EXACT FINITE AND GAUGE-YUKAWA UNIFIED THEORIES AND THEIR PREDICTIONS*

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The recent developments in the soft supersymmetry breaking (SSB) sector of Gauge-Yukawa and Finite Unified Theories permit the derivation of exact renormalization group invariant results also in this sector of the theory. Of particular interest is a RGI sum rule for the soft scalar masses holding to all-orders in perturbation theory. In the case of Finite Unified Theories the sum rule ensures the all-loop finiteness also in their SSB sector and in this way are promoted to completely finite ones. Using the sum rule we investigate the minimal supersymmetric Gauge-Yukawa and two Finite-Gauge-Yukawa SU(5) models. The characteristic features of these models are: a) the old agreement of the top quark mass prediction remains unchanged, b) the lightest Higgs boson is predicted to be around 120 GeV, c) the s-spectrum starts above several hundreds of GeV.

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1. Introduction

In the recent years the theoretical endeavours that attempt to achieve a deeper understanding of Nature have to present a series of successes in developing frameworks that aim to describe the fundamental theory at the Planck scale. However, the essence of all theoretical efforts in Elementary Particle Physics is to understand the present day free parameters of the Standard Model (SM) in terms of a few fundamental ones, *i.e.* to achieve *reduction of couplings*. It is sad to recall that all recent celebrated theoretical successes did not offer anything in the understanding of the free parameters of the SM, and in the best case they just manage to accomodate in a rather poor way earlier ideas for Physics Beyond the SM, such as Grand Unified Theories (GUTs) and supersymmetry. In our recent studies [1–8], we have developed a complementary strategy in searching for a more fundamental theory possibly at the Planck scale, whose basic ingredients are GUTs and supersymmetry, but its consequences certainly go beyond the known ones. Our method consists of hunting for renormalization group invariant (RGI) relations holding below the Planck scale, which in turn are preserved down to the GUT scale. This programme, called Gauge–Yukawa unification scheme, applied in the dimensionless couplings of supersymmetric GUTs, such as gauge and Yukawa couplings, had already noticable successes by predicting correctly, among others, the top quark mass in the finite and in the minimal $N = 1$ supersymmetric SU(5) GUTs. An impressive aspect of the RGI relations is that one can guarantee their validity to all-orders in perturbation theory by studying the uniqueness of the resulting relations at one-loop, as was proven in the early days of the programme of *reduction of couplings* [9]. Even more remarkable is the fact that it is possible to find RGI relations among couplings that guarantee finiteness to all-orders in perturbation theory [10, 11].

Although supersymmetry seems to be an essential feature for a successful realization of the above programme, its breaking has to be understood too, since it has the ambition to supply the SM with predictions for several of its free parameters. Indeed, the search for RGI relations has been extended to the soft supersymmetry breaking sector (SSB) of these theories [5, 12], which involves parameters of dimension one and two. More recently a very interesting progress has been made [13–18] concerning the renormalization properties of the SSB parameters based conceptually and technically on the work of Ref. [19]. In Ref. [19] the powerful supergraph method [20] for studying supersymmetric theories has been applied to the softly broken ones by using the "spurion" external space-time independent superfields [21]. In the latter method a softly broken supersymmetric gauge theory is considered as a supersymmetric one in which the various parameters such as couplings and masses have been promoted to external superfields that acquire "vacuum expectation values". Based on this method the relations among the soft term renormalization and that of an unbroken supersymmetric theory have been derived. In particular the β-functions of the parameters of the softly broken theory are expressed in terms of partial differential operators involving the dimensionless parameters of the unbroken theory. The key point in the strategy of Refs. [16–18] in solving the set of coupled differential equations so as to be able to express all parameters in a RGI way, was to transform the partial differential operators involved to total derivative operators. This is indeed possible to be done on the RGI surface which is defined by the solution of the reduction equations.

