it is very important that the energy of the e^+e^- machine can be optimized so that only very few thresholds are crossed at a time. Another important feature is the availability of beam polarization as well as a possibility of running in e^-e^- [11] or in $e\gamma$ and

$\gamma\gamma$ [12] modes. Making judicious choices of these features, the confusing mixing of many final states, unavoidable at the LHC, with the cascade decays might be avoided and analyses restricted to a specific subset of processes performed. The measurements that can be performed in the Higgs sector are discussed in the talk by Zerwas [5]. Here I will discuss some methods of extracting SUSY parameters from the gaugino (chargino/neutralino) and sfermion sectors. In contrast to many earlier analyses [13], we will not elaborate on global fits but rather we will discuss attempts at "measuring" the fundamental parameters. Such attempts generically involve two steps:

A: from the observed quantities: cross sections, asymmetries *etc.*
$\Longrightarrow$ determine the physical parameters: the masses, mixings and couplings of sparticles

B: from the physical parameters
$\Longrightarrow$ extract the Lagrangian parameters: M_i, μ, $\tan\beta$, A^u, $m_{\tilde{Q}}$ *etc.*

Each step can suffer from both experimental problems and theoretical ambiguities. Concentrating first on the theoretical ones, recently these two steps have been fully realized for the chargino sector [14] and the work on exploiting the neutralinos is in progress [15]. Similar strategies have been developed for sleptons and squarks [16,17]. An alternative approach for the step B, based only on the masses of some of the charginos and neutralinos, can be found in [18].

3. Determining the Lagrangian parameters

3.1. Charginos: extracting $\tan\beta$, M_2 *and* μ

The spin–1/2 superpartners of the W boson and charged Higgs boson, $\tilde{W}^\pm$ and $\tilde{H}^\pm$, mix to form chargino mass eigenstates $\tilde{\chi}^\pm_{1,2}$. Their masses $m_{\tilde{\chi}^\pm_{1,2}}$ and the mixing angles $\phi_{\rm L}, \phi_{\rm R}$ are determined by the elements of the chargino mass matrix in the $(\tilde{W}^-, \tilde{H}^-)$ basis [6]

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2} m_W c_\beta \\ \sqrt{2} m_W s_\beta & \mu \end{pmatrix} \tag{9}$$

which is given in terms of fundamental parameters: M_2, μ, and $\tan\beta = v_2/v_1$; $s_\beta = \sin\beta$, $c_\beta = \cos\beta$. As outlined above, we will discuss first how to determine the chargino masses and mixing angles [step A] and then the procedure of extracting M_2, μ, and $\tan\beta = v_2/v_1$ [step B].

Charginos are produced in e^+e^- collisions, either in diagonal or in mixed pairs

$$e^+e^- \;\to\; \tilde{\chi}^+_i \; \tilde{\chi}^-_j \; .$$

With the second chargino $\tilde{\chi}_2^\pm$ expected to be significantly heavier than the first one, at LEP2 or even in the first phase of e^+e^- linear colliders, the chargino $\tilde{\chi}_1^\pm$ may be, for some time, the only chargino state that can be studied experimentally in detail. Therefore, we concentrate on the diagonal pair production of the lightest chargino $\tilde{\chi}_1^+\tilde{\chi}_1^-$ in e^+e^- collisions. Next, assuming an upgrade in energy, we consider additional informations available from $\tilde{\chi}_1^\pm\ \tilde{\chi}_2^\mp$ and $\tilde{\chi}_2^+\ \tilde{\chi}_2^-$ production processes.

Two different matrices acting on the left- and right-chiral $(\tilde{W}, \tilde{H})$ states are needed to diagonalize the asymmetric mass matrix (9). The two (positive) eigenvalues are given by

$$m^2_{\tilde{\chi}^\pm_{1,2}} = \tfrac{1}{2}\left[M_2^2 + \mu^2 + 2m_W^2 \mp \Delta\right] , \tag{10}$$

where

$$\Delta = \left[(M_2^2 + \mu^2 + 2m_W^2)^2 - 4(M_2\mu - m_W^2 \sin 2\beta)^2\right]^{1/2} . \tag{11}$$

The left- and right-chiral components of the mass eigenstate $\tilde{\chi}_1^-$ are related to the wino and higgsino components in the following way,

$$\begin{aligned} \tilde{\chi}^-_{1L} &= \tilde{W}^-_{\rm L} \cos\phi_{\rm L} + \tilde{H}^-_{1L} \sin\phi_{\rm L} , \\ \tilde{\chi}^-_{1R} &= \tilde{W}^-_{\rm R} \cos\phi_{\rm R} + \tilde{H}^-_{2R} \sin\phi_{\rm R} \end{aligned} \tag{12}$$

with the rotation angles given by

$$\begin{aligned} \cos 2\phi_{\rm L} &= -(M_2^2 - \mu^2 - 2m_W^2 \cos 2\beta)/\Delta , \\ \sin 2\phi_{\rm L} &= -2\sqrt{2} m_W (M_2 \cos\beta + \mu \sin\beta)/\Delta , \\ \cos 2\phi_{\rm R} &= -(M_2^2 - \mu^2 + 2m_W^2 \cos 2\beta)/\Delta , \\ \sin 2\phi_{\rm R} &= -2\sqrt{2} m_W (M_2 \sin\beta + \mu \cos\beta)/\Delta . \end{aligned} \tag{13}$$

As usual, we take $\tan\beta$ positive, M_2 positive and μ of either sign.

