ELECTROWEAK SYMMETRY BREAKING AND THE HIGGS SECTOR*

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These three lectures review the state of our understanding of electroweak interactions and the search for the agent of electroweak symmetry breaking. The themes of the lectures are *(i)* the electroweak theory and its experimental status, *(ii)* the standard-model Higgs boson, and *(iii)* aspects of electroweak theory beyond the standard $SU(2)_L \otimes U(1)_Y$ model.

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1. Introduction

The central challenge in particle physics is to explore the 1-TeV scale and elucidate the nature of electroweak symmetry breaking. A key element in this quest is the search for the Higgs boson, the agent of electroweak symmetry breaking in the standard electroweak theory.

Uncovering the secrets of the Higgs sector is the focus of much present and future experimental research. The LEP 2 experiments are searching now for a light Higgs boson and for low-scale supersymmetry. At the Tevatron, CDF and DØ will begin next year a high-luminosity run with considerable sensitivity to new physics, and offer promise for decisive light-Higgs searches in the future. The Large Hadron Collider at CERN will bring extensive explorations of TeV-scale physics beginning in about 2005. Linear colliders now on the drawing boards would offer complementary possibilities for the study of electroweak symmetry breaking.

These three lectures offer a short course in the current state of electroweak symmetry breaking and the Higgs sector. The subject is vast, so

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many important topics will receive only a schematic treatment. Complementary views of the electroweak panorama are to be found in other recent lecture notes [1–7].

2. The electroweak theory

2.1. Brief résumé and perspective

Let us review the essential elements of the $\mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ electroweak theory [8–10]. The electroweak theory takes three crucial clues from experiment:

- The existence of left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_{\mathrm{L}} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_{\mathrm{L}} \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_{\mathrm{L}}$$

  and

$$\begin{pmatrix} u \\ d' \end{pmatrix}_{\mathrm{L}} \qquad \begin{pmatrix} c \\ s' \end{pmatrix}_{\mathrm{L}} \qquad \begin{pmatrix} t \\ b' \end{pmatrix}_{\mathrm{L}} ;$$

- The universal strength of the weak interactions;
- The idealization that neutrinos are massless.

To save writing, we shall construct the electroweak theory as it applies to a single generation of leptons. In this form, it is neither complete nor consistent: anomaly cancellation requires that a doublet of color-triplet quarks accompany each doublet of color-singlet leptons. However, the needed generalizations are simple enough to make that we need not write them out.

To incorporate electromagnetism into a theory of the weak interactions, we add to the $\mathrm{SU}(2)_{\mathrm{L}}$ family symmetry suggested by the first two experimental clues a $\mathrm{U}(1)_Y$ weak-hypercharge phase symmetry. We begin by specifying the fermions: a left-handed weak isospin doublet

$$\mathsf{L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_{\mathrm{L}} \tag{2.1}$$

with weak hypercharge $Y_{\mathrm{L}} = -1$, and a right-handed weak isospin singlet

$$\mathsf{R} \equiv e_{\mathrm{R}} \tag{2.2}$$

with weak hypercharge $Y_{\mathrm{R}} = -2$.

The electroweak gauge group, $\mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$, implies two sets of gauge fields: a weak isovector $\vec{b}_\mu$, with coupling constant g, and a weak isoscalar

$\mathcal{A}_\mu$, with coupling constant g'. Corresponding to these gauge fields are the field-strength tensors

$$F^\ell_{\mu\nu} = \partial_\nu b^\ell_\mu - \partial_\mu b^\ell_\nu + g\varepsilon_{jk\ell} b^j_\mu b^k_\nu \ , \tag{2.3}$$

for the weak-isospin symmetry, and

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu \ , \tag{2.4}$$

for the weak-hypercharge symmetry. We may summarize the interactions by the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}} \ , \tag{2.5}$$

with

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^\ell_{\mu\nu} F^{\ell\mu\nu} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}, \tag{2.6}$$

and

$$\begin{aligned} \mathcal{L}_{\text{leptons}} \ &= \ \overline{\mathsf{R}}\, i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2} \mathcal{A}_\mu Y \right) \mathsf{R} \\ &+ \ \overline{\mathsf{L}}\, i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2} \mathcal{A}_\mu Y + i\frac{g}{2} \vec{\tau} \cdot \vec{b}_\mu \right) \mathsf{L} \, . \end{aligned} \tag{2.7}$$

The $\mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ gauge symmetry forbids a mass term for the electron in the matter piece (2.7). Moreover, the theory we have described contains four massless electroweak gauge bosons, namely $\mathcal{A}_\mu$, b^1_μ, b^2_μ, and b^3_μ, whereas Nature has but one: the photon. To give masses to the gauge bosons and constituent fermions, we must hide the electroweak symmetry.

The most apt analogy for the hiding of the electroweak gauge symmetry is found in superconductivity. In the Ginzburg–Landau description [11] of the superconducting phase transition, a superconducting material is regarded as a collection of two kinds of charge carriers: normal, resistive carriers, and superconducting, resistanceless carriers.

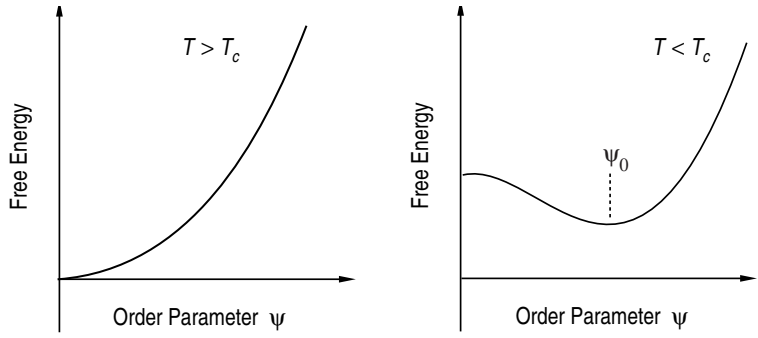
In the absence of a magnetic field, the free energy of the superconductor is related to the free energy in the normal state through

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha \, |\psi|^2 + \beta \, |\psi|^4 \quad , \tag{2.8}$$

where α and β are phenomenological parameters and $|\psi|^2$ is an order parameter that measures the density of superconducting charge carriers. The parameter β is non-negative, so that the free energy is bounded from below.

Above the critical temperature for the onset of superconductivity, the parameter α is positive and the free energy of the substance is supposed to be an increasing function of the density of superconducting carriers, as

shown in figure 1(a). The state of minimum energy, the vacuum state, then corresponds to a purely resistive flow, with no superconducting carriers active. Below the critical temperature, the parameter α becomes negative and the free energy is minimized when $\psi = \psi_0 \neq 0$, as illustrated in Fig. 1(b).



(a) (b)

Fig. 1. Ginzburg–Landau description of the superconducting phase transition.

This is a nice cartoon description of the superconducting phase transition, but there is more. In an applied magnetic field $\vec{H}$, the free energy is

$$G_{\rm super}(\vec{H}) = G_{\rm super}(0) + \frac{\vec{H}^2}{8\pi} + \frac{1}{2m^\star}|-i\hbar\nabla\psi - (e^\star/c)\vec{A}\psi|^2 \ , \qquad (2.9)$$

where $e^\star$ and $m^\star$ are the charge (-2 units) and effective mass of the superconducting carriers. In a weak, slowly varying field $\vec{H} \approx 0$, when we can approximate $\psi \approx \psi_0$ and $\nabla\psi \approx 0$, the usual variational analysis leads to the equation of motion,

$$\nabla^2\vec{A} - \frac{4\pi e^\star}{m^\star c^2}|\psi_0|^2\,\vec{A} = 0 \ , \qquad (2.10)$$

the wave equation of a massive photon. In other words, the photon acquires a mass within the superconductor. This is the origin of the Meissner effect, the exclusion of a magnetic field from a superconductor. More to the point for our purposes, it shows how a symmetry-hiding phase transition can lead to a massive gauge boson.

To give masses to the intermediate bosons of the weak interaction, we take advantage of a relativistic generalization of the Ginzburg–Landau phase transition known as the Higgs mechanism [12]. We introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad (2.11)$$

with weak hypercharge $Y_\phi = +1$. Next, we add to the Lagrangian new (gauge-invariant) terms for the interaction and propagation of the scalars,

$$\mathcal{L}_{\rm scalar} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi), \tag{2.12}$$

where the gauge-covariant derivative is

$$\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_\mu\,, \tag{2.13}$$

and the potential interaction has the form

$$V(\phi^\dagger\phi) = \mu^2(\phi^\dagger\phi) + |\lambda|\,(\phi^\dagger\phi)^2. \tag{2.14}$$

We are also free to add a Yukawa interaction between the scalar fields and the leptons,

$$\mathcal{L}_{\rm Yukawa} = -G_e\left[\overline{\mathsf{R}}(\phi^\dagger \mathsf{L}) + (\overline{\mathsf{L}}\phi)\mathsf{R}\right]. \tag{2.15}$$

We then arrange their self-interactions so that the vacuum state corresponds to a broken-symmetry solution. The electroweak symmetry is spontaneously broken if the parameter $\mu^2 < 0$. The minimum energy, or vacuum state, may then be chosen to correspond to the vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \tag{2.16}$$

where the numerical value of

$$v = \sqrt{-\mu^2/\,|\lambda|} = \left(G_F\sqrt{2}\right)^{-\frac{1}{2}} \approx 246\ \text{GeV} \tag{2.17}$$

is fixed by the low-energy phenomenology of charged-current interactions.

As a result of spontaneous symmetry breaking, the weak bosons acquire masses, as auxiliary scalars assume the role of the third (longitudinal) degrees of freedom of what had been massless gauge bosons. Specifically, the mediator of the charged-current weak interaction, $W^\pm = (b_1 \mp ib_2)/\sqrt{2}$, acquires a mass characterized by $M_W^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W$, where θ_W is the weak mixing angle. The mediator of the neutral-current weak interaction, $Z = b_3\cos\theta_W - \mathcal{A}\sin\theta_W$, acquires a mass characterized by $M_Z^2 = M_W^2/\cos^2\theta_W$. After spontaneous symmetry breaking, there remains an unbroken $\mathrm{U}(1)_{\rm em}$ phase symmetry, so that electromagnetism is mediated by a massless photon, $A = \mathcal{A}\cos\theta_W + b_3\sin\theta_W$, coupled to the electric charge $e = gg'/\sqrt{g^2+g'^2}$. As a vestige of the spontaneous breaking of the symmetry, there remains a massive, spin-zero particle, the Higgs boson. The mass of the Higgs scalar is given symbolically as $M_H^2 = -2\mu^2 > 0$, but we have no

prediction for its value. Though what we take to be the work of the Higgs boson is all around us, the Higgs particle itself has not yet been observed.

The fermions (the electron in our abbreviated treatment) acquire masses as well; these are determined not only by the scale of electroweak symmetry breaking, v, but also by their Yukawa interactions with the scalars. The mass of the electron is set by the dimensionless coupling constant $G_e = m_e\sqrt{2}/v \approx 3 \times 10^{-6}$, which is both small and — so far as we now know — arbitrary.