On the phenomenological side there exist some serious developments too. Previously an appealing "universal" set of soft scalar masses was asummed in the SSB sector of supersymmetric theories, given that apart from economy and simplicity (1) they are part of the constraints that preserve finiteness up to two-loops [22, 23], (2) they are RGI up to two-loops in more general supersymmetric gauge theories, subject to the condition known as $P = 1/3\ Q$ [12] and (3) they appear in the attractive dilaton dominated supersymmetry breaking superstring scenarios [24]. However, further studies have exhibited a number of problems all due to the restrictive nature of the "universality" assumption for the soft scalar masses. For instance (a) in finite unified theories the universality predicts that the lightest supersymmetric particle is a charged particle, namely the superpartner of the τ lepton $\tilde{\tau}$ (b) the MSSM with universal soft scalar masses is inconsistent with the attractive radiative electroweak symmetry breaking [25] and (c) which is the worst of all, the universal soft scalar masses lead to charge and/or colour breaking minima deeper than the standard vacuum [26]. Therefore, there have been attempts to relax this constraint without loosing its attractive features. First an interesting observation was made that in $N = 1$ Gauge–Yukawa unified theories there exists a RGI sum rule for the soft scalar masses at lower orders; at one-loop for the non-finite case [6] and at two-loops for the finite case [7]. The sum rule manages to overcome the above unpleasant phenomenological consequences. Moreover it was proven [18] that the sum rule for the soft scalar massses is RGI to all-orders for both the general as well as for the finite case. Finally the exact β-function for the soft scalar masses in the Novikov-Shifman-Vainstein-Zakharov (NSVZ) scheme [27] for the softly broken supersymmetric QCD has been obtained. Armed with the above tools and results we are in a position to study the spectrum of the full finite and minimal supersymmetric SU(5) models in terms of few free parameters with emphasis on the predictions for the masses of the lightest Higgs and LSP and on the constraints imposed by having a large $\tan\beta$.

2. Reduction of couplings and finiteness in $N = 1$ SUSY gauge theories

A RGI relation among couplings, $\Phi(g_1, \cdots, g_N) = 0$, has to satisfy the partial differential equation (PDE) $\mu\, d\Phi/d\mu = \sum_{i=1}^{N} \beta_i\, \partial\Phi/\partial g_i = 0$, where β_i is the β-function of g_i. There exist $(N-1)$ independent Φ's, and finding the complete set of these solutions is equivalent to solve the so-called reduction equations (REs), $\beta_g\,(dg_i/dg) = \beta_i$, $i = 1, \cdots, N$, where g and β_g are the primary coupling and its β-function. Using all the $(N-1)\,\Phi$'s to impose RGI relations, one can in principle express all the couplings in terms of a single coupling g. The complete reduction, which formally preserves perturbative renormalizability, can be achieved by demanding a power series solution, whose uniqueness can be investigated at the one-loop level. The completely reduced theory contains only one independent coupling with the corresponding β-function. This possibility of coupling unification is attractive, but it can be too restrictive and hence unrealistic. In practice one may use fewer Φ's as RGI constraints.

It is clear by examining specific examples, that the various couplings in supersymmetric theories have easily the same asymptotic behaviour. Therefore searching for a power series solution to the REs is justified. This is not the case in non-supersymmetric theories.

Let us then consider a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g. The superpotential of the theory is given by

$$W \;=\; \frac{1}{2}\, m^{ij}\, \Phi_i\, \Phi_j + \frac{1}{6}\, C^{ijk}\, \Phi_i\, \Phi_j\, \Phi_k \;, \tag{1}$$

where m^{ij} and C^{ijk} are gauge invariant tensors and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G .

The one-loop β-function of the gauge coupling g is given by

$$\beta_g^{(1)} \;=\; \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i l(R_i) - 3\, C_2(G)\right] \;, \tag{2}$$

where $l(R_i)$ is the Dynkin index of R_i and $C_2(G)$ is the quadratic Casimir of the adjoint representation of the gauge group G . The β-functions of C^{ijk}, by virtue of the non-renormalization theorem, are related to the anomalous dimension matrix γ_i^j of the matter fields Φ_i as

$$\beta_C^{ijk} = \frac{d}{dt}\, C^{ijk} = C^{ijp} \;\sum_{n=1} \frac{1}{(16\pi^2)^n}\, \gamma_p^{k(n)} + (k \leftrightarrow i) + (k \leftrightarrow j) \;. \tag{3}$$

At one-loop level the γ_i^j are given by

$$\gamma_i^{j(1)} = \frac{1}{2} C_{ipq}\, C^{jpq} - 2\, g^2\, C_2(R_i) \delta_i^j \ , \tag{4}$$

where $C_2(R_i)$ is the quadratic Casimir of the representation R_i, and $C^{ijk} = C^*_{ijk}$.