Light charginos are produced in pairs in e^+e^- collisions through s-channel γ and Z, and t-channel sneutrino exchange. The production cross section will thus depend on the chargino mass $m_{\tilde{\chi}_1^\pm}$, the sneutrino mass $m_{\tilde{\nu}}$ and the mixing angles, Eq. (13), which determine the couplings of the chargino states to the Z and the sneutrino. The unpolarized total cross section for $m_{\tilde{\chi}_1^\pm} = 95$ GeV is illustrated in Fig. 1 for representative cases of dominant higgsino, gaugino or mixed content of the lightest chargino state. The sharp rise near threshold should allow a precise determination of the chargino mass. The sensitivity to the sneutrino mass with the typical destructive interference in the gaugino and mixed cases necessitates the knowledge of this parameter [19].

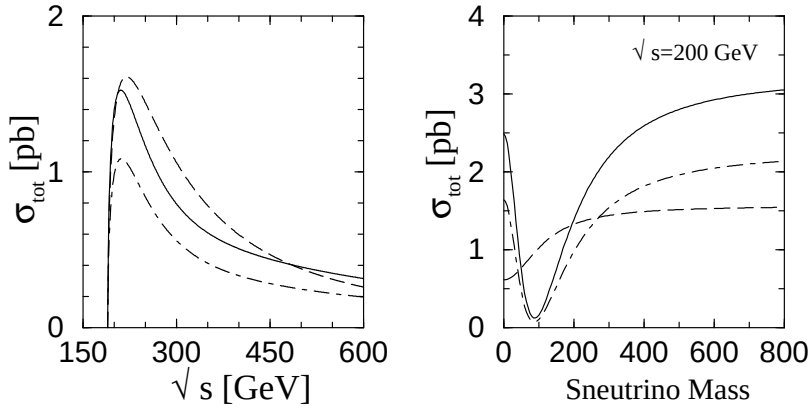

Fig. 1. Total cross section for the chargino pair production for a representative set of M_2, μ: solid line for the gaugino case, dashed line for the higgsino case, dot-dashed line for the mixed case. In the left panel $m_{\tilde{\nu}} = 200$ GeV. (Taken from [14].)

Charginos are not stable and each will decay directly to a pair of matter fermions (leptons or quarks) and the (stable) lightest neutralino $\tilde{\chi}_1^0$. The decay proceeds through the exchange of a W boson (charged Higgs exchange is suppressed for light fermions) or scalar partners of leptons or quarks. The decay matrix elements will depend on further parameters like the scalar masses and couplings to the neutralino. In addition, the presence of two invisible neutralinos in the final state of the process, $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0(f_1\bar{f}_2)(\bar{f}_3 f_4)$, makes it impossible to measure directly the chargino production angle Θ in the laboratory frame. Integrating over this angle and also over the invariant masses of the fermionic systems $(f_1\bar{f}_2)$ and $(\bar{f}_3 f_4)$, one can write the differential cross section in the following form:

$$\frac{d^4\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0(f_1\bar{f}_2)(\bar{f}_3 f_4))}{d\cos\theta^* d\phi^* d\cos\bar{\theta}^* d\bar{\phi}^*} = \frac{\alpha^2\beta}{124\pi s}\mathcal{B}\,\bar{\mathcal{B}}\,\Sigma(\theta^*, \phi^*, \bar{\theta}^*, \bar{\phi}^*)\,, \qquad (14)$$

where α is the fine structure constant, β the velocity of the chargino in the c.m. frame. For the $\tilde{\chi}_1^-$ decay we have $\mathcal{B} = Br(\tilde{\chi}_1^- \to \tilde{\chi}_1^0 f_1\bar{f}_2)$, θ^* is the polar angle of the $f_1\bar{f}_2$ system in the $\tilde{\chi}_1^-$ rest frame with respect to the chargino's flight direction in the lab frame, and ϕ^* is the azimuthal angle with respect to the production plane; quantities with a bar refer to the $\tilde{\chi}_1^+$ decay. The differential cross section $\Sigma(\theta^*, \phi^*, \bar{\theta}^*, \bar{\phi}^*)$ is expressed in terms of sixteen independent angular combinations of helicity production amplitudes

$$\begin{aligned}\Sigma \;=\; & \Sigma_{\rm unpol} + \kappa\cos\theta^*\mathcal{P} + \bar{\kappa}\cos\bar{\theta}^*\bar{\mathcal{P}} + \cos\theta^*\cos\bar{\theta}^*\kappa\bar{\kappa}\mathcal{Q} \\ & + \sin\theta^*\sin\bar{\theta}^*\cos(\phi^* + \bar{\phi}^*)\kappa\bar{\kappa}\mathcal{Y} + \ldots \end{aligned} \qquad (15)$$