2.2. Experimental update

It will be helpful for orientation to recall some of the recent precision electroweak measurements as presented at the DPF99 Conference in Los Angeles [13,14]. We will go looking for trouble in §2.4 below, but the overall assessment is that electroweak observables are in accord with the predictions of the standard model at the level of 0.1% [15,16]. The degree of agreement is summarized pictorially in figure 2 (*cf.* Table 1 of Ref. [16]). Taken together, the Z^0-pole data from the LEP experiments and SLD yield a weak mixing parameter

$$\sin^2\theta_W^{\rm eff} = 0.23128 \pm 0.00022. \tag{2.18}$$

Direct measurements at LEP 2 and the Tevatron give the W-boson mass

$$M_W = (80.39 \pm 0.06)\ \text{GeV}/c^2\ , \tag{2.19}$$

while the "world-average" top-quark mass from CDF and DØ is

$$m_t = (174.3 \pm 5.1)\ \text{GeV}/c^2\ . \tag{2.20}$$

The NuTeV experiment at Fermilab has reported a competitive indirect determination of the W mass, inferred from measurements of the $\nu_\mu N$ and $\bar\nu_\mu N$ cross sections. They find

$$M_W = (80.26 \pm 0.11)\ \text{GeV}/c^2\ . \tag{2.21}$$

Thanks to a new evaluation of the finite part of the $\mathcal{O}(\alpha^2)$ correction to the muon lifetime [17], we have a new determination of the Fermi constant measured in muon decay,

$$G_\mu = (1.16637 \pm 0.00001) \times 10^{-5}\ \text{GeV}^{-2}\ . \tag{2.22}$$

Bennett and Wieman (Boulder) have reported a new determination of the weak charge of Cesium by measuring the transition polarizability for the 6S-7S transition [18]. The new value,

$$Q_W(\text{Cs}) = -72.06 \pm 0.28\ (\text{exp}) \pm 0.34\ (\text{theory}), \tag{2.23}$$

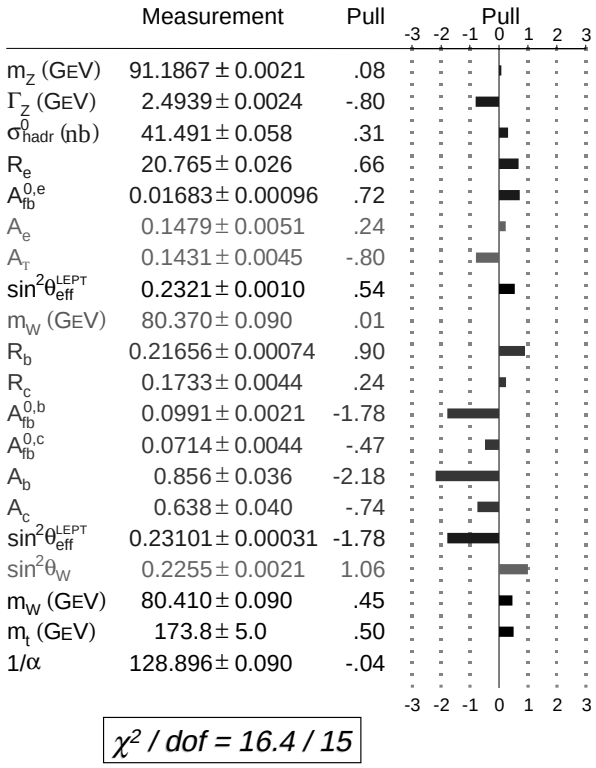


Fig. 2. Precision electroweak measurements and the pulls they exert on a global fit to the standard model, from Ref. [15].

represents a seven-fold improvement in the experimental error and a significant reduction in the theoretical uncertainty. It differs by 2.5 standard deviations from the prediction of the standard model. We are left with the traditional situation in which elegant measurements of parity nonconservation in atoms are on the edge of incompatibility with the standard model.

From the wealth of particle searches and cross-section measurements at LEP 2, let us simply remark that no anomalies whatever have been noted in the reactions

$$e^+e^- \to \left\{ \begin{array}{c} W^+W^- \\ Z^0Z^0 \\ \ell^+\ell^- \\ q\bar{q} \end{array} \right. \quad . \tag{2.24}$$

Similarly, the overall conclusion from HERA is that the neutral-current and charged-current cross sections measured in e^+p collisions have the expected character and reproduce the known values of M_W and M_Z.

2.3. Experimental clues about M_H

The success of the electroweak theory means that it makes sense to use standard-model fits to the electroweak observables to determine "best" values for the parameters that are not yet directly constrained by experiment. Over the past decade, the greatest sensitivity has been to the value of the top-quark mass, and fits to the electroweak observables gave early indications for the great mass of the top quark [19]. Now that the top-quark mass is known rather well from Tevatron experiments, we can interrogate the quantum corrections to electroweak observables for the best value of the Higgs-boson mass. In detail, the inferences depend upon the data set selected and the values adopted for the "known" parameters, including the value of the fine structure constant $\alpha(M_Z^2)$ evaluated at the Z^0 pole. The consensus of the fits is that, *within the standard electroweak theory,* the Higgs boson may be just around the corner. In the global fit of Erler and Langacker [16], which is representative of other work, the best-fit value for the mass of the standard-model Higgs boson is

$$M_H = 107^{+67}_{-45}\ \mathrm{GeV}/c^2\ , \tag{2.25}$$

and the 95% CL upper limit is $M_H \lesssim 255\ \mathrm{GeV}/c^2$. A very interesting question is, how are these constraints relaxed in specific theories other than the standard model?

2.4. Some experimental issues

Suppose, in the face of the spectacular successes of the electroweak theory, we go looking for trouble. Where might we find it? The heavy top quark gives rise to the theoretical suspicion that anomalies are most likely to show themselves in the third generation of quarks and leptons. As it happens, the only suggestive anomaly in precision measurements on the Z^0 pole involves b quarks. The forward-backward asymmetry for $b\bar{b}$ events measured at LEP and the left-right forward-backward asymmetry for $b\bar{b}$ events measured at SLD indicate a three-standard-deviation difference from the standard model for

$$A_b = \frac{L_b^2 - R_b^2}{L_b^2 + R_b^2}\ , \tag{2.26}$$

where L_b and R_b are the left-handed and right-handed chiral couplings of the Z to b quarks. At tree level in the standard model, they take the values

$$\begin{aligned} L_b^{\mathrm{theory}} &= -1 + \tfrac{2}{3}\sin^2\theta_W \approx -0.846\ , \\ R_b^{\mathrm{theory}} &= \tfrac{2}{3}\sin^2\theta_W \approx 0.154\ , \end{aligned} \tag{2.27}$$

Current measurements imply that

$$A_b^{\rm exp} = (0.94 \pm 0.02) A_b^{\rm theory} \quad . \tag{2.28}$$

We must reconcile this apparent discrepancy with the good agreement between the quantity $R_b = \Gamma(Z^0 \to b\bar{b})/\Gamma(Z^0 \to \text{hadrons})$, which is sensitive to the combination $L_b^2 + R_b^2$. The current data say that

$$R_b^{\rm exp} = (1.004 \pm 0.004) R_b^{\rm theory} \ , \tag{2.29}$$

which implies that

$$L_b^2 + R_b^2 = 0.7432 \pm 0.0040 \ . \tag{2.30}$$

We can solve (2.28) and (2.30) simultaneously; choosing the appropriate signs, we find

$$\begin{aligned} L_b^{\rm exp} &= -0.836 \pm 0.004 \ , \\ R_b^{\rm exp} &= 0.2117 \pm 0.0176 \ . \end{aligned} \tag{2.31}$$

Expressed as deviations from the standard model, we have

$$\begin{aligned} \delta L_b &\equiv L_b^{\rm theory} - L_b^{\rm exp} = -0.010 \pm 0.004 \ , \\ \delta R_b &\equiv R_b^{\rm theory} - R_b^{\rm exp} = -0.0577 \pm 0.0176 \ . \end{aligned} \tag{2.32}$$

Neither effect is titanic! However, the suggestion that $\delta R_b / R_b^{\rm theory} \approx -40\%$ (whereas $\delta L_b / L_b^{\rm theory} \approx 1\%$) can be taken as an indication that if we want to look for trouble, the right-handed b coupling is the place to look. If this anomaly is real, we might expect to observe flavor-changing neutral-current transitions $b \to s$, $b \to d$, and $s \to d$.

2.5. An assessment

Experiments over the past twenty-five years have brought us numerous confirmations of the $\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$ electroweak theory: the existence of neutral currents, the necessity of charm, and the existence and properties of the weak gauge bosons $W^\pm$ and Z^0. Experiment has also given essential guidance to the form of the evolving standard model through the discovery of the third generation of leptons (ν_τ, τ) and quarks (t, b). And, finally, experiment has given us a number of significant surprises that have shaped both experimental and theoretical opportunities: the narrowness of J/ψ and ψ', the unexpectedly long B lifetime, the large degree of B^0–$\bar{B}^0$ mixing, the extreme heaviness of the top quark, and — very likely — evidence of neutrino oscillations.

Ten years of precision measurements have found no significant deviations from the predictions of the electroweak theory. A series of quite remarkable experiments, not to mention the accompanying evolution in theorectical calculations, have tested the quantum corrections of the electroweak theory — loop effects — to a precision of one per mil. The net result of this prodigious effort is that we have no found no evidence for new physics ... yet.

It is remarkable that the resulting theory has been tested at distances ranging from about 10^{-17} cm to about 4×10^{20} cm, especially when we consider that classical electrodynamics has its roots in the tabletop experiments that gave us Coulomb's law. These basic ideas were modified in response to the quantum effects observed in atomic experiments. High-energy physics experiments both inspired and tested the unification of weak and electromagnetic interactions. At distances longer than common experience, electrodynamics — in the form of the statement that the photon is massless — has been tested in measurements of the magnetic fields of the planets. With additional assumptions, the observed stability of the Magellanic clouds provides evidence that the photon is massless over distances of about 10^{22} cm [20].

The extraordinary success of the electroweak theory leaves us with these urgent questions: Is the electroweak theory true? Can it be complete?

3. The Standard-Model Higgs boson

3.1. Why the Higgs boson must exist

How can we be sure that a Higgs boson, or something very like it, will be found? One path to the *theoretical* discovery of the Higgs boson involves its role in the cancellation of high-energy divergences. An illuminating example is provided by the reaction

$$e^+e^- \to W^+W^-, \tag{3.1}$$

which is described in lowest order by the four Feynman graphs in figure 3. The contributions of the direct-channel γ- and Z^0-exchange diagrams of Figs. 3(a) and (b) cancel the leading divergence in the $J = 1$ partial-wave amplitude of the neutrino-exchange diagram in figure 3(c). This is the famous "gauge cancellation" observed in experiments at LEP 2 and the Tevatron.

However, the $J = 0$ partial-wave amplitude, which exists in this case because the electrons are massive and may therefore be found in the "wrong" helicity state, grows as $s^{1/2}$ for the production of longitudinally polarized gauge bosons. The resulting divergence is precisely cancelled by the Higgs boson graph of figure 3(d). If the Higgs boson did not exist, something else

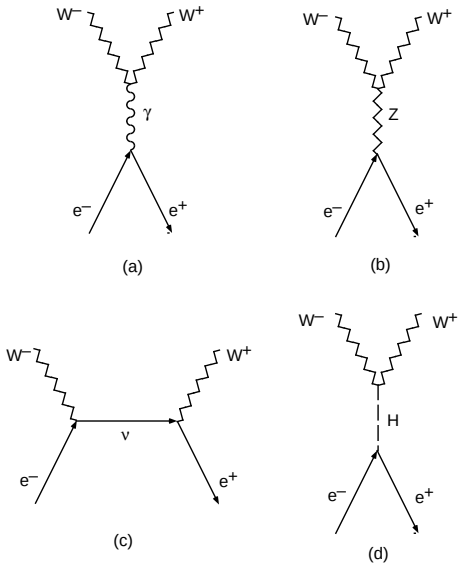


Fig. 3. Lowest-order contributions to the $e^+e^- \to W^+W^-$ scattering amplitude.

would have to play this role. From the point of view of S-matrix analysis, the Higgs-electron-electron coupling must be proportional to the electron mass, because "wrong-helicity" amplitudes are always proportional to the fermion mass.