As one can see from Eqs. (2) and (4) all the one-loop β-functions of the theory vanish if $\beta_g^{(1)}$ and $\gamma_i^{j(1)}$ vanish, *i.e.*

$$\sum_i \ell(R_i) = 3 C_2(G)\,, \qquad \frac{1}{2} C_{ipq} C^{jpq} = 2\delta_i^j g^2 C_2(R_i)\,. \tag{5}$$

A very interesting result is that the conditions (5) are necessary and sufficient for finiteness at the two-loop level.

The one- and two-loop finiteness conditions (5) restrict considerably the possible choices of the irreps. R_i for a given group G as well as the Yukawa couplings in the superpotential (1). Note in particular that the finiteness conditions cannot be applied to the supersymmetric standard model (SSM), since the presence of a U(1) gauge group is incompatible with the condition (5), due to $C_2[\mathrm{U}(1)] = 0$. This naturally leads to the expectation that finiteness should be attained at the grand unified level only, the SSM being just the corresponding, low-energy, effective theory.

A natural question to ask is what happens at higher loop orders. There exists a very interesting theorem [10] which guarantees the vanishing of the β-functions to all orders in perturbation theory, if we demand reduction of couplings, and that all the one-loop anomalous dimensions of the matter field in the completely and uniquely reduced theory vanish identically.

3. Soft Supersymmetry Breaking — sum rule of soft scalar masses

The above described method of reducing the dimensionless couplings has been extended [5] to the Soft Supersymmetry Breaking (SSB) dimensionful parameters of $N = 1$ supersymmetric theories. In addition it was found [6] that RGI SSB scalar masses in Gauge-Yukawa unified models satisfy a universal sum rule. Here we will describe first how the use of the available two-loop RG functions and the requirement of finiteness of the SSB parameters up to this order leads to the soft scalar-mass sum rule [7].

Consider the superpotential given by (1) along with the Lagrangian for SSB terms

$$-\mathcal{L}_{\mathrm{SB}} \ = \ \frac{1}{6}\, h^{ijk}\, \phi_i \phi_j \phi_k + \frac{1}{2}\, b^{ij}\, \phi_i \phi_j + \frac{1}{2}\, (m^2)_i^j\, \phi^{*\,i} \phi_j + \frac{1}{2}\, M\, \lambda\lambda + \mathrm{H.c.}\,, \tag{6}$$

where the ϕ_i are the scalar parts of the chiral superfields Φ_i , λ are the gauginos and M their unified mass. Since we would like to consider only finite theories here, we assume that the gauge group is a simple group and the one-loop β function of the gauge coupling g vanishes. We also assume that the reduction equations admit power series solutions of the form

$$C^{ijk} \; = \; g \sum_{n=0} \rho^{ijk}_{(n)} g^{2n} \; , \tag{7}$$

According to the finiteness theorem of Ref. [10], the theory is then finite to all orders in perturbation theory, if, among others, the one-loop anomalous dimensions $\gamma_i^{j\,(1)}$ vanish. The one- and two-loop finiteness for h^{ijk} can be achieved by

$$h^{ijk} \; = \; -MC^{ijk} + \ldots = -M\rho^{ijk}_{(0)}\, g + O(g^5) \; . \tag{8}$$

Now, to obtain the two-loop sum rule for soft scalar masses, we assume that the lowest order coefficients $\rho^{ijk}_{(0)}$ and also $(m^2)^i_j$ satisfy the diagonality relations

$$\rho_{ipq(0)}\rho^{jpq}_{(0)} \; \propto \; \delta^j_i \text{ for all } p \text{ and } q \;\; \text{and} \;\; (m^2)^i_j = m^2_j \delta^i_j \; , \tag{9}$$

respectively. Then we find the following soft scalar-mass sum rule

$$(\; m^2_i + m^2_j + m^2_k \;)/MM^\dagger \; = \; 1 + \frac{g^2}{16\pi^2}\, \Delta^{(1)} + O(g^4) \tag{10}$$

for i, j, k with $\rho^{ijk}_{(0)} \neq 0$, where $\Delta^{(1)}$ is the two-loop correction

$$\Delta^{(1)} \; = \; -2\sum_l [(m^2_l/MM^\dagger) - (1/3)]\; T(R_l), \tag{11}$$

which vanishes for the universal choice in accordance with the previous findings of Ref. [23].