Out of the sixteen terms, corresponding to the unpolarized, 2×3 polarization components and 3×3 spin–spin correlations in the production process, only 7 are independent (neglecting small effects from the Z-boson width and loop corrections) and $\kappa = -\bar{\kappa}$ in the CP-invariant theory. The polarization component $\mathcal{P}$ coming from the $\tilde{\chi}_1^-$ system, for example, reads

$$\mathcal{P} = \tfrac{1}{4}\int d\cos\Theta \sum_{\sigma=\pm}\left[|A_{\sigma;++}|^2 + |A_{\sigma;+-}|^2 - |A_{\sigma;-+}|^2 - |A_{\sigma;--}|^2\right] , \quad (16)$$

where $2\pi\alpha A_{\sigma;\lambda\lambda'}$ is the helicity amplitude with $\sigma;\lambda\lambda'$ denoting the helicities of the electron and $\tilde{\chi}_1^-\tilde{\chi}_1^+$ pair, respectively. All the complicted dependence on the chargino decay dynamics (neutralino and sfermion masses and their couplings) is contained in the spin analysis-powers κ and $\bar{\kappa}$.

The crucial observation of [14] is that all explicitly written terms in Eq. (3.1) can be extracted and three κ-independent physical observables, $\Sigma_{\rm unpol}, \mathcal{P}^2/\mathcal{Q}$ and $\mathcal{P}^2/\mathcal{Y}$, constructed. Indeed, it is possible by means of kinematical projections, since $\cos\theta^*, \cos\bar{\theta}^*$ and $\sin\theta^*\sin\bar{\theta}^*\cos(\phi^*+\bar{\phi}^*)$ are fully determined by the measurable parameters $E, |\vec{p}|$ (the energy and momentum of each of the decay systems $f_i\bar{f}_j$ in the laboratory frame) and the chargino mass. As a result, the chargino properties can be determined *independently* of the other sectors of the model[2]. The measurements of the cross section and either of the ratios $\mathcal{P}^2/\mathcal{Q}$ or $\mathcal{P}^2/\mathcal{Y}$ can be interpreted as contour lines in the plane $\{\cos 2\phi_{\rm L}, \cos 2\phi_{\rm R}\}$ which intersect with large angles so that a high precision in the resolution can be achieved. A representative example for the determination of $\cos 2\phi_{\rm L}$ and $\cos 2\phi_{\rm R}$ is shown in Fig. 2. The mass of the light chargino is set to $m_{\tilde{\chi}_1^\pm} = 95$ GeV, and the "measured" cross section, $\mathcal{P}^2/\mathcal{Q}$ and $\mathcal{P}^2/\mathcal{Y}$ at $\sqrt{s} = 500$ GeV are taken to be

$$\sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-) = 0.37 \ \text{ pb}, \quad \mathcal{P}^2/\mathcal{Q} = -0.24, \quad \mathcal{P}^2/\mathcal{Y} = -3.66 \quad (17)$$

in the left panel of Fig. 2. The three contour lines meet at a single point $\{\cos 2\phi_{\rm L}, \cos 2\phi_{\rm R}\} = \{-0.8, -0.5\}$. The sneutrino mass is set to $m_{\tilde{\nu}} = 250$ GeV. Note that the $m_{\tilde{\nu}}$ can be determined together with the mixing angles by requiring a consistent solution from the "measured quantities" σ, $\mathcal{P}^2/\mathcal{Q}$ and $\mathcal{P}^2/\mathcal{Y}$ at several values of incoming energy, as exemplified in both panels of Fig. 2 for $\sqrt{s} = 500$ and 300 GeV.

If polarized beams are available, the left-right asymmetry $A_{\rm LR}$ can provide an alternative way to extract the mixing angles (or serve as a consistency check). This is also demonstrated in Fig. 2, where contour lines for

[2] Actually to determine the kinematical variables, $\cos\theta^*$ *etc.*, the knowledge of $m_{\tilde{\chi}_1^0}$ is also needed, which can be extracted from the energy distributions of final state particles, see later. However, it must be stressed that the above procedure does not depend on the details of decay dynamics nor on the structure of (potentially more complex) neutralino and sfermion sectors.