Let us underline this result. If the gauge symmetry were unbroken, there would be no Higgs boson, no longitudinal gauge bosons, and no extreme divergence difficulties. But there would be no viable low-energy phenomenology of the weak interactions. The most severe divergences of individual diagrams are eliminated by the gauge structure of the couplings among gauge bosons and leptons. A lesser, but still potentially fatal, divergence arises because the electron has acquired mass — because of the Higgs mechanism. Spontaneous symmetry breaking provides its own cure by supplying a Higgs boson to remove the last divergence. A similar interplay and compensation must exist in any satisfactory theory.

3.2. Bounds on M_H

The Standard Model does not give a precise prediction for the mass of the Higgs boson. We can, however, use arguments of self-consistency to place plausible lower and upper bounds on the mass of the Higgs particle in the minimal model. Unitarity arguments [21] lead to a conditional upper bound on the Higgs boson mass. It is straightforward to compute the amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, and to make a partial-wave

decomposition, according to

$$\mathcal{M}(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta) \; . \tag{3.2}$$

Most channels "decouple," in the sense that partial-wave amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies), for any value of the Higgs boson mass M_H. Four channels are interesting:

$$W_{\rm L}^+ W_{\rm L}^- \quad Z_{\rm L}^0 Z_{\rm L}^0/\sqrt{2} \quad HH/\sqrt{2} \quad H Z_{\rm L}^0 \, , \tag{3.3}$$

where the subscript L denotes the longitudinal polarization states, and the factors of $\sqrt{2}$ account for identical particle statistics. For these, the s-wave amplitudes are all asymptotically constant (*i.e.*, well-behaved) and proportional to $G_F M_H^2$. In the high-energy limit[1],

$$\lim_{s \gg M_H^2} (a_0) \to \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix} \; . \tag{3.4}$$

Requiring that the largest eigenvalue respect the partial-wave unitarity condition $|a_0| \leq 1$ yields

$$M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F} \right)^{1/2} = 1 \text{ TeV}/c^2 \tag{3.5}$$

as a condition for perturbative unitarity.

If the bound is respected, weak interactions remain weak at all energies, and perturbation theory is everywhere reliable. If the bound is violated, perturbation theory breaks down, and weak interactions among $W^\pm$, Z, and H become strong on the 1-TeV scale. This means that the features of strong interactions at GeV energies will come to characterize electroweak gauge boson interactions at TeV energies. We interpret this to mean that new phenomena are to be found in the electroweak interactions at energies not much larger than 1 TeV.

[1] It is convenient to calculate these amplitudes by means of the Goldstone-boson equivalence theorem [22], which reduces the dynamics of longitudinally polarized gauge bosons to a scalar field theory with interaction Lagrangian given by $\mathcal{L}_{\rm int} = -\lambda v h (2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2$, with $1/v^2 = G_F\sqrt{2}$ and $\lambda = G_F M_H^2/\sqrt{2}$.

It is worthwhile to note in passing that the threshold behavior of the partial-wave amplitudes for gauge-boson scattering follows generally from chiral symmetry [23]. The partial-wave amplitudes a_{IJ} of definite isospin I and angular momentum J are given by

$$\begin{aligned} a_{00} &\approx \quad G_F s/8\pi\sqrt{2} \quad \text{attractive,} \\ a_{11} &\approx \quad G_F s/48\pi\sqrt{2} \quad \text{attractive,} \\ a_{20} &\approx -G_F s/16\pi\sqrt{2} \quad \text{repulsive.} \end{aligned} \tag{3.6}$$

The electroweak theory itself provides another reason to expect that discoveries will not end with the Higgs boson. Scalar field theories make sense on all energy scales only if they are noninteracting, or "trivial" [24]. The vacuum of quantum field theory is a dielectric medium that screens charge. Accordingly, the effective charge is a function of the distance or, equivalently, of the energy scale. This is the famous phenomenon of the running coupling constant.

In $\lambda\phi^4$ theory (compare the interaction term in the Higgs potential), it is easy to calculate the variation of the coupling constant λ in perturbation theory by summing bubble graphs like this one:

. (3.7)

The coupling constant $\lambda(\mu)$ on a physical scale μ is related to the coupling constant on a higher scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2}\log\left(\Lambda/\mu\right) \ . \tag{3.8}$$

This perturbation-theory result is reliable only when λ is small, but lattice field theory allows us to treat the strong-coupling regime.

In order for the Higgs potential to be stable (*i.e.*, for the energy of the vacuum state not to race off to $-\infty$), $\lambda(\Lambda)$ must not be negative. Therefore we can rewrite (3.8) as an inequality,

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2}\log\left(\Lambda/\mu\right) \ . \tag{3.9}$$

This gives us an *upper bound*,

$$\lambda(\mu) \leq 2\pi^2/3\log\left(\Lambda/\mu\right) \ , \tag{3.10}$$

on the coupling strength at the physical scale μ. If we require the theory to make sense to arbitrarily high energies — or short distances — then we must take the limit $\Lambda \to \infty$ while holding μ fixed at some reasonable physical scale. In this limit, the bound (3.10) forces $\lambda(\mu)$ to zero. The scalar field theory has become free field theory; in theorist's jargon, it is trivial.

We can rewrite the inequality (3.10) as a bound on the Higgs-boson mass. Rearranging and exponentiating both sides gives the condition

$$\Lambda \leq \mu \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right) \ . \tag{3.11}$$

Choosing the physical scale as $\mu = M_H$, and remembering that, before quantum corrections,

$$M_H^2 = 2\lambda(M_H)v^2 \ , \tag{3.12}$$

where $v = (G_F\sqrt{2})^{-1/2} \approx 246$ GeV is the vacuum expectation value of the Higgs field times $\sqrt{2}$, we find that

$$\Lambda \leq M_H \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right) \ . \tag{3.13}$$

For any given Higgs-boson mass, there is a maximum energy scale $\Lambda^\star$ at which the theory ceases to make sense. The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies.

This perturbative analysis breaks down when the Higgs-boson mass approaches 1 TeV/c^2 and the interactions become strong. Lattice analyses [25] indicate that, for the theory to describe physics to an accuracy of a few percent up to a few TeV, the mass of the Higgs boson can be no more than about 710 ± 60 GeV/c^2. Another way of putting this result is that, if the elementary Higgs boson takes on the largest mass allowed by perturbative unitarity arguments, the electroweak theory will be living on the brink of instability.

A lower bound is obtained by computing [26] the first quantum corrections to the classical potential (2.14). Requiring that $\langle\phi\rangle_0 \neq 0$ be an absolute minimum of the one-loop potential up to a scale Λ yields the vacuum-stability condition

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2}(2M_W^4 + M_Z^4 - 4m_t^4)\log(\Lambda^2/v^2) \ . \tag{3.14}$$

The upper and lower bounds plotted in figure 4 are the results of full two-loop calculations [27]. There I have also indicated the upper bound on M_H derived from precision electroweak measurements *in the framework of the*

standard electroweak theory. If the Higgs boson is relatively light — which would itself require explanation — then the theory can be self-consistent up to very high energies. If the electroweak theory is to make sense all the way up to a unification scale $\Lambda^\star = 10^{16}$ GeV, then the Higgs-boson mass must lie in the interval $145\ \mathrm{GeV}/c^2 \lesssim M_W \lesssim 170\ \mathrm{GeV}/c^2$ [28].

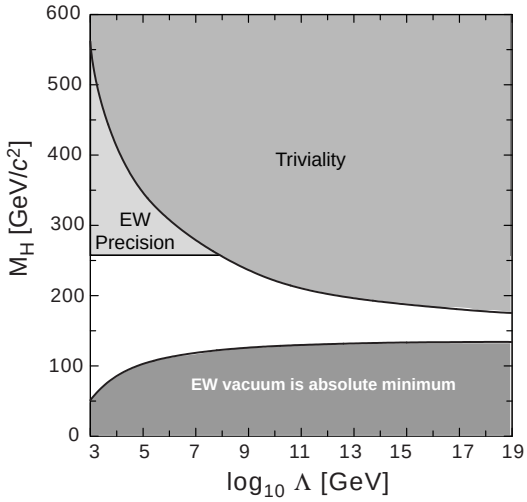


Fig. 4. Bounds on the Higgs-boson mass that follow from requirements that the electroweak theory be consistent up to the energy scale Λ. The upper bound follows from triviality conditions; the lower bound follows from the requirement that $V(v) < V(0)$. Also shown is the range of masses permitted at the 95% confidence level by precision measurements.

3.3. Higgs-boson properties

Once we assume a value for the Higgs-boson mass, it is a simple matter to compute the rates for Higgs-boson decay into pairs of fermions or weak bosons [29]. For a fermion with color N_c, the partial width is

$$\Gamma(H \to f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_\mathrm{c} \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2} , \tag{3.15}$$

which is proportional to M_H in the limit of large Higgs mass. The partial width for decay into a W^+W^- pair is

$$\Gamma(H \to W^+W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}}(1-x)^{1/2}(4 - 4x + 3x^2) , \tag{3.16}$$

where $x \equiv 4M_W^2/M_H^2$. Similarly, the partial width for decay into a pair of Z^0 bosons is

$$\Gamma(H \to Z^0 Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}}(1-x')^{1/2}(4-4x'+3x'^2)\,, \tag{3.17}$$

where $x' \equiv 4M_Z^2/M_H^2$. The rates for decays into weak-boson pairs are asymptotically proportional to M_H^3 and $\frac{1}{2}M_H^3$, respectively, the factor $\frac{1}{2}$ arising from weak isospin. In the final factors of (3.16) and (3.17), $2x^2$ and $2x'^2$, respectively, arise from decays into transversely polarized gauge bosons. The dominant decays for large M_H are into pairs of longitudinally polarized weak bosons.

Branching fractions for decay modes that hold promise for the detection of a light Higgs boson are displayed in figure 5. In addition to the $f\bar{f}$ and VV modes that arise at tree level, I have included the $\gamma\gamma$ mode that proceeds through loop diagrams. Though rare, the $\gamma\gamma$ channel offers an important target for LHC experiments.

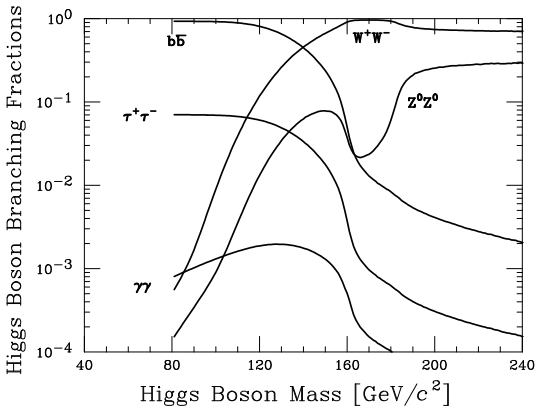


Fig. 5. Branching fractions for the prominent decay modes of a light Higgs boson.

Figure 6 shows the partial widths for the decay of a Higgs boson into the dominant W^+W^- and Z^0Z^0 channels and into $t\bar{t}$, for $m_t = 175$ GeV/c^2. Whether the $t\bar{t}$ mode will be useful to confirm the observation of a heavy Higgs boson, or merely drains probability from the ZZ channel favored for a heavy-Higgs search, is a question for detailed detector simulations.

Below the W^+W^- threshold, the total width of the standard-model Higgs boson is rather small, typically less than 1 GeV. Far above the threshold for decay into gauge-boson pairs, the total width is proportional to M_H^3. At masses approaching 1 TeV/c^2, the Higgs boson is an ephemeron, with a perturbative width approaching its mass. The Higgs-boson total width is plotted as a function of M_H in figure 7.