If we know higher-loop β-functions explicitly, we can follow the same procedure and find higher-loop RGI relations among SSB terms. However, the β-functions of the soft scalar masses are explicitly known only up to two loops. In order to obtain higher-loop results, we need something else instead of knowledge of explicit β-functions, e.g. some relations among β-functions.

The recent progress made using the spurion technique [20, 21] leads to the following all-loop relations among SSB β-functions, [13–17]

$$\begin{aligned}
\beta_M &= 2\mathcal{O}\left(\frac{\beta_g}{g}\right) , && (12)\\
\beta_h^{ijk} &= \gamma^i{}_l h^{ljk} + \gamma^j{}_l h^{ilk} + \gamma^k{}_l h^{ijl} \\
&\quad -2\gamma^i_{1\,l} C^{ljk} - 2\gamma^j_{1\,l} C^{ilk} - 2\gamma^k_{1\,l} C^{ijl} , && (13)\\
(\beta_{m^2})^i{}_j &= \left[\Delta + X\frac{\partial}{\partial g}\right]\gamma^i{}_j , && (14)\\
\mathcal{O} &= \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial C^{lmn}}\right) , && (15)\\
\Delta &= 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2\frac{\partial}{\partial g^2} + \tilde{C}_{lmn}\frac{\partial}{\partial C_{lmn}} + \tilde{C}^{lmn}\frac{\partial}{\partial C^{lmn}} , && (16)
\end{aligned}$$

where $(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j$, $C_{lmn} = (C^{lmn})^*$, and

$$\tilde{C}^{ijk} = (m^2)^i{}_l C^{ljk} + (m^2)^j{}_l C^{ilk} + (m^2)^k{}_l C^{ijl} . \qquad (17)$$

It was also found [17] that the relation

$$h^{ijk} = -M(C^{ijk})' \equiv -M\frac{dC^{ijk}(g)}{d\ln g} , \qquad (18)$$

among couplings is all-loop RGI. Furthermore, using the all-loop gauge β-function of Novikov *et al.* [27] given by

$$\beta_g^{\mathrm{NSVZ}} = \frac{g^3}{16\pi^2}\left[\frac{\sum_l T(R_l)(1-\gamma_l/2) - 3C(G)}{1 - g^2 C(G)/8\pi^2}\right] , \qquad (19)$$

it was found the all-loop RGI sum rule [18],

$$\begin{aligned}
m_i^2 + m_j^2 + m_k^2 &= |M|^2\left\{\frac{1}{1-g^2C(G)/(8\pi^2)}\frac{d\ln C^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln C^{ijk}}{d(\ln g)^2}\right\} \\
&\quad + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2}\frac{d\ln C^{ijk}}{d\ln g} . && (20)
\end{aligned}$$

In addition the exact-β function for m^2 in the NSVZ scheme has been obtained [18] for the first time and is given by

$$\begin{aligned}
\beta_{m_i^2}^{\mathrm{NSVZ}} &= \left[|M|^2\left\{\frac{1}{1-g^2C(G)/(8\pi^2)}\frac{d}{d\ln g} + \frac{1}{2}\frac{d^2}{d(\ln g)^2}\right\}\right. \\
&\quad \left. + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2}\frac{d}{d\ln g}\right] \gamma_i^{\mathrm{NSVZ}} . && (21)
\end{aligned}$$

Surprisingly enough, the all-loop result (20) coincides with the superstring result for the finite case in a certain class of orbifold models [7] if $d\ln C^{ijk}/d\ln g = 1$.

4. Gauge–Yukawa–unified theories

In this section we will look at concrete SU(5) models, where the reduction of couplings in the dimensionless and dimensionful sector has been achieved.