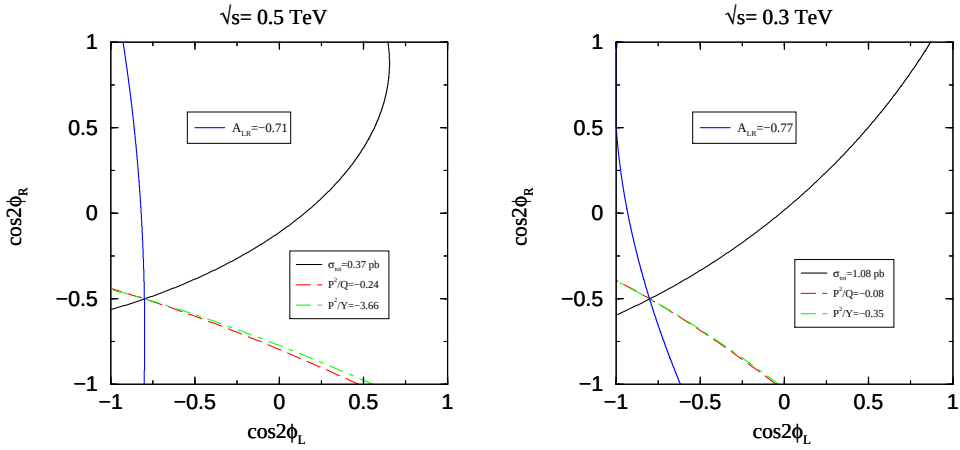

Fig. 2. Contours for the "measured values" of the total cross section (solid line), $\mathcal{P}^2/\mathcal{Q}$, and $\mathcal{P}^2/\mathcal{Y}$ (dot-dashed line) for $m_{\tilde{\chi}_1^\pm} = 95$ GeV [$m_{\tilde{\nu}} = 250$ GeV]. Superimposed are contour lines (solid, almost vertical lines) for the "measured" LR asymmetry.

the "measured" values of $A_{\rm LR}$ are also shown. Moreover, with right-handed electron beams one can turn off the sneutrino exchange in the production process and since at high energy the γ and Z "demix" back to the W_3^0 and B^0 gauge bosons, only the higgsino component of the chargino is selected. Thus the polarization alone will give us the composition of charginos. In short, the step A can be fully realized for the lightest charginos.

Let us now discuss the step B and describe briefly how to determine the Lagrangian parameters M_2, μ and $\tan\beta$ from $m_{\tilde{\chi}_1^\pm}$, $\cos 2\phi_{\rm L}$ and $\cos 2\phi_{\rm R}$. It is most transparently achieved by introducing the two triangular quantities

$$p = \cot(\phi_{\rm R} - \phi_{\rm L}) \quad \text{and} \quad q = \cot(\phi_{\rm R} + \phi_{\rm L})\,. \tag{18}$$

They are expressed in terms of the measured values $\cos 2\phi_{\rm L}$ and $\cos 2\phi_{\rm R}$ up to a discrete ambiguity due to undetermined signs of $\sin 2\phi_{\rm L}$ and $\sin 2\phi_{\rm R}$

$$\begin{aligned} p^2 + q^2 &= \frac{2(\sin^2 2\phi_{\rm L} + \sin^2 2\phi_{\rm R})}{(\cos 2\phi_{\rm L} - \cos 2\phi_{\rm R})^2}\,, \\ pq &= \frac{\cos 2\phi_{\rm L} + \cos 2\phi_{\rm R}}{\cos 2\phi_{\rm L} - \cos 2\phi_{\rm R}}\,, \\ p^2 - q^2 &= \frac{4\sin 2\phi_{\rm L} \sin 2\phi_{\rm R}}{(\cos 2\phi_{\rm L} - \cos 2\phi_{\rm R})^2}\,. \end{aligned} \tag{19}$$

Solving then Eqs (13) for $\tan\beta$ one finds at most two possible solutions, and

using

$$\begin{aligned} M_2 &= m_W[(p+q)s_\beta - (p-q)c_\beta]/\sqrt{2}\,, \\ \mu &= m_W[(p-q)s_\beta - (p+q)c_\beta]/\sqrt{2}\,, \end{aligned} \tag{20}$$

we arrive at $\tan\beta$, M_2 and μ up to a two-fold ambiguity. For example, taking the "measured values" from Eq. (17), the following results are found in [14]

$$[\tan\beta; M_2, \mu] = \begin{cases} [1.06;\ 83\text{GeV},\ -59\text{GeV}] \\ \\ [3.33;\ 248\text{GeV},\ 123\text{GeV}] \end{cases} . \tag{21}$$

To summarize, from the lightest chargino pair production, the measurements of the total production cross section and either the angular correlations among the chargino decay products ($\mathcal{P}^2/\mathcal{Q}$, $\mathcal{P}^2/\mathcal{Y}$) or the LR asymmetry, the step A can be realized and the physical parameters $m_{\tilde{\chi}_1^\pm}$, $\cos 2\phi_{\rm L}$ and $\cos 2\phi_{\rm R}$ determined unambiguously. Then the fundamental parameters $\tan\beta$, M_2 and μ are extracted (step B) up to a two-fold ambiguity.