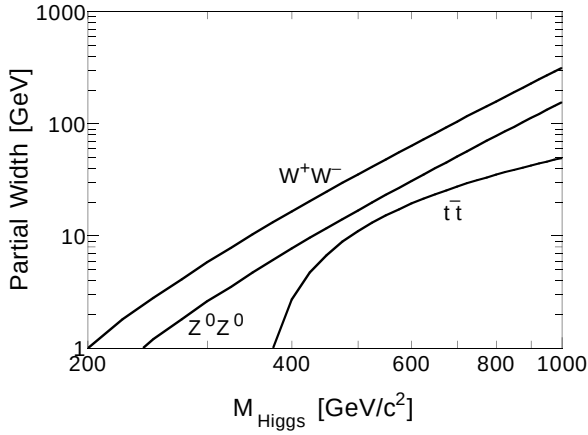


Fig. 6. Partial widths for the prominent decay modes of a heavy Higgs boson.

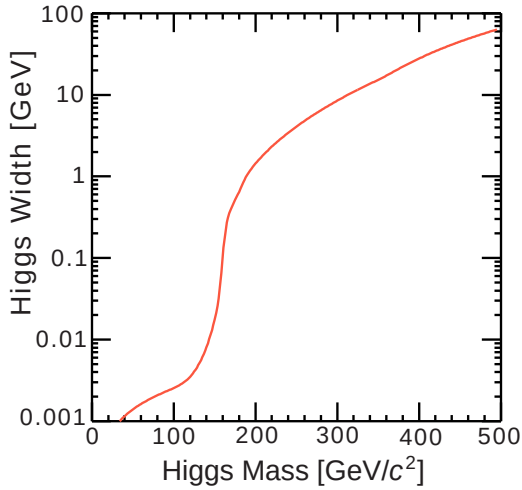


Fig. 7. Higgs-boson total width as a function of mass.

3.4. Higgs-boson searches [30]

3.4.1. e^+e^- collisions at LEP

Because the standard-model Higgs boson couples to fermion mass, the cross section for the reaction $e^+e^- \to H \to$ all is minute ($\propto m_e^2$). With any remotely conceivable luminosity, even the narrowness of a light Higgs boson is not enough to make it visible. This circumstance sets aside a traditional strength of electron-positron colliders: pole physics. Instead, the most promising reaction for Higgs-boson physics at an e^+e^- collider is

associated production,

$$e^+e^- \to HZ\,, \tag{3.18}$$

that corresponds to the Feynman diagram in figure 8, which has no small couplings. The cross section [31],

$$\sigma = \frac{\pi\alpha^2}{8\sqrt{s}} \frac{K(K^2+3M_Z^2)[1+(1-4x_W)^2]}{(s-M_Z^2)^2 \quad x_W^2(1-x_W)^2}\,, \tag{3.19}$$

where K is the c.m. momentum of the Higgs boson and $x_W \equiv \sin^2\theta_W$, approaches about ten percent of $\sigma(e^+e^- \to \mu^+\mu^-)$.

Searching in the observable channels of the reaction (3.18), the four LEP 2 experiments are sensitive nearly to the kinematical boundary

$$M_H^{\max} = \sqrt{s} - M_Z\,. \tag{3.20}$$

Recent running at $\sqrt{s} = 189$ GeV leads to upper limits that lie within a few GeV/c^2 of $M_H^{\max}$ [32]. If the Higgs boson is established at LEP 2, it should be possible to determine its mass within a few hundred MeV/c^2 [33].

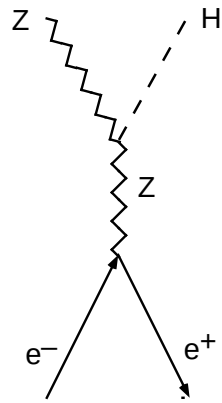


Fig. 8. Lowest-order contributions to the $e^+e^- \to HZ^0$ scattering amplitude.

Well above threshold, the angular distribution of Higgs production is characteristic of the CP character of the Higgs boson. For the CP-even standard-model Higgs boson,

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta} \propto \sin^2\theta\,, \tag{3.21}$$

while for a CP-odd Higgs boson, the production angular distribution is

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta} \propto 1+\cos^2\theta\,. \tag{3.22}$$

The angular distribution will be a powerful diagnostic — once a Higgs boson is observed. For other techniques to determine the parity of a Higgs particle, see [34].

3.4.2. e^+e^- collisions at a linear collider

At e^+e^- linear colliders above the LEP 2 energy scale, the most promising reactions for the production of Higgs bosons are (3.18) plus the gauge-boson–fusion reactions

$$e^+e^- \to \begin{cases} \nu\bar{\nu}H & (W^+W^- \text{ fusion}) \\ e^+e^-H & (Z^0Z^0 \text{ fusion}) \end{cases} \quad . \tag{3.23}$$

The capabilities of linear colliders for the Higgs-boson search (and for a rich variety of other investigations) have been summarized by Murayama and Peskin [35] and in the report of the ECFA/DESY Linear Collider Working Group [36]. Thorough searches and incisive determinations of Higgs-boson couplings are possible. It is plausible that, by measuring the $e^+e^- \to HZ$ excitation curve for a light Higgs boson, M_H could be determined as well as M_W. A typical estimate is

$$\delta M_H \approx 60 \text{ MeV}/c^2 \sqrt{\frac{100 \text{ fb}^{-1}}{\mathcal{L}}} \text{ , for } M_H = 100 \text{ GeV}/c^2 \text{ .} \tag{3.24}$$

By shining high-power, high-repetition-rate, eV-energy lasers onto the e^+ and e^- (or e^- and e^-) beams in a linear collider, it is possible to create a $\gamma\gamma$ collider with definite polarization and useful luminosity. Such an instrument would be well suited to the study of the formation reaction

$$\gamma\gamma \to H \to b\bar{b} \ , \tag{3.25}$$

with a rate proportional to $\Gamma(H \to \gamma\gamma)\Gamma(H \to b\bar{b})/\Gamma(H \to \text{all})$. Knowing the $b\bar{b}$ branching ratio, one therefore has a direct determination of the $H \to \gamma\gamma$ coupling, an important diagnostic for the physics of electroweak symmetry breaking.

3.4.3. A $\mu^+\mu^-$ Higgs factory

In common with the electron, the muon is an elementary lepton at our current limits of resolution. Its energy is not shared among many partons, so the muon is a more efficient delivery vehicle for high energies than is the composite proton. Because the muon is so massive, synchrotron radiation does not represent a barrier to small, high-energy, circular machines — as it does for electrons.

Beyond the suggestion of these practical advantages, muons offer a possibly decisive physics advantage. The great seduction of a First Muon Collider is that the cross section for the reaction $\mu^+\mu^- \to H$, direct-channel formation of the Higgs boson, is larger than the cross section for $e^+e^- \to H$ by a factor $(m_\mu/m_e)^2 \approx 42,750$. This is a very large factor. The tantalizing question is whether it is large enough to make possible a "Higgs factory" with the luminosities that may be achieved in $\mu^+\mu^-$ colliders. In e^+e^- collisions, as we have remarked, the s-channel formation cross section is hopelessly small. That is why the associated-production reaction $e^+e^- \to HZ$ has become the preferred search mode at LEP 2.

The properties of the muon also raise challenges to the construction and exploitation of a $\mu^+\mu^-$ collider. The muon is not free: it doesn't come out of a bottle like the proton or boil off a metal plate like the electron. On the other hand, it is readily produced in the decay $\pi \to \mu\nu$. Still, gathering large numbers of muons in a dense beam is a formidable engineering challenge, and the focus of much of the R&D effort over the next few years. The muon is also not stable, but decays with a lifetime of 2.2 μs into $\mu^- \to e^-\bar{\nu}_e\nu_\mu$. We must act fast to capture, cool, accelerate, and use muons, and must be able to replenish the supply quickly. Multiply 2.2 μs by whatever Lorentz (γ) factor you like for a muon collider, it is still a very short time.

The important possibility that a $\mu^+\mu^-$ collider can operate as a Higgs factory has been studied extensively [37, 38]. If the Higgs boson is light ($M_H \lessapprox 2M_W$), and therefore narrow, then the muon's large mass makes it thinkable that the reactions

$$\mu^+\mu^- \to H \to b\bar{b} \text{ and other modes}$$

will occur with a large rate that will enable a comprehensive study of the properties of the Higgs boson. We assume that a light Higgs boson has been found, and that its mass has been determined with an uncertainty of $\pm(100\text{-}200)$ MeV/c^2 [33]. Then suppose that an optimized machine is built with $\sqrt{s} = M_H$.

The muon's mass confers another important instrumental advantage: the momentum spread of a muon collider is naturally small, and can be made extraordinarily small. The Higgs factory [39] would operate in two modes:

- modest luminosity (0.05 fb^{-1}/year) and high momentum resolution ($\sigma_p/p = 3 \times 10^{-5}$);
- standard luminosity (0.6 fb^{-1}/year) and normal momentum resolution ($\sigma_p/p = 10^{-3}$).

At high resolution, the spread in c.m. energy is comparable to the natural width of a light Higgs boson: $\sigma_{\sqrt{s}} \approx$ a few MeV $\approx \Gamma(H \to \text{ all})$. At normal

resolution, $\sigma_{\sqrt{s}} \gg \Gamma(H \to \text{ all})$. It is worth remarking that the Higgs factory would be small, with a circumference of just 380 meters, and that the number of turns a muon makes in one lifetime is 820.

The first order of business is to run in high-resolution mode to determine the Higgs-boson mass with exquisite precision. The procedure contemplated is to scan a large number of points (determined by $2\Delta M_H/\sigma_{\sqrt{s}} \approx 100$), each with enough integrated luminosity to establish a three-standard-deviation excess. If each point requires an integrated luminosity 0.0015 fb^{-1}, then the scan requires 100×0.0015 fb$^{-1} = 0.15$ fb^{-1}, about three nominal years of running. The reward is that, after the scan, the Higgs-boson mass will be known with an uncertainty of $\Delta M_H \approx \sigma_{\sqrt{s}} \approx 2$ MeV/c^2, which is quite stunning.

Extended running in the form of a three-point scan of the Higgs-boson line at $\sqrt{s} = M_H, M_H \pm \sigma_{\sqrt{s}}$ would then make possible an unparalleled exploration of Higgs-boson properties. With an integrated luminosity of 0.4 fb^{-1} one may contemplate precisions of $\Delta M_H \approx 0.1$ MeV/c^2, $\Delta\Gamma_H \approx 0.5$ MeV $\approx \frac{1}{6}\Gamma_H$, $\Delta(\sigma \cdot B(H \to b\bar{b})) \approx 3\%$, and $\Delta(\sigma \cdot B(H \to WW^\star)) \approx 15\%$.

These are impressive measurements indeed. The width of the putative Higgs boson is an important discriminant for supersymmetry, for it can range from the standard-model value to considerably larger values. Within the minimal supersymmetric extension of the standard model (MSSM), the ratio of the $b\bar{b}$ and $WW^\star$ yields is essentially determined by M_A, the mass of the CP-odd Higgs boson. In the decoupling limit, $M_A \to \infty$, the MSSM reproduces the standard-model ratio. Deviations indicate that A is light. In the most optimistic scenario, this measurement could determine M_A well enough to guide the development of a second (CP-odd) Higgs factory using the reaction $\mu^+\mu^- \to A$.

Again, these remarkable measurements exact a high price. At a luminosity of 0.05 fb^{-1}/year, it takes 8 years to accumulate 0.40 fb^{-1} *after the scan* to determine M_H within machine resolution. It is plain that this program becomes considerably more compelling if the Higgs-factory luminosity can be raised by a factor of 2 or 3 — or more! Let us note finally that the flux of decay electrons challenges the operation of silicon detectors close to the interaction point [40].