4.1. Finite unified models

A predictive Gauge–Yukawa unified SU(5) model which is finite to all orders, in addition to the requirements mentioned already, should also have the following properties:

1. One-loop anomalous dimensions are diagonal, *i.e.*, $\gamma_i^{(1)\,j} \propto \delta_i^j$, according to the assumption (9).
2. Three fermion generations, $\overline{\mathbf{5}}_i$ $(i = 1,2,3)$, obviously should not couple to $\mathbf{24}$. This can be achieved for instance by imposing $B-L$ conservation.
3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and anti-quintet, which couple to the third generation.

In the following we discuss two versions of the all-order finite model.

A: The model of Ref. [1].

B: A slight variation of the model **A**, whose differences from **A** will become clear in the following.

The superpotential which describes the two models takes the form [1, 7]

$$\begin{aligned} W \;=\; & \sum_{i=1}^{3} \left[\, \frac{1}{2} g_i^u\, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d\, \mathbf{10}_i \overline{\mathbf{5}}_i\, \overline{H}_i \,\right] + g_{23}^u\, \mathbf{10}_2 \mathbf{10}_3 H_4 \\ & + g_{23}^d\, \mathbf{10}_2 \overline{\mathbf{5}}_3\, \overline{H}_4 + g_{32}^d\, \mathbf{10}_3 \overline{\mathbf{5}}_2\, \overline{H}_4 + \sum_{a=1}^{4} g_a^f\, H_a\, \mathbf{24}\, \overline{H}_a + \frac{g^\lambda}{3}\,(\mathbf{24})^3 \;, \end{aligned} \tag{22}$$

where H_a and $\overline{H}_a$ $(a = 1, \ldots, 4)$ stand for the Higgs quintets and anti-quintets.

The non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ for the models $\{\mathbf{A}\ ,\ \mathbf{B}\}$ are:

$$(g_1^u)^2 = \left\{\frac{8}{5}, \frac{8}{5}\right\} g^2\ , (g_1^d)^2 = \left\{\frac{6}{5}, \frac{6}{5}\right\} g^2\ , (g_2^u)^2 = (g_3^u)^2 = \left\{\frac{8}{5}, \frac{4}{5}\right\} g^2\ ,$$
$$(g_2^d)^2 = (g_3^d)^2 = \left\{\frac{6}{5}, \frac{3}{5}\right\} g^2\ , (g_{23}^u)^2 = \left\{0, \frac{4}{5}\right\} g^2\ , (g_{23}^d)^2 = (g_{32}^d)^2 = \left\{0, \frac{3}{5}\right\} g^2\ ,$$
$$(g^\lambda)^2 = \frac{15}{7} g^2\ , (g_2^f)^2 = (g_3^f)^2 = \left\{0, \frac{1}{2}\right\} g^2\ , (g_1^f)^2 = 0\ , (g_4^f)^2 = \{1, 0\}\, g^2\ . \tag{23}$$

According to the theorem of Ref. [10] these models are finite to all orders. After the reduction of couplings the symmetry of W is enhanced [1, 7].

The main difference of the models **A** and **B** is that three pairs of Higgs quintets and anti-quintets couple to the **24** for **B** so that it is not necessary to mix them with H_4 and $\overline{H}_4$ in order to achieve the triplet-doublet splitting after the symmetry breaking of SU(5).

In the dimensionful sector, the sum rule gives us the following boundary conditions at the GUT scale [7]:

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 \ = \ m_{H_d}^2 + m_{\overline{\mathbf{5}}}^2 + m_{\mathbf{10}}^2 = M^2 \ \text{ for } \ \mathbf{A}\ , \tag{24}$$

$$\begin{aligned} m_{H_u}^2 + 2m_{\mathbf{10}}^2 &= M^2\ ,\ m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}\ , \\ m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 &= \frac{4M^2}{3} \quad \text{for} \ \ \mathbf{B}, \end{aligned} \tag{25}$$

where we use as free parameters $m_{\overline{\mathbf{5}}} \equiv m_{\overline{\mathbf{5}}_3}$ and $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ for the model **A**, and $m_{\mathbf{10}}$ for **B**, in addition to M.