If the collider energy is sufficient to produce the two chargino states in pairs, the above ambiguity can be removed [20]. The new ingredient in this case is the knowledge of the heavier chargino mass. Like for the ligter one, $m_{\tilde{\chi}_2^\pm}$ can be determined very precisely from the sharp rise of the production cross sections $\sigma(e^+e^- \to \tilde{\chi}_i^- \tilde{\chi}_j^+)$. Then the value of $\tan\beta$ is uniquely determined in terms of the mass difference of two chargino states, $\Delta = m^2_{\tilde{\chi}_2^\pm} - m^2_{\tilde{\chi}_1^\pm}$, and two mixing angles as follows

$$\tan\beta = \sqrt{\frac{4m_W^2 + \Delta\,(\cos 2\phi_{\rm R} - \cos 2\phi_{\rm L})}{4m_W^2 - \Delta\,(\cos 2\phi_{\rm R} - \cos 2\phi_{\rm L})}}\,. \tag{22}$$

Using the convention $M_2 > 0$, the gaugino mass parameter M_2 and the modulus of the higgsino mass parameter are given by

$$\begin{aligned} M_2 &= \frac{1}{2}\sqrt{2(m^2_{\tilde{\chi}_2^\pm} + m^2_{\tilde{\chi}_1^\pm} - 2m_W^2) - \Delta\,(\cos 2\phi_{\rm R} + \cos 2\phi_{\rm L})}\,, \\ |\mu| &= \frac{1}{2}\sqrt{2(m^2_{\tilde{\chi}_2^\pm} + m^2_{\tilde{\chi}_1^\pm} - 2m_W^2) + \Delta\,(\cos 2\phi_{\rm R} + \cos 2\phi_{\rm L})}\,. \end{aligned} \tag{23}$$

The sign of μ is then determined by the sign of the following expression

$$\text{sign}(\mu) = \text{sign}[\Delta^2 - (M_2^2 - \mu^2)^2 - 4m_W^2(M_2^2 + \mu^2) - 4m_W^4 \cos^2 2\beta]\,. \tag{24}$$

Before leaving the chargino sector, let us note that from the energy distribution of the final particles in the decay of the charginos $\tilde{\chi}_1^{\pm}$, the mass of the lightest neutralino $\tilde{\chi}_1^0$ can be measured [16]. This, as we will see in the next subsection, allows us to derive the parameter M_1 in the CP–invariant theories so that the neutralino mass matrix, too, can be reconstructed in a model-independent way.

3.2. And neutralinos: extracting also M_1

The spin-1/2 superpartners of the neutral electroweak gauge bosons and neutral Higgs bosons mix to form four neutralino mass eigenstates $\tilde{\chi}_{1,2,3,4}^0$. Their masses $m_{\tilde{\chi}_i^0}$ and the mixing angles are determined by the elements of the neutralino mass matrix given by ($s_W = \sin\theta_W$, $c_W = \cos\theta_W$) [6]

$$\mathcal{M}_N = \begin{bmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{bmatrix} . \quad (25)$$

Since $\mathcal{M}_N$ is symmetric, an orthogonal matrix $\mathcal{N}$ can be constructed that transforms $\mathcal{M}_N$ to a (positive) diagonal matrix. This mathematical problem can be solved analytically [21]. Due to the large ensemble of four neutralinos, however, the analysis is much more complex than in the chargino case. In particular, the step B, *i.e.* the analytical reconstruction of the fundamental SUSY parameters, is more complicated although, after measuring the parameters M_2, μ and $\tan\beta$ from the chargino production, the only additional parameter in the neutralino mass matrix is M_1.

Neutralinos are produced in e^+e^- collisions either in diagonal or non-diagonal pairs. The lightest neutralino $\tilde{\chi}_1^0$ is generally expected to be the lightest SUSY particle (LSP) and therefore stable in the R-parity preserving model. As a result, the production of the lightest neutralino pairs is difficult to identify and exploit experimentally. Therefore we consider production processes where at least one of the neutralinos is not an LSP, for example $\tilde{\chi}_1^0\tilde{\chi}_2^0$ or $\tilde{\chi}_2^0\tilde{\chi}_2^0$. These processes are generated by the s-channel Z exchange and the t- and u-channel selectron $\tilde{e}_{\rm L,R}$ exchanges. The transition matrix elements will then depend not only on the neutralino properties but on the selectron masses as well. The heavier neutralino $\tilde{\chi}_2^0$ will decay into the LSP and a fermion pair, leptons or quarks, $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 f\bar{f}$, throught the exchange of a Z boson or scalar partners of the fermion (the neutral Higgs boson exchange is negligible for light fermions). The decay products will serve as a signature of the production process and from the fast rise of the cross sections the masses $m_{\tilde{\chi}_i^0}$ can be measured precisely.