3.4.4. $\bar{p}p$ collisions at the Tevatron

The cross sections for Higgs-boson production at the Tevatron are shown in figure 9 [41]. The values — no larger than a few picobarns — highlight the need for large integrated luminosity and favorable branching fractions. At the same time, many processes become accessible once the integrated luminosity exceeds a few fb^{-1}.

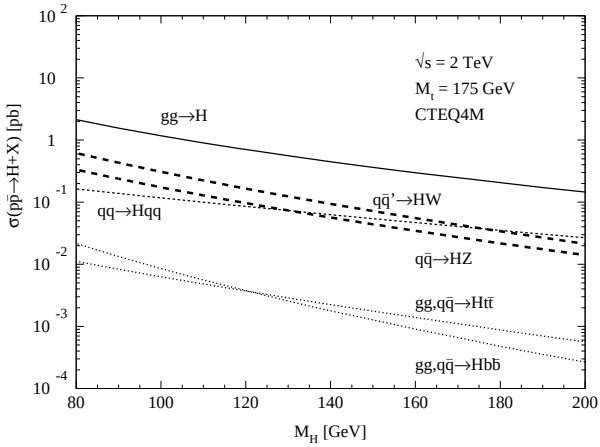


Fig. 9. Cross sections for Higgs-boson production in 2-TeV $\bar{p}p$ collisions, from Ref. [41].

The most promising channel for searches at the Tevatron will be the $b\bar{b}$ mode, for which the branching fraction exceeds about 50% throughout the region preferred by supersymmetry and the precision electroweak data. At the Tevatron, the direct production of a light Higgs boson in gluon-gluon fusion $gg \to H \to b\bar{b}$ is swamped by the ordinary QCD production of $b\bar{b}$ pairs. Even with an integrated luminosity of 30 fb^{-1}, the experiments anticipate only $<$ 1-σ excess, with plausible invariant-mass resolution. It will be possible to calibrate the $b\bar{b}$ mass resolution over the region of the Higgs search in Run II, which aims to accumulate 2 fb^{-1}: the electroweak production of $Z^0 \to b\bar{b}$ should stand well above background and be clearly observable in Run II [42].

The high background in the $b\bar{b}$ channel means that special topologies must be employed to improve the ratio of signal to background and the significance of an observation. The high luminosities that can be contemplated for a future run argue that the associated-production reactions

$$\begin{array}{rcl} \bar{p}p & \to & HW + \text{anything} \\ & & \quad \big\lfloor \to \ell\nu \\ & & \big\lfloor \to b\bar{b} \end{array} \tag{3.26}$$

and

$$\begin{array}{rcl} \bar{p}p & \to & HZ + \text{anything} \\ & & \quad \big\lfloor \to \ell^+\ell^- + \nu\bar{\nu} \\ & & \big\lfloor \to b\bar{b} \end{array} \tag{3.27}$$

are plausible candidates for a Higgs discovery at the Tevatron [43].

The prospects for exploiting these topologies were explored in detail in connection with the Run II Supersymmetry / Higgs Workshop at Fermilab [44]. Taking into account what is known, and what might conservatively be expected, about sensitivity, mass resolutions, and background rejection, these investigations show that it is unlikely that a standard-model Higgs boson could be observed in Tevatron Run II. (Note, however, that the ability to use $W \to q\bar{q}$ decays would markedly increase the sensitivity.) The prospects are much brighter for Run III. Indeed, the sensitivity to a light Higgs boson is what motivates the integrated luminosity of 30 fb^{-1} specified for Run III.

The detection strategy evolved in the Supersymmetry/Higgs Workshop involves combining the HZ and HW signatures of (3.26) and (3.27), and adding the data from the CDF and DØ detectors. Prospects are summarized in figure 10, which shows as a function of the Higgs-boson mass the luminosity required for exclusion at 95% confidence level (dashed line), three-standard-deviation evidence (thin solid line), and five-standard-deviation discovery (thick solid line). We see that an integrated luminosity of 2 fb^{-1}, expected in Run II, is insufficient for a convincing observation of a standard-model Higgs boson with a mass too large to be observed at LEP 2. However, a 95% CL exclusion is possible up to about 125 GeV/c^2. On the other hand, about 10 fb^{-1} would permit detailed study of a standard-model Higgs boson discovered at LEP 2. If the Higgs boson lies beyond the reach of LEP 2, $M_H \gtrsim$(100–105) GeV/c^2, then a 5-σ discovery will be possible in a future Run III of the Tevatron (30 fb^{-1}) for masses up to about (125–130) GeV/c^2. This

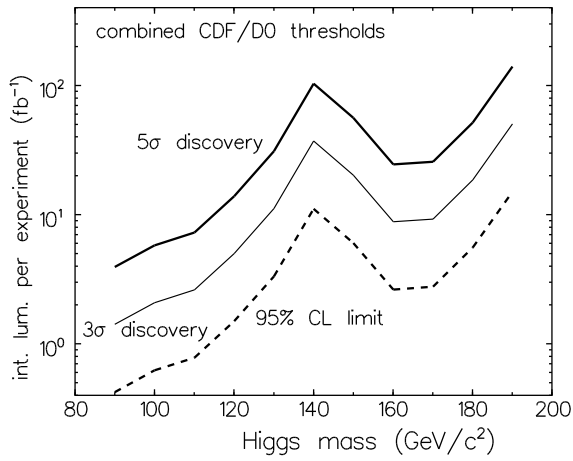


Fig. 10. Integrated luminosity projected for the detection of a standard-model Higgs boson at the Tevatron Collider.

prospect is the most powerful incentive we have for Run III. Over the range of masses accessible in associated production at the Tevatron, it should be possible to determine the mass of the Higgs boson to $\pm(1\text{–}3)$ GeV/c^2.

Recent studies [45] suggest that it may be possible to extend the reach of the Tevatron significantly by making use of the real-W–virtual-W (WW^*) decay modes for Higgs boson produced in the elementary reaction $gg \to H$. As we saw in the discussion leading up to figure 6, the WW^* channel has the largest branching fraction for $M_H \gtrsim 140$ GeV/c^2. According to the analysis summarized in figure 10, the large cross section $\times$ branching fraction of the $gg \to H \to WW^*$ mode extends the 3-σ detection sensitivity of Run III into the region 145 GeV/$c^2 \lesssim M_H \lesssim 180$ GeV/c^2. This is an extremely exciting opportunity, and it is important that the WW^* proposal receive independent critical analysis. For the moment, it appears that the determination of the Higgs-boson mass would have limited precision, perhaps $\delta M_H \approx 30$ GeV/c^2 [46]. This question also requires additional study.

3.4.5. *pp* collisions at the LHC

Many significant advances have informed preparations for experiments at the Large Hadron Collider. These include new or enhanced detector components and improved integration of individual elements into a high-performance detector, refined Monte Carlo tools, the evolution of new techniques for computing multiparton amplitudes, and progress in accelerator technology. The capabilities of the LHC experiments to search for, and study, the Higgs boson are thoroughly documented in the Technical Proposals [47]. I will confine myself here to a few summary comments.

A 5-σ discovery is possible up to $M_H \approx 800$ GeV/c^2 in a combination of the channels

$$\begin{array}{lll} H & \to & Z\,Z \\ & & \quad \hookrightarrow \ell^+\ell^- \\ & & \hookrightarrow \ell^+\ell^-, \end{array} \tag{3.28}$$

$$\begin{array}{ll} H & W \\ & \quad \hookrightarrow b\bar{b} \\ \hookrightarrow \ell\nu & \end{array} \tag{3.29}$$

and

$$H \to \gamma\gamma \text{ or perhaps } \tau^+\tau^-.$$

The reach of LHC experiments can be extended by making use of the channels

$$\begin{array}{lll} H & \to & Z\,Z \\ & & \quad \hookrightarrow \ell^+\ell^- \text{ or } \nu\bar{\nu} \\ & & \hookrightarrow \text{jet jet,} \end{array} \tag{3.30}$$

and

$$\begin{array}{lll} H & \to & W\,W \\ & & \quad \hookrightarrow \ell\nu \\ & & \hookrightarrow \text{jet jet.} \end{array} \tag{3.31}$$

For Higgs-boson masses below about 300 GeV/c^2, it should be possible to determine the Higgs mass to 100-300 MeV/c^2 [33]. For a recent exposition of the prospects for Higgs-boson searches from LEP to the LHC, see the lectures by Dittmar [48].

4. Higgs physics beyond the Standard Model

In this final lecture, I want to review some indications for physics beyond the standard model, and explore some possibilities for the new phenomena we might encounter. To begin, I want to revisit a longstanding, but usually unspoken, challenge to the completeness of the electroweak theory as we have defined it: the vacuum energy problem. I do so not only for its intrinsic interest, but also to raise the question, "Which problems of completeness and consistency do we worry about at a given moment?" It is perfectly acceptable science — indeed, it is often essential — to put certain problems aside, in the expectation that we will return to them at the right moment. What is important is never to forget that the problems are there, even if we do not allow them to paralyze us. Then I will return to the significance of the 1-TeV scale, and move on to brief comments on supersymmetry and technicolor. The final topic of this lecture is the problem of fermion masses, which is undoubtedly linked to the question of electroweak symmetry breaking, but calls for new insights that will go beyond the standard model.

4.1. The vacuum energy problem

For our simple choice (2.14) of the Higgs potential, the value of the potential at the minimum is

$$V(\langle\phi^\dagger\phi\rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda|\, v^4}{4} < 0. \tag{4.1}$$

Identifying $M_H^2 = -2\mu^2$, we see that the Higgs potential contributes a field-independent constant term,

$$\varrho_H \equiv \frac{M_H^2 v^2}{8}. \tag{4.2}$$

I have chosen the notation ϱ_H because the constant term in the Lagrangian plays the role of a vacuum energy density. When we consider gravitation, adding a vacuum energy density $\varrho_{\rm vac}$ is equivalent to adding a cosmological constant term to Einstein's equation. Although recent observations [49] raise the intriguing possibility that the cosmological constant may be different from zero, the essential observational fact is that the vacuum energy density must be very tiny indeed [50],

$$\varrho_{\rm vac} \lesssim 10^{-46}\ {\rm GeV}^4\ . \tag{4.3}$$

Therein lies the puzzle: if we take $v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246$ GeV from (2.17) and insert the current experimental lower bound [32] $M_H \gtrsim 95\ {\rm GeV}/c^2$ into (4.2), we find that the contribution of the Higgs field to the vacuum energy density is

$$\varrho_H \gtrsim 7.6 \times 10^7\ {\rm GeV}^4, \tag{4.4}$$

some 54 orders of magnitude larger than the upper bound inferred from the cosmological constant.

What are we to make of this mismatch, which has been apparent [51–54] for nearly a quarter of a century? The fact that $\varrho_H \gg \varrho_{\rm vac}$ means that the smallness of the cosmological constant needs to be explained. In a unified theory of the strong, weak, and electromagnetic interactions, other (heavy!) Higgs fields have nonzero vacuum expectation values that may give rise to still greater mismatches. At a fundamental level, we can therefore conclude that a spontaneously broken gauge theory of the strong, weak, and electromagnetic interactions — or merely of the electroweak interactions — cannot be complete. Either we must find a separate principle that zeroes the vacuum energy density of the Higgs field, or we may suppose that a proper quantum theory of gravity, in combination with the other interactions, will resolve the puzzle of the cosmological constant. In an interesting paper that prefigured the idea of "large" extra dimensions, van der Bij [55] has argued that because gravity and the Higgs field are both universal, they must be linked, perhaps in a spontaneously broken gravity in which the standard-model Higgs boson is the origin of the Planck mass.