4.2. The minimal supersymmetric SU(5) model

Next let us consider the minimal supersymmetric SU(5) model. The field content is minimal. Neglecting the CKM mixing, one starts with six Yukawa and two Higgs couplings. We then require GYU to occur among the Yukawa couplings of the third generation and the gauge coupling. We also require the theory to be completely asymptotically free. In the one-loop approximation, the GYU yields $g_{t,b}^2 \ = \ \sum_{m,n=1}^{\infty} \kappa_{t,b}^{(m,n)}\ h^m\ f^n\ g^2$ (h and f are related to the Higgs couplings), where h is allowed to vary from 0 to 15/7, while f may vary from 0 to a maximum which depends on h and vanishes at $h = 15/7$. As a result, it was obtained [2]: $0.97\, g^2 \lesssim g_t^2 \lesssim 1.37\, g^2\ ,\ \ 0.57\, g^2 \lesssim g_b^2 = g_\tau^2 \lesssim 0.97\, g^2$. It was found [4, 8] that consistency with proton decay requires $g_t^2,\ g_b^2$ to be very close to the left hand side values in the inequalities.

In this model, the reduction of parameters implies that at the GUT scale the SSB terms are proportional to the gaugino mass, which thus characterizes the scale of supersymmetry breaking [5].

5. Predictions of low energy parameters

Since the gauge symmetry is spontaneously broken below $M_{\rm GUT}$, the finiteness and gauge-Yukawa unification conditions do not restrict the renormalization property at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (23), the $h = -MC$ relation (8) and the soft scalar-mass sum rule (10) at $M_{\rm GUT}$, as applied in the various models. So we examine the evolution of these parameters according to their renormalization group equations at two-loop for dimensionless parameters and at one-loop for dimensionful ones with the relevant boundary conditions. Below $M_{\rm GUT}$ their evolution is assumed to be governed by the MSSM. We further assume a unique supersymmetry breaking scale M_s so that below M_s the SM is the correct effective theory.

The predictions for the top quark mass M_t are ~ 183 and ~ 174 GeV in models **A** and **B** respectively, and ~ 181 GeV for the minimal SU(5) model. Comparing these predictions with the most recent experimental value $M_t = (173.8 \pm 5.2)$ GeV, and recalling that the theoretical values for M_t may suffer from a correction of less than $\sim 4\%$ [8], we see that they are consistent with the experimental data. In addition the value of $\tan\beta$ is obtained as $\tan\beta = 54$ and 48 for models **A** and **B** respectively, and $\tan\beta = 48$ for the minimal SU(5) model.

In the SSB sector, besides the constraints imposed by reduction of couplings and finiteness, we also look for solutions which are compatible with radiative electroweak symmetry breaking. As it has been mentioned, in the minimal SU(5) model the SSB sector contains only one independent parameter, the gaugino mass M, which characterizes the scale of supersymmetry breaking. The lightest supersymmetric particle is found to be a neutralino of ~ 220 GeV for $M(M_{\rm GUT}) \sim 0.5$ TeV. In Fig. 1 we present the dependence of the lightest Higgs mass m_h on the gaugino mass M.

Concerning the SSB sector of the finite theories **A** and **B**, besides the gaugino mass we have two and one more free parameters respectively, as previously mentioned. Thus, we look for the parameter space in which the lighter $\tilde{\tau}$ mass squared $m^2_{\tilde{\tau}}$ is larger than the lightest neutralino mass squared m^2_{χ} (which is the LSP). In the case where all the soft scalar masses are universal at the unfication scale, there is no region of $M_s = M$ below O(few) TeV in which $m^2_{\tilde{\tau}} > m^2_{\chi}$ is satisfied. But once the universality condition is relaxed this problem can be solved naturally (provided the sum rule). More specifically, using the sum rule (10) and imposing the conditions a) successful