Additional informations can be obtained by analysing the angular correlations among the decay products, like in the chargino sector. In the case of $\tilde{\chi}_2^0\tilde{\chi}_2^0$, the method developed for the chargino case can be applied directly. One can attempt to separate the production from the decay processes and determine the $Z\tilde{\chi}_i^0\tilde{\chi}_j^0$ and $e\tilde{e}\tilde{\chi}_i^0$ couplings (expressed as known combinations of the mixing matrix elements $\mathcal{N}_{ij}$ [6]). Such a separation is interesting for the hadronic or $\mu^+\mu^-$ decay modes of $\tilde{\chi}_2^0$ ($\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 q\bar{q}$ or $\tilde{\chi}_1^0\mu^+\mu^-$) because independent information on the neutralino couplings to electron-selectron and quark-squark or $\mu^\pm\tilde{\mu}^\mp$ from the production and decay processes, respectively, can be inferred. For the $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 e^+e^-$ the production/decay separation might be useful only from the point of view of consistency checks. In the $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production case, only one neutralino decays and such a separation is not possible due to a limited number of measurable kinematical variables. Nevertheless some information on couplings can be extracted. The prescription for the step A in the neutralino sector still awaits a detailed analysis [15].

However, the knowledge of $m_{\tilde{\chi}_1^0}$ in addition to the measurements performed in the chargino sector is sufficient to pinpoint the value of M_1, the only new parameter. This particular problem has recently been considered in [18], where the emphasis has been put on the step B, namely to what extent the reconstruction of the Lagrangian parameters through a controllable analytical procedure, including all possible ambiguities, is possible if three of the chargino and neutralino masses and $\tan\beta$ were known. Two cases have been analysed:

S_1: the two charginos and one neutralino masses are input,

S_2: one chargino and two neutralino masses are input.

In case S_1, a closed analytical procedure to determine M_1 has been found. The crucial observation is to use the four independent linear combinations of the entries of $\mathcal{M}_N$ which are invariant under similarity transformations, and thus relate them simply to the four eigenvalues of $\mathcal{M}_N$. As a result, any of the neutralino masses taken as input, for example $\tilde{\chi}_1^0$, in addition to μ, M_2 and $\tan\beta$ allows the set of these consistency relations to be solved for the other three neutralino masses. Then the M_1 parameter is determined as

$$M_1 = -\frac{P_{ij}^2 + P_{ij}(\mu^2 + m_Z^2 + M_2 S_{ij} - S_{ij}^2) + \mu m_Z^2 M_2 s_W^2 \sin 2\beta}{P_{ij}(S_{ij} - M_2) + \mu(c_W^2 m_Z^2 \sin 2\beta - \mu M_2)}\,, \qquad (26)$$

where

$$S_{ij} \equiv \tilde{m}_i + \tilde{m}_j, \qquad P_{ij} \equiv \tilde{m}_i\tilde{m}_j\,,$$

$i \neq j$, and $\tilde{m}_i = \varepsilon_i m_{\tilde{\chi}_i^0}$ (the mass parameters can be negative, for the details we refer to [18]). As an example of the numerical result of such a procedure, the sensitivity to a chargino mass with the other chargino and the neutralino masses fixed is shown in Fig. 3.

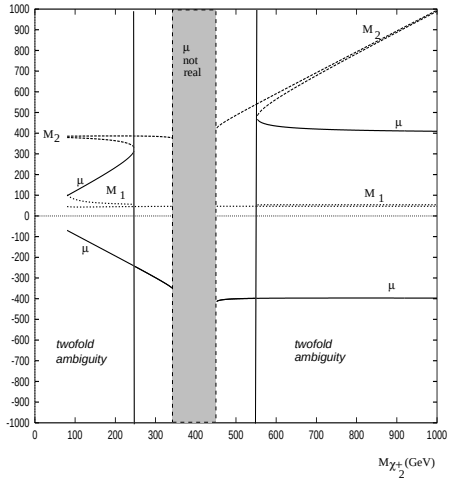

Fig. 3. μ, M_1 and M_2 (with the "higgsino-like" convention $|\mu| \leq M_2$) as functions of $m_{\tilde{\chi}_2^+}$ for fixed $m_{\tilde{\chi}_1^+} = 400$ GeV, $m_{\tilde{\chi}_2^0} = 50$ GeV, and $\tan\beta = 2$ (taken from Ref. [18]).

In case S_2, the above consistency relations can be reformulated in terms of two quadratic equatiuons for M_2 and M_1 at a given value of μ (and $\tan\beta$). Without any additional theoretical input, a numerical (iterative) procedure is used to obtain at most four distinct solutions for μ, M_1 and M_2 for a given set of $\tilde{\chi}_1^\pm$ and two neutralino masses.

3.3. Sfermions: extracting $m_{\tilde{f}_\mathrm{L}}$, $m_{\tilde{f}_\mathrm{R}}$ and A^f

For each fermion chirality $f_{\mathrm{L,R}}$ supersymmetry predicts a corresponding sfermion $\tilde{f}_{\mathrm{L,R}}$. Since SUSY is broken, the chiral left and right sfermions $\tilde{f}_\mathrm{L}$ and $\tilde{f}_\mathrm{R}$ may aquire different mass terms and they can mix. The mass eigenstates and mixing are determined by the mass matrices (for a given sfermion flavor $\tilde{f}$)

$$M_{\tilde{f}}^2 = \begin{bmatrix} m_{\tilde{f}_\mathrm{L}}^2 + m_f^2 & m_f(A^f - \mu r_f) \\ m_f(A^f - \mu r_f) & m_{\tilde{f}_\mathrm{R}}^2 + m_f^2 \end{bmatrix} \tag{27}$$