The vacuum energy problem must be an important clue. But to what?

4.2. Why is the electroweak scale small?

In the first two lectures, we have outlined the electroweak theory, emphasized that the need for a Higgs boson (or substitute) is quite general, and reviewed the properties of the standard-model Higgs boson. By considering a thought experiment, gauge-boson scattering at very high energies, we found a first signal for the importance of the 1-TeV scale. Now, let us explore another path to the 1-TeV scale.

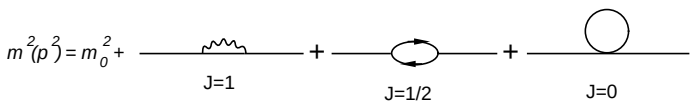
The $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ electroweak theory does not explain how the scale of electroweak symmetry breaking is maintained in the presence of quantum corrections. The problem of the scalar sector can be summarized neatly as follows [56]. The Higgs potential is

$$V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| \, (\phi^\dagger \phi)^2 \; . \tag{4.5}$$

With μ^2 chosen to be less than zero, the electroweak symmetry is spontaneously broken down to the U(1) of electromagnetism, as the scalar field acquires a vacuum expectation value that is fixed by the low-energy phenomenology,

$$\langle \phi \rangle_0 = \sqrt{-\mu^2/2|\lambda|} \equiv (G_F \sqrt{8})^{-1/2} \approx 175 \text{ GeV} \; . \tag{4.6}$$

Beyond the classical approximation, scalar mass parameters receive quantum corrections from loops that contain particles of spins $J = 1, 1/2$, and 0:



(4.7)

The loop integrals are potentially divergent. Symbolically, we may summarize the content of (4.7) as

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \cdots \; , \tag{4.8}$$

where Λ defines a reference scale at which the value of m^2 is known, g is the coupling constant of the theory, and the coefficient C is calculable in any particular theory. Instead of dealing with the relationship between observables and parameters of the Lagrangian, we choose to describe the variation of an observable with the momentum scale. In order for the mass shifts induced by radiative corrections to remain under control (*i.e.*, not to greatly exceed the value measured on the laboratory scale), either Λ must

be small, so the range of integration is not enormous, or new physics must intervene to cut off the integral.

If the fundamental interactions are described by an $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ gauge symmetry, *i.e.*, by quantum chromodynamics and the electroweak theory, then the natural reference scale is the Planck mass,

$$\Lambda \sim M_\mathrm{Planck} = \left(\frac{\hbar c}{G_\mathrm{Newton}}\right)^{1/2} \approx 1.22 \times 10^{19}\ \mathrm{GeV}\ . \tag{4.9}$$

In a unified theory of the strong, weak, and electromagnetic interactions, the natural scale is the unification scale,

$$\Lambda \sim M_U \approx 10^{15}\text{–}10^{16}\ \mathrm{GeV}\ . \tag{4.10}$$

Both estimates are very large compared to the scale of electroweak symmetry breaking (4.6). We are therefore assured that new physics must intervene at an energy of approximately 1 TeV, in order that the shifts in m^2 not be much larger than (4.6).

Only a few distinct scenarios for controlling the contribution of the integral in (4.8) can be envisaged. The supersymmetric solution is especially elegant. Exploiting the fact that fermion loops contribute with an overall minus sign (because of Fermi statistics), supersymmetry balances the contributions of fermion and boson loops. In the limit of unbroken supersymmetry, in which the masses of bosons are degenerate with those of their fermion counterparts, the cancellation is exact:

$$\sum_{i=\substack{\mathrm{fermions}\\+\mathrm{bosons}}} C_i \int dk^2 = 0\ . \tag{4.11}$$

If the supersymmetry is broken (as it must be in our world), the contribution of the integrals may still be acceptably small if the fermion-boson mass splittings ΔM are not too large. The condition that $g^2 \Delta M^2$ be "small enough" leads to the requirement that superpartner masses be less than about 1 TeV/c^2.

A second solution to the problem of the enormous range of integration in (4.8) is offered by theories of dynamical symmetry breaking such as technicolor. In technicolor models, the Higgs boson is composite, and new physics arises on the scale of its binding, $\Lambda_\mathrm{TC} \simeq O(1\ \mathrm{TeV})$. Thus the effective range of integration is cut off, and mass shifts are under control.

A third possibility is that the gauge sector becomes strongly interacting. This would give rise to WW resonances, multiple production of gauge bosons, and other new phenomena at energies of 1 TeV or so. It is likely

that a scalar bound state — a quasi-Higgs boson — would emerge with a mass less than about 1 TeV/c^2 [57].

We cannot avoid the conclusion that some new physics must occur on the 1-TeV scale.

4.3. Supersymmetry

The search for supersymmetry was discussed extensively here in Sierra Nevada by Daniel Treille [58] and Daniel Denegri, so I will restrict myself to a few general remarks about the motivation for supersymmetry on the electroweak scale, and its connection with string theory [59–61].

One of the best phenomenological motivations for supersymmetry on the 1-TeV scale is that the minimal supersymmetric extension of the standard model so closely approximates the standard model itself. A nice illustration of the small differences between predictions of supersymmetric models and the standard model is the compilation of pulls prepared by Erler and Pierce [62], which is shown in figure 11. This is a nontrivial property of new physics beyond the standard model, and a requirement urged on us by the unbroken quantitative success of the established theory. On the aesthetic — or theoretical — side, supersymmetry is the maximal — indeed, unique — extension of Poincaré invariance. It also offers a path to the incorporation of gravity, since local supersymmetry leads directly to supergravity. As a practical matter, supersymmetry on the 1-TeV scale offers a solution to the naturalness problem, and allows a fundamental scalar to exist at low energies.

When we combine supersymmetry with unification of the fundamental forces, we obtain a satisfactory prediction for the weak mixing parameter, $\sin^2\theta_W$, and a simple picture of coupling-constant unification [63]. Adding an assumption of universality, we are led naturally to a picture in which the top mass is linked with the electroweak scale, so that $m_t \approx v/\sqrt{2}$. Finally, the assumption of R-parity leads to a stable lightest supersymmetric particle, which is a natural candidate for the dark matter of the Universe.

Supersymmetry doubles the spectrum of fundamental particles. We know that supersymmetry must be significantly broken in Nature, because the electron is manifestly not degenerate in mass with its scalar partner, the selectron. It is interesting to contemplate just how different the world would have been if the selectron, not the electron, were the lightest charged particle and therefore the stable basis of everyday matter [64]. If atoms were selectronic, there would be no Pauli principle to dictate the integrity of molecules. As Dyson [65] and Lieb [66] demonstrated, transforming electrons and nucleons from fermions to bosons would cause all molecules to shrink into an insatiable undifferentiated blob. Luckily, there is no analogue

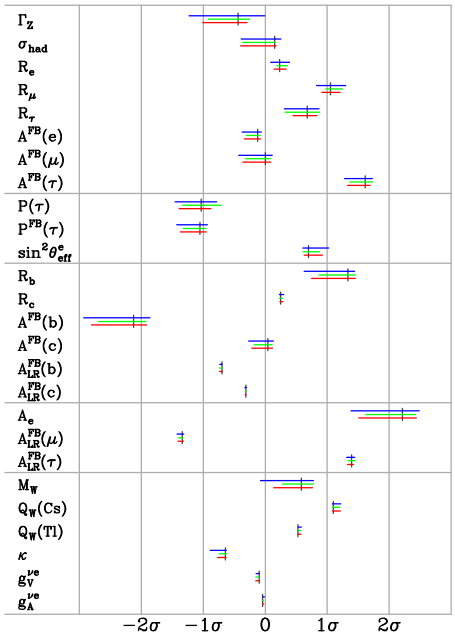


Fig. 11. The range of best fit predictions of precision observables in the supergravity model (upper horizontal lines), the $\mathbf{5} \oplus \mathbf{5}^*$ gauge-mediated model (middle lines), the $\mathbf{10} \oplus \mathbf{10}^*$ gauge-mediated model (lower lines), and in the standard model at its global best fit value (vertical lines), in units of standard deviation, from Ref. [62].

of chiral symmetry to guarantee naturally small squark and slepton masses. So while supersymmetry menaces us with an amorphous death, it is likely that a full understanding of supersymmetry will enable us to explain why we live in a universe ruled by the exclusion principle.

Many theorists take a step beyond supersymmetry to string theory, the only known consistent theory of quantum gravity [67, 68]. String theory aspires to unite all the fundamental interactions in one (and only one?) theory with few free parameters. If successful, this program might explain the standard-model gauge group, unified extensions to the $\mathrm{SU(3)_C} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$ gauge symmetry, and the fermion content of the standard model. The defining ambition of string theory is to reconcile quantum mechanics and the implications of the uncertainty principle with general relativity and its guiding notion of a smooth spacetime.

String theory makes several generic predictions for physics beyond the standard model: additional U(1) subgroups of the unifying group lead to new gauge bosons, and additional colored fermions augment the spectrum of fundamental constituents. It also requires general relativity and, for con-

sistency, extra spacetime dimensions, some of which might be detectably large [69–72]. And string theory requires supersymmetry, though not necessarily on the 1-TeV scale.

In spite of what doubters often say, there *is* experimental support for string theory from accelerator experiments. Superstrings predicted gravity in 1974 [73], and LEP accelerator physicists detected tidal forces in 1993 [74]. What more empirical evidence could one demand?

In a supersymmetric theory, two Higgs doublets are required to give masses to fermions with weak isospin $I_3 = \frac{1}{2}$ and $I_3 = -\frac{1}{2}$. Let us designate the two doublets as Φ_1 and Φ_2. Before supersymmetry is broken, the scalar potential has the form

$$V = \mu^2(\Phi_1^2 + \Phi_2^2) + \frac{g^2 + g'^2}{8}(\Phi_1^2 + \Phi_2^2)^2 + \frac{g^2}{2}\left|\Phi_1^* \cdot \Phi_2\right|^2 \ . \tag{4.12}$$

By adding all possible soft supersymmetry-breaking terms, we raise the possibility that the electroweak symmetry will be broken. We choose

$$\begin{aligned} \langle \Phi_1 \rangle_0 &= v_1 > 0 \ , \\ \langle \Phi_2 \rangle_0 &= v_2 > 0 \ , \end{aligned} \tag{4.13}$$

with $v_1^2 + v_2^2 = v^2$ and

$$\frac{v_2}{v_1} \equiv \tan\beta \ . \tag{4.14}$$

After the $W^\pm$ and Z^0 acquire masses, five spin-zero degrees of freedom remain as massive spin-zero particles: the lightest scalar h^0, a heavier neutral scalar H^0, two charged scalars $H^\pm$, and a neutral pseudoscalar A^0. At tree level, we may express all the (pseudo)scalar masses in terms of M_A and $\tan\beta$, to find

$$M^2_{h^0,H^0} = \frac{1}{2}\left\{M_A^2 + M_Z^2 \mp \left[(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta\right]^{1/2}\right\} \ , \tag{4.15}$$

and

$$M^2_{H^\pm} = M_W^2 + M_A^2 \ . \tag{4.16}$$

At tree level, there is a simple mass hierarchy, given by

$$\begin{aligned} M_{h^0} &< M_Z|\cos 2\beta| \\ M_{H^0} &> M_Z \\ M_{H^\pm} &> M_W \ , \end{aligned} \tag{4.17}$$

but there are very important *positive* loop corrections to $M^2_{h^0}$ (proportional to $G_F m_t^4$) that were overlooked in the earliest calculations. These loop corrections change the mass predictions very significantly.

The results of a full, modern calculation [75] are shown in figure 12. There we see that the mass of the lightest Higgs scalar is largest for large values of the pseudoscalar mass M_A and in the limit of large $\tan\beta$.