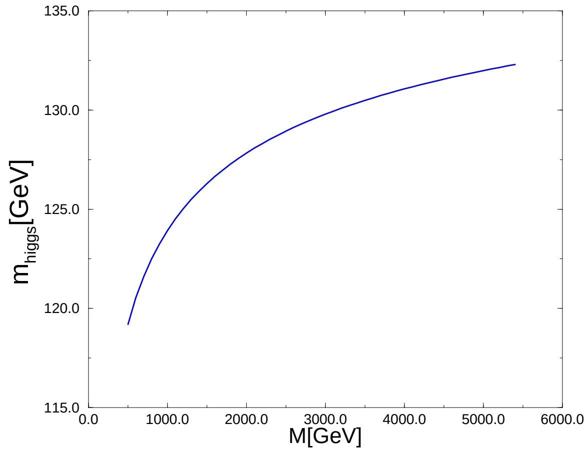

Fig. 1. The M dependence of m_h for the minimal SU(5) model

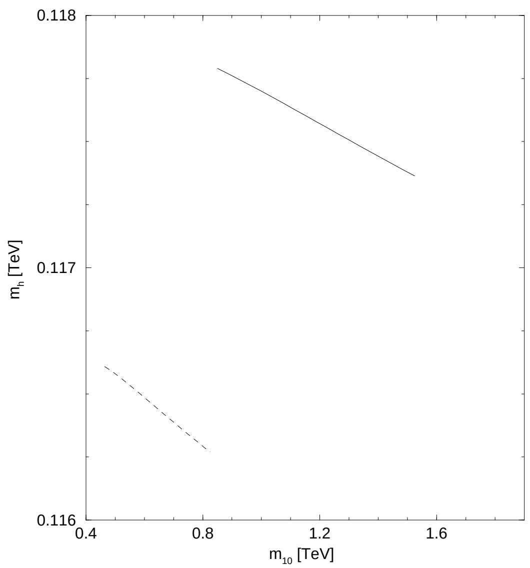

Fig. 2. m_h as function of $m_{\mathbf{10}}$ for $M = 0.8$ (dashed) 1.0 (solid) TeV for the finite model **B**.

radiative electroweak symmetry breaking b) $m_{\tilde{\tau}^2} > 0$ and c) $m_{\tilde{\tau}^2} > m_{\chi^2}$, we find a comfortable parameter space for both models (although model **B** requires large $M \sim 1$ TeV).

In Fig. 2 we present the $m_{\mathbf{10}}$ dependence of m_h for for $M = 0.8$ (dashed) 1.0 (solid) TeV for the finite Model **B**, which shows that the value of m_h is rather stable. Similar results hold also for Model **A**.

In Tables I, II, and III we present representative examples of the values obtained for the sparticle spectra in each of the models. The value of the lightest Higgs physical mass M_h has already the one-loop radiative corrections included, evaluated at the appropriate scale [28, 29].

TABLE I

A representative example of the predictions for the s-spectrum for the finite model **A** with $M = 1.0$ TeV, $m_{\overline{\mathbf{5}}} = 0.8$ TeV and $m_{\mathbf{10}} = 0.6$ TeV.

$m_\chi = m_{\chi_1}$ (TeV)	0.45	$m_{\tilde{b}_2}$ (TeV)	1.76
m_{χ_2} (TeV)	0.84	$m_{\tilde{\tau}} = m_{\tilde{\tau}_1}$ (TeV)	0.63
m_{χ_3} (TeV)	1.49	$m_{\tilde{\tau}_2}$ (TeV)	0.85
m_{χ_4} (TeV)	1.49	$m_{\tilde{\nu}_1}$ (TeV)	0.88
$m_{\chi_1^\pm}$ (TeV)	0.84	m_A (TeV)	0.64
$m_{\chi_2^\pm}$ (TeV)	1.49	$m_{H^\pm}$ (TeV)	0.65
$m_{\tilde{t}_1}$ (TeV)	1.57	m_H (TeV)	0.65
$m_{\tilde{t}_2}$ (TeV)	1.77	m_h (TeV)	0.122
$m_{\tilde{b}_1}$ (TeV)	1.54		

TABLE II

A representative example of the predictions of the s-spectrum for the finite model **B** with $M = 1$ TeV and $m_{\mathbf{10}} = 0.65$ TeV.