with

$$m_{\tilde{f}_\mathrm{L}}^2 = m_{\tilde{Q}}^2 + m_Z^2 \cos 2\beta\,(T_f^3 - e_f \sin^2\theta_W), \tag{28}$$

$$m^2_{\tilde{f}_{\rm R}} = m^2_{\tilde{F}'} + e_f m_Z^2 \cos 2\beta \sin^2\theta_W, \tag{29}$$

where e_f and T_f^3 are the charge and the third component of the weak isospin of the sfermion $\tilde{f}$, m_f is the mass of the corresponding fermion, $m_{\tilde{F}'} = m_{\tilde{u}}$, $m_{\tilde{u}}$ for $\tilde{f}_{\rm R} = \tilde{u}_{\rm R}$, $\tilde{d}_{\rm R}$, respectively, and $r_f = 1/\tan\beta$ for up- and $r_f = \tan\beta$ for down-type sfermions. The matrices (27) are diagonalized by orthogonal transformations with mixing angles θ_f defined by

$$\sin 2\theta_f = \frac{2m_f(A^f - \mu r_f)}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}} \quad , \quad \cos 2\theta_f = \frac{m^2_{\tilde{f}_{\rm L}} - m^2_{\tilde{f}_{\rm R}}}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}} \tag{30}$$

and the masses of the sfermion eigenstates are given by

$$m^2_{\tilde{f}_{1,2}} = m_f^2 + \frac{1}{2}\left[m^2_{\tilde{f}_{\rm L}} + m^2_{\tilde{f}_{\rm R}} \mp \sqrt{(m^2_{\tilde{f}_{\rm L}} - m^2_{\tilde{f}_{\rm R}})^2 + 4m_f^2(A^f - \mu r_f)^2}\right]. \tag{31}$$

Since the mixing term is of order m_f, it can be substantial only for the third generation sfermions (for sbottom and stau if $\tan\beta$ is large), with an important consequence of lowering the mass of the lighter eigenstate $m_{\tilde{f}_1}$. As a result, the lighter stop $\tilde{t}_1$ is expected to be the lightest scalar fermion.

Sfermions are produced in pairs in e^+e^- collisions

$$e^+e^- \to \tilde{f}_i \bar{\tilde{f}}_j \tag{32}$$

through s-channel γ and Z exchange; only the selectron production receives an additional t-channel neutralino exchange contribution. Since the gauge boson couplings respect chirality, the nondiagonal $\tilde{f}_1\bar{\tilde{f}}_2$ production can occur only for nontrivial mixing.

It is important first to verify experimentally the chiral nature of produced sfermions. This can be easily done at e^+e^- colliders by using polarized beams (not available at LHC) or by reconstructing the polarization of the final state fermions from sfermion decays (too difficult in hadron collisions). As an example, consider the pair production of right-handed staus which most probably will decay into the LSP neutralinos and τ's. The signature is the same as that of W pair production with the W's decaying into $\tau\nu_\tau$. However, at high-energy the Z and γ "demix" back to the W_3^0 and B^0 (hypercharge). Since the former does not couple to right-handed states, only the hypercharge boson is exchanged in right sfermion production. Therefore the background W pair production can be suppressed by choosing right-handed electrons. Moreover, as a result of hypercharge assignements $Y(e_{\rm L}) = -1$ and $Y(e_{\rm R}) = -2$, the signal cross section with right-handed e^- beams will be by factor 4 larger than with the left-handed e^- beams. The beam polarization therefore is a very powerful tool: allows us not only to tag the nature

of the stau (right-handed) independently of its decays and increase the signal cross section, but also suppress the background. All these has been checked by the full simulation of the Japanese group [22]. In addition, reconstruction of the τ polarization in the decay process $\tilde{\tau} \to \tilde{\chi}_1^0 \tau$ will play an important role in exploring the Yukawa couplings. The $\tilde{\tau}\tau\tilde{\chi}_1^0$ coupling depends on the neutralino composition. The interaction involving gaugino component ($\tilde{B}$ or $\tilde{W}$) is proportional to gauge couplings and is chiral conserving, whereas the interaction involving higgsino component ($\tilde{H}_{1,2}$) is proportional to τ Yukawa coupling $Y_\tau \sim m_\tau / \cos\beta$ and chiral flipping. Thus the polarization of τ lepton from $\tilde{\tau}$ decays depends on the ratio of the chirality flipping and chirality conserving interactions, and consequently on $\tan\beta$. For a detailed discussion of stau production we refer to [23].

Once the sfermion production has been optimized, one can either infer the sfermion mass from a threshold scan (which is independent of the decay) or (as in chargino case) the measurement of the fermion energy spectrum will give both the $m_{\tilde{f}}$ and the LSP mass. A combined fit for a low luminosity option of 10 fb^{-1} and 85% polarization of the electron beam shows that a precision of order a few percent for sfermion masses can easily be obtained [16].