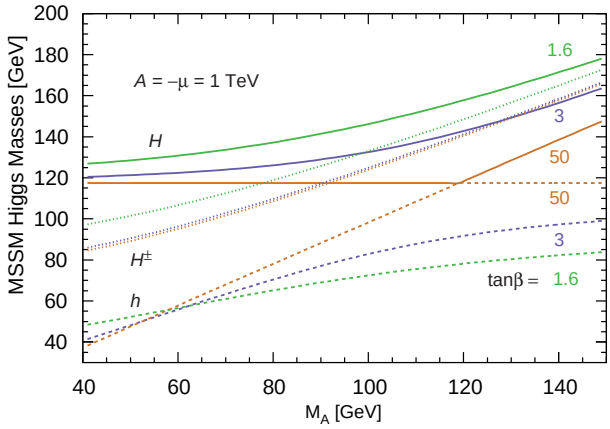


Fig. 12. Higgs-boson masses in the minimal supersymmetric standard model as a function of M_A for the case $A = -\mu = M_S = 1$ TeV, for a top-quark mass $m_t = 175$ GeV/c^2, from Ref. [75].

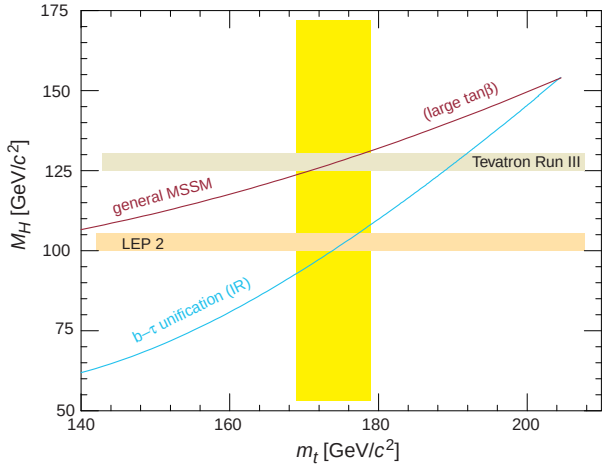


Fig. 13. Upper bounds on the mass of the lightest Higgs boson, as a function of the top-quark mass, in two variants of the minimal supersymmetric standard model. The upper curve refers to a general MSSM, in the large-tan β limit; the lower curve corresponds to an infrared-fixed-point scenario with b-τ unification, from Ref. [76].

Because the minimal supersymmetric standard model (MSSM) implies *upper bounds* on the mass of the lightest scalar h^0, it sets attractive targets for experiment. Two such upper bounds are shown as functions of the top-quark mass in figure 13. The large-tan β limit of a general MSSM yields the upper curve; an infrared-fixed-point scheme with b-τ unification produces an upper bound characterized by the lower curve. The vertical band shows the current information on m_t. We see that the projected sensitivity of LEP 2 experiments covers the full range of lightest-Higgs masses that occur in the infrared-fixed-point scheme. The sensitivity promised by Run III of the Tevatron gives full coveralge of h^0 masses in the MSSM. These are very intriguing experimental possibilities. For further discussion, consult the LEP 2 Yellow Book [75] and the Proceedings of the Tevatron Supersymmetry/Higgs Workshop [44].

4.4. New strong dynamics

Dynamical symmetry breaking offers a different solution to the naturalness problem of the electroweak theory: in technicolor, there are no elementary scalars. We hope that solving the dynamics that binds elementary fermions into a composite Higgs boson and other WW resonances will bring addition predictive power. It is worth saying that technicolor is a far more ambitious program than global supersymmetry. It doesn't merely seek to finesse the hierarchy problem, it aims to predict the mass of the Higgs surrogate. Against the aesthetic appeal of supersymmetry we can weigh technicolor's excellent pedigree. As we have seen in Section 2.1, the Higgs mechanism of the standard model is the relativistic generalization of the Ginzburg–Landau description of the superconducting phase transition. Dynamical symmetry breaking schemes — technicolor and its relatives — are inspired by the Bardeen–Cooper–Schrieffer theory of superconductivity, and seek to give a similar microscopic description of electroweak symmetry breaking.

The dynamical-symmetry-breaking approach realized in technicolor theories is modeled upon our understanding of the superconducting phase transition [77–79]. The macroscopic order parameter of the Ginzburg–Landau phenomenology corresponds to the wave function of superconducting charge carriers, which acquires a nonzero vacuum expectation value in the superconducting state. The microscopic Bardeen–Cooper–Schrieffer theory [80] identifies the dynamical origin of the order parameter with the formation of bound states of elementary fermions, the Cooper pairs of electrons. The basic idea of technicolor is to replace the elementary Higgs boson with a fermion-antifermion bound state. By analogy with the superconducting phase transition, the dynamics of the fundamental technicolor gauge inter-

actions among technifermions generate scalar bound states, and these play the role of the Higgs fields.

The elementary fermions — electrons — and gauge interactions — QED — needed to generate the scalar bound states are already present in the case of superconductivity. Could a scheme of similar economy account for the transition that hides the electroweak symmetry? Consider an $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ theory of massless up and down quarks. Because the strong interaction is strong, and the electroweak interaction is feeble, we may treat the $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ interaction as a perturbation. For vanishing quark masses, QCD has an exact $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{SU}(2)_\mathrm{R}$ chiral symmetry. At an energy scale $\sim \Lambda_\mathrm{QCD}$, the strong interactions become strong, fermion condensates appear, and the chiral symmetry is spontaneously broken to the familiar flavor symmetry:

$$\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{SU}(2)_\mathrm{R} \to \mathrm{SU}(2)_\mathrm{V} \ . \tag{4.18}$$

Three Goldstone bosons appear, one for each broken generator of the original chiral invariance. These were identified by Nambu [81] as three massless pions.

The broken generators are three axial currents whose couplings to pions are measured by the pion decay constant f_π. When we turn on the $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ electroweak interaction, the electroweak gauge bosons couple to the axial currents and acquire masses of order $\sim g f_\pi$. The mass-squared matrix,

$$\mathcal{M}^2 = \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{pmatrix} \frac{f_\pi^2}{4} \ , \tag{4.19}$$

(where the rows and columns correspond to W^+, W^-, W_3, and $\mathcal{A}$) has the same structure as the mass-squared matrix for gauge bosons in the standard electroweak theory. Diagonalizing the matrix (4.19), we find that $M_W^2 = g^2 f_\pi^2/4$ and $M_Z^2 = (g^2 + g'^2) f_\pi^2/4$, so that

$$\frac{M_Z^2}{M_W^2} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2 \theta_W} \ . \tag{4.20}$$

The photon emerges massless.

The massless pions thus disappear from the physical spectrum, having become the longitudinal components of the weak gauge bosons. Unfortunately, the mass acquired by the intermediate bosons is far smaller than required for a successful low-energy phenomenology; it is only [82] $M_W \approx 30$ MeV/c^2.

The minimal technicolor model of Weinberg [83] and Susskind [84] transcribes the same ideas from QCD to a new setting. The technicolor gauge

group is taken to be $\mathrm{SU}(N)_{\mathrm{TC}}$ (usually $\mathrm{SU}(4)_{\mathrm{TC}}$), so the gauge interactions of the theory are generated by

$$\mathrm{SU}(4)_{\mathrm{TC}} \otimes \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y \ . \tag{4.21}$$

The technifermions are a chiral doublet of massless color singlets

$$\begin{pmatrix} U \\ D \end{pmatrix}_{\mathrm{L}} \quad U_{\mathrm{R}},\ D_{\mathrm{R}} \ . \tag{4.22}$$

With the electric charge assignments $Q(U) = \frac{1}{2}$ and $Q(D) = -\frac{1}{2}$, the theory is free of electroweak anomalies. The ordinary fermions are all technicolor singlets.

In analogy with our discussion of chiral symmetry breaking in QCD, we assume that the chiral TC symmetry is broken,

$$\mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{SU}(2)_{\mathrm{R}} \otimes \mathrm{U}(1)_{\mathrm{V}} \to \mathrm{SU}(2)_{\mathrm{V}} \otimes \mathrm{U}(1)_{\mathrm{V}} \ . \tag{4.23}$$

Three would-be Goldstone bosons emerge. These are the technipions

$$\pi_T^+, \quad \pi_T^0, \quad \pi_T^-, \tag{4.24}$$

for which we are free to *choose* the technipion decay constant as

$$F_\pi = \left(G_F\sqrt{2}\right)^{-1/2} = 246\ \mathrm{GeV} \ . \tag{4.25}$$

This amounts to choosing the scale on which technicolor becomes strong. When the electroweak interactions are turned on, the technipions become the longitudinal components of the intermediate bosons, which acquire masses

$$\begin{aligned} M_W^2 &= g^2 F_\pi^2/4 &&= \frac{\pi\alpha}{G_F\sqrt{2}\sin^2\theta_W} \\ M_Z^2 &= \left(g^2 + g'^2\right) F_\pi^2/4 &&= M_W^2/\cos^2\theta_W \end{aligned} \ , \tag{4.26}$$

that have the canonical Standard Model values, thanks to our choice (4.25) of the technipion decay constant.

Technicolor shows how the generation of intermediate boson masses could arise without fundamental scalars or unnatural adjustments of parameters. It thus provides an elegant solution to the naturalness problem of the Standard Model. However, it has a major deficiency: it offers no explanation for the origin of quark and lepton masses, because no Yukawa couplings are generated between Higgs fields and quarks or leptons. Consequently, technicolor serves as a reminder that there are two problems of mass: explaining the masses of the gauge bosons, which demands an understanding

of electroweak symmetry breaking; and accounting for the quark and lepton masses, which requires not only an understanding of electroweak symmetry breaking but also a theory of the Yukawa couplings that set the scale of fermion masses in the standard model. We can be confident that the origin of gauge-boson masses will be understood on the 1-TeV scale. We do not know where we will decode the pattern of the Yukawa couplings; I describe a possible approach in §4.5.

To generate fermion mass, we may embed technicolor in a larger extended technicolor gauge group $G_{\rm ETC} \supset G_{\rm TC}$ that couples the quarks and leptons to technifermions [85]. If the $G_{\rm ETC}$ gauge symmetry is broken down to $G_{\rm TC}$ at a scale $\Lambda_{\rm ETC}$, then the quarks and leptons can acquire masses

$$m \sim \frac{g^2_{\rm ETC} F^3_\pi}{\Lambda^2_{\rm ETC}} \,. \tag{4.27}$$

The generation of fermion mass is where all the experimental threats to technicolor arise. The rich particle content of ETC models generically leads to quantum corrections that are in conflict with precision electroweak measurements [86]. Moreover, if quantum chromodynamics is a good model for the chiral-symmetry breaking of technicolor, then extended technicolor produces flavor-changing neutral currents at uncomfortably large levels. We conclude that QCD must not provide a good template for the technicolor interaction.

How could TC dynamics be different from QCD dynamics? Most modern implementations of dynamical symmetry breaking invoke multiple scales [87] to reconcile the generation of fermion masses with the constraints on flavor-changing neutral currents. In traditional ETC, the TC interaction is precociously asympototically free. If instead the technicolor interaction remains strong all the way from F_π to $\Lambda_{\rm ETC}$, the link (4.27) between F_π, $\Lambda_{\rm ETC}$, and fermion masses is modified to

$$m \sim \frac{g^2_{\rm ETC} F^2_\pi}{\Lambda_{\rm ETC}} \,, \tag{4.28}$$

so that the scale $\Lambda_{\rm ETC}$ that produces the observed fermion masses can be much larger than before. A high ETC scale in turn suppresses flavor-changing neutral-current processes to acceptable levels.