$m_\chi = m_{\chi_1}$ (TeV)	0.45	$m_{\tilde{b}_2}$ (TeV)	1.70
m_{χ_2} (TeV)	0.84	$m_{\tilde{\tau}} = m_{\tilde{\tau}_1}$ (TeV)	0.47
m_{χ_3} (TeV)	1.30	$m_{\tilde{\tau}_2}$ (TeV)	0.67
m_{χ_4} (TeV)	1.31	$m_{\tilde{\nu}_1}$ (TeV)	0.88
$m_{\chi_1^\pm}$ (TeV)	0.84	m_A (TeV)	0.73
$m_{\chi_2^\pm}$ (TeV)	1.31	$m_{H^\pm}$ (TeV)	0.73
$m_{\tilde{t}_1}$ (TeV)	1.51	m_H (TeV)	0.73
$m_{\tilde{t}_2}$ (TeV)	1.73	m_h (TeV)	0.118
$m_{\tilde{b}_1}$ (TeV)	1.56		

Finally, we calculate BR($b \to s\gamma$) [30], whose experimental value is $1 \times 10^{-4} < \mathrm{BR}(b \to s\gamma) < 4 \times 10^{-4}$. The SM predicts $\mathrm{BR}(b \to s\gamma) = 3.1 \times 10^{-4}$. This imposes a further restriction in our parameter space, namely $M \sim 1$ TeV if $\mu < 0$ for all three models. This restriction is less strong in the case that $\mu > 0$. For example, the minimal model with $M = 1$ TeV leads to $\mathrm{BR}(b \to s\gamma) = 3.8 \times 10^{-4}$ for $\mu < 0$.

TABLE III

A representative example of the predictions of the s-spectrum for the minimal SU(5) model with $M = 1.0$ TeV.

$m_\chi = m_{\chi_1}$ (TeV)	0.45	$m_{\tilde{b}_2}$ (TeV)	1.88
m_{χ_2} (TeV)	0.84	$m_{\tilde{\tau}} = m_{\tilde{\tau}_1}$ (TeV)	0.92
m_{χ_3} (TeV)	1.73	$m_{\tilde{\tau}_2}$ (TeV)	1.10
m_{χ_4} (TeV)	1.73	$m_{\tilde{\nu}_1}$ (TeV)	1.43
$m_{\chi_1^\pm}$ (TeV)	0.84	m_A (TeV)	0.70
$m_{\chi_2^\pm}$ (TeV)	1.73	$m_{H^\pm}$ (TeV)	0.70
$m_{\tilde{t}_1}$ (TeV)	1.69	m_H (TeV)	0.70
$m_{\tilde{t}_2}$ (TeV)	1.89	m_h (TeV)	0.120
$m_{\tilde{b}_1}$ (TeV)	1.70		

6. Conclusions

The programme of searching for exact RGI relations among dimensionless couplings in supersymmetric GUTs, started few years ago, has now supplemented with the derivation of similar relations involving dimensionful parameters in the SSB sector of these theories. In the earlier attempts it was possible to derive RGI relations among gauge and Yukawa couplings of supersymmetric GUTs, which could lead even to all-loop finiteness under certain conditions. These theoretically attractive theories have been shown not only to be realistic but also to lead to a successful prediction of the top quark mass. The new theoretical developments include the existence of a RGI sum rule for the soft scalar masses in the SSB sector of $N = 1$ supersymmetric gauge theories exhibiting gauge-Yukawa unification. The all-loop sum rule substitutes now the universal soft scalar masses and overcomes its phenomenological problems. Of particular theoretical interest is the fact that the finite unified theories, which could be made all-loop finite in the supersymmetric sector can now be made completely *finite*. In addition it is interesting to note that the sum rule coincides with that of a certain class of string models in which the massive string modes are organized into $N = 4$ supermultiplets. Last but not least in Ref. [18], the exact β-function for the soft scalar masses in the NSVZ scheme was obtained for the first time. On the other hand the above theories have a remarkable predictive power leading to testable predictions of their spectrum in terms of very few parameters. In addition to the prediction of the top quark mass, which holds unchanged the characteristic features that will judge the viability of these models in the future are 1) the lightest Higgs mass is found to be around 120 GeV and the s-spectrum starts beyond several hundreds of GeV. Therefore the next important test of Gauge-Yukawa and Finite Unified theories will be given with the measurement of the Higgs mass, for which these models show an appreciable stability, which is alarmingly close to the IR quasi fixed point prediction of the MSSM for large $\tan\beta$ [31].

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