A case study of $e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1$ with the aim of determining the SUSY parameters has been performed by the Vienna group [17] at $\sqrt{s} = 500$ GeV and $\mathcal{L} = 50$ fb^{-1}. The input $m_{\tilde{t}_1} = 180$ GeV and left-right stop mixing angle $|\cos\theta_{\tilde{t}}| = 0.57$ corresponds to the minimum of the cross section. The cross sections at tree level for these parameters are $\sigma_{\rm L} = 48.6$ fb and $\sigma_{\rm R} = 46.1$ fb for 90% left- and right-polarized e^- beam, respectively. Based on detailed studies the experimental errors on these cross sections are estimated to be $\Delta\sigma_{\rm L} = \pm\ 6$ fb and $\Delta\sigma_{\rm R} = \pm\ 4.9$ fb. Figure 4 shows the resulting error bands and the corresponding error ellipse in the $m_{\tilde{t}_1}$–$\cos\theta_{\tilde{t}}$ plane. The experimental accuracy for the stop mass and mixing angle are $m_{\tilde{t}_1} = 180 \pm 7$ GeV, $|\cos\theta_{\tilde{t}}| = 0.57 \pm 0.06$.

Additional experimental input is needed, however, to determine the fundamental parameters. The Vienna group decided to exploit the sbottom system. Assuming that $\tan\beta$ is low and the $\tilde{b}_{\rm L}$–$\tilde{b}_{\rm R}$ mixing can be neglected, *i.e.* $\cos\theta_{\tilde{b}} = 1$, and taking $\tilde{b}_1 = \tilde{b}_{\rm L} = 200$ GeV, $\tilde{b}_2 = \tilde{b}_{\rm R} = 220$ GeV, the cross sections and the expected experimental errors are $\sigma_{\rm L}(e^+e^- \to \tilde{b}_1\bar{\tilde{b}}_1) = 61.1 \pm 6.4$ fb, $\sigma_{\rm R}(e^+e^- \to \tilde{b}_2\bar{\tilde{b}}_2) = 6 \pm 2.6$ fb for the 90% left- and right-polarized e^- beams. The resulting experimental errors are $m_{\tilde{b}_1} = 200 \pm 4$ GeV, $m_{\tilde{b}_2} = 220 \pm 10$ GeV. With these results the mass of the heavier stop can be calculated and is found to be $m_{\tilde{t}_2} = 289 \pm 15$ GeV. Verifying this prediction experimentally will test the MSSM.

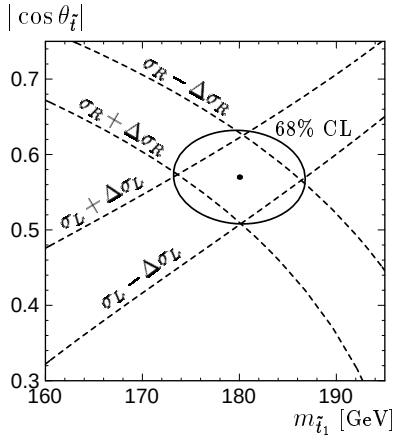

Fig. 4. Error bands (dashed) and the corresponding error ellipse as a function of $m_{\tilde{t}_1}$ and $|\cos\theta_{\tilde{t}}|$ for the tree–level cross sections of $e^+e^- \to \tilde{t}_1\tilde{t}_1$ at $\sqrt{s} = 500$ GeV with 90% left- and right-polarized electron beam (taken from Ref. [17]).

To complete the step B, $\mu = -200$ GeV, $\tan\beta = 2$ and $m_t = 175$ GeV have been taken assuming that μ and $\tan\beta$ are known from other experiments (from chargino sector, for example). The soft-supersymmetry breaking parameters of the stop and sbottom systems can then be determined up to a two-fold ambiguity: $m_{\tilde{Q}} = 195 \pm 4$ GeV, $m_{\tilde{u}} = 138 \pm 26$ GeV, $m_{\tilde{d}} = 219 \pm 10$ GeV, $A^t = -236 \pm 38$ GeV if $\cos\theta_{\tilde{t}} > 0$, and $A^t = 36 \pm 38$ GeV if $\cos\theta_{\tilde{t}} < 0$.

4. Conclusions

In this talk I have tried to illustrate the discovery power and precision tools developed to explore supersymmetry at future e^+e^- linear colliders. The LC is an excellent machine for supersymmetry because a systematic, model-independent determination of the supersymmetry parameters is possible within a discovery reach that is limited by the available center-of-mass energy. Although we only considered real-valued parameters, some of the material presented here goes through unaltered if phases are allowed [14,20] even though extra information will still be needed to determine those phases.

It should be stressed that the strategies presented here are just at the theoretical level. A more realistic simulation of the experimental measurements of physical observables and related errors is still needed to assess fully the physics potential of LC. Nevertheless, if the LC and detectors are built and work as expected, I have no doubt that the actual measurements will be better than anything I have presented here — provided supersymmetry is discovered! After all, nobody beats experimentalists with real data.

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