How could the β function of the technicolor interaction be small over a broad range of energy scales? To cancel the antiscreening contribution of gauge bosons, it is necessary to introduce either many fermions, or fermions in higher representations of the gauge group. Adding more fermions enlarges the chiral symmetries and thus increases the number of (pseudo-)Goldstone bosons that arise. The enriched spectrum of such a model could contain light resonances. Eichten, Lane, and Womersley [88] have investigated a model

in which the mesons built of techniquarks are relatively light. The spectrum includes technipions $\pi_{\rm T}^+$, $\pi_{\rm T}^0$, and $\pi_{\rm T}^-$ with masses around 100 GeV/c^2, and technivector mesons $\rho_{\rm T}$ and $\omega_{\rm T}$ with masses around 250 GeV/c^2. They have shown that new signatures,

$$\begin{array}{rcl} \omega_T & \to & \gamma\pi_T^0 \\ & & \big\lfloor\!\to b\bar{b} \end{array} \tag{4.29}$$

and

$$\begin{array}{rcl} \rho_T^+ & \to & W^+\pi_T^0 \\ & & \big\lfloor\!\to b\bar{b} \end{array} \tag{4.30}$$

hold promise for incisive searches in Run II of the Tevatron Collider.

4.5. The problem of fermion masses

The example of technicolor serves to emphasize that solving the origin of electroweak symmetry breaking will not necessarily give us insight into the origin of fermion masses. How are we to make sense of the puzzling pattern of quark masses, for example, shown in figure 14?

Like coupling constants, masses depend upon the momentum scale on which they are defined. The values plotted in figure 14 are defined in the modified–minimal-subtraction ($\overline{\rm MS}$) scheme, and evaluated at the mass of each quark (for c, b, t) or at 1 GeV (for u, d, s).

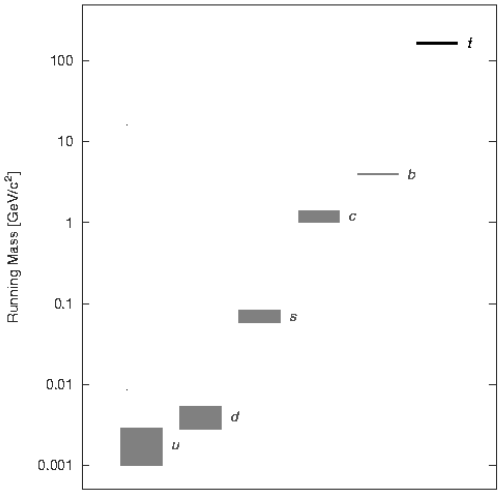


Fig. 14. Running masses $[m_q(m_q)]$ of the quarks in the $\overline{\rm MS}$ scheme.

The running of quark and lepton masses makes it evident that the pattern we perceive in our low-energy experiments is influenced not only by the underlying pattern, but also by the evolution from the scale on which masses are set down to our scale. It is then no surprise that the pattern we discern seems irrational. Perhaps an underlying order might show itself on some other scale.

It is helpful to examine this idea in a specific framework. A simple and convenient example is provided by the SU(5) unified theory of quarks and leptons, and of the strong, weak, and electromagnetic interactions. In the SU(5) theory, spontaneous symmetry breaking occurs in two steps. First, a **24** of scalars breaks

$$\mathrm{SU}(5) \to \mathrm{SU}(3)_{\mathrm{c}} \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y \,, \tag{4.31}$$

and gives extremely large masses to the leptoquark gauge bosons $X^{\pm 4/3}$ and $Y^{\pm 1/3}$. The **24** does not occur in the $\bar{\mathsf{L}}\mathsf{R}$ products

$$\begin{aligned} \mathbf{5}^* \otimes \mathbf{10} &= \mathbf{5} \oplus \mathbf{45} \\ \mathbf{10} \otimes \mathbf{10} &= \mathbf{5}^* \oplus \mathbf{45}^* \oplus \mathbf{50}^* \end{aligned} \tag{4.32}$$

that generate fermion masses, so the quarks and leptons escape large masses at tree level. In a second step, a **5** of scalars (which contains the standard-model Higgs doublet) breaks

$$\mathrm{SU}(3)_{\mathrm{c}} \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y \to \mathrm{SU}(3)_{\mathrm{c}} \otimes \mathrm{U}(1)_{\mathrm{em}} \,, \tag{4.33}$$

and endows the fermions with mass. This pattern of spontaneous symmetry breaking relates quark and lepton masses at the unification scale, predicting that

$$\left.\begin{aligned} m_e &= m_d \\ m_\mu &= m_s \\ m_\tau &= m_b \end{aligned}\right\} \text{ at the unification scale } M_U. \tag{4.34}$$

The masses of the charge-$\frac{2}{3}$ quarks, m_u, m_{c}, m_t, are separate parameters.

Within the SU(5) framework, it is straightforward to compute the evolution of the masses from M_U to another scale μ [89]. In leading logarithmic approximation, we find that

$$\begin{aligned} \ln\left[m_{u,c,t}(\mu)\right] \;\approx\; & \ln\left[m_{u,c,t}(M_U)\right] + \frac{12}{33-2n_f} \ln\left(\frac{\alpha_3(\mu)}{\alpha_U}\right) \\ & + \frac{27}{88-8n_f} \ln\left(\frac{\alpha_2(\mu)}{\alpha_U}\right) - \frac{3}{10n_f} \ln\left(\frac{\alpha_1(\mu)}{\alpha_U}\right) \,, \end{aligned} \tag{4.35}$$

$$\ln\left[m_{d,s,b}(\mu)\right] \approx \ln\left[m_{d,s,b}(M_U)\right] + \frac{12}{33-2n_f}\ln\left(\frac{\alpha_3(\mu)}{\alpha_U}\right) + \frac{27}{88-8n_f}\ln\left(\frac{\alpha_2(\mu)}{\alpha_U}\right) + \frac{3}{20n_f}\ln\left(\frac{\alpha_1(\mu)}{\alpha_U}\right) , \quad (4.36)$$

and

$$\ln\left[m_{e,\mu,\tau}(\mu)\right] \approx \ln\left[m_{e,\mu,\tau}(M_U)\right] + \frac{27}{88-8n_f}\ln\left(\frac{\alpha_2(\mu)}{\alpha_U}\right) - \frac{27}{20n_f}\ln\left(\frac{\alpha_1(\mu)}{\alpha_U}\right) , \quad (4.37)$$

where n_f is the number of quark or lepton flavors. The running of the quark masses receives contributions from the color, weak-isospin, and weak-hypercharge interactions, whereas the color force does not influence the evolution of lepton masses. Accordingly, the quark masses run significantly, while the lepton masses run much less. [To simplify a bit, I have omitted the Higgs-boson contributions, which are important in the case of the heavy top quark[2]].

The classic intriguing prediction of the SU(5) theory involves the masses of the b quark and the τ lepton, which are degenerate at the unification point. To compare m_b and m_τ on other scales, we combine (4.36) and (4.37) to write

$$\ln\left[\frac{m_b(\mu)}{m_\tau(\mu)}\right] \approx \ln\left[\frac{m_b(M_U)}{m_\tau(M_U)}\right] + \frac{12}{33-2n_f}\ln\left(\frac{\alpha_3(\mu)}{\alpha_U}\right) - \frac{3}{2n_f}\ln\left(\frac{\alpha_1(\mu)}{\alpha_U}\right) . \quad (4.38)$$

The first term on the right-hand side vanishes, according to (4.34). If we consider the case $n_f = 6$ with $1/\alpha_U = 40$, $1/\alpha_s(\mu) = 5$, and $1/\alpha_1(\mu) = 65$, the evolution of m_b and m_τ is shown in figure 15. At the low scale, we compute

$$m_b = 2.91 m_\tau \approx 5.16 \text{ GeV}/c^2 , \quad (4.39)$$

in suggestive agreement with what we know from experiment. The same procedure would lead to predictions for the first and second generations, at a scale $\mu \approx 1$ GeV, namely

$$\underbrace{\frac{m_s}{m_d}}_{\approx 20} = \underbrace{\frac{m_\mu}{m_e}}_{\approx 200} , \quad (4.40)$$

which is less successful.

[2] A proper treatment would also take account of fermion (especially top!) threshold effects, and of the large $Ht\bar{t}$ couplings [90].

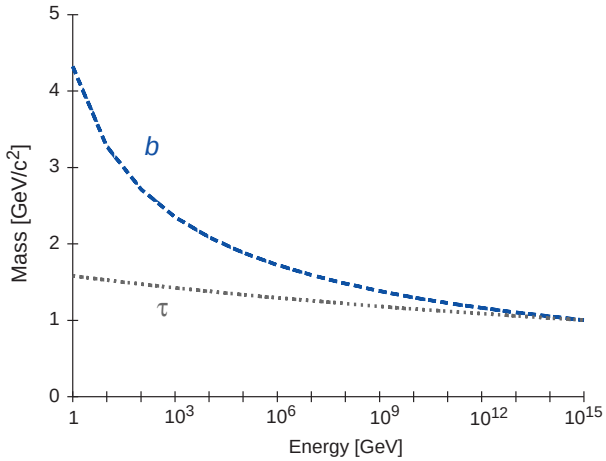


Fig. 15. Evolution of the τ-lepton and b-quark masses from the unification scale to low energies.

A more elaborate symmetry-breaking scheme — for example, adding a **45** of scalars — can change the relationship between the electron mass and the down-quark mass at the unification scale. A scheme that results in

$$m_s = \tfrac{1}{3} m_\mu, \quad m_d = 3m_e \quad \text{at } M_U \tag{4.41}$$

leads to the low-energy predictions

$$\left.\begin{aligned} m_s &\approx \tfrac{4}{3} m_\mu \\ m_d &\approx 12 m_e \end{aligned}\right\} \quad \text{at } \mu \approx 1 \text{ GeV} . \tag{4.42}$$

In the 1990s, this kind of analysis has given rise to a new cottage industry for understanding the pattern of fermion masses — including neutrino masses. Begin with a plausible unified theory, *e.g.*, supersymmetric SU(5), with its advantages for coupling-constant unification and the low-energy prediction for the weak mixing parameter $\sin^2 \theta_W$, or supersymmetric SO(10), to include massive neutrinos. Then find "textures," simple patterns of Yukawa matrices that lead to successful predictions for masses and mixing angles. Interpret these in terms of patterns of symmetry breaking. Finally, seek a derivation — or at least some motivation — for the explicit entry. I think this is a very interesting strategy; whether progress will be rapid or slow is less obvious to me. But I draw reassurance and encouragement from the fact that some schemes fail on their predictions for m_t or $|V_{cb}|$. The fact that failure is possible gives meaning to success.

To be sure, there are other plausible approaches to the origin of fermion masses, including composite models, schemes that assign a special role to

the top quark, other mechanisms that communicate mass radiatively to the quarks and leptons, and — within the framework of string theory — the idea that the pattern of fermion masses is determined by the topology of extradimensional space. But the general lesson I would draw from the exercise we have just undertaken is that a baffling pattern of masses observed at low energies may well arise from a simple and comprehensible pattern at high scales. For a sampler of recent work along these lines, see [91–94].

5. Concluding remarks

Understanding the mechanism of electroweak symmetry breaking is the urgent challenge for particle physics over the next decade. Searches for the Higgs boson and for electroweak physics beyond the standard model give form to the experimental programs at LEP 2, the Tevatron Collider, and the Large Hadron Collider, and guide the explorations of new accelerator projects. Many possibilities for decisive observations lie before us. Looking beyond the exploration of the 1-TeV scale we see the problem of fermion mass, which is intimately linked to electroweak symmetry breaking, and may also lead us to higher scales. I am very optimistic about the prospects for both theoretical and experimental progress. I believe that within the next decade we will complete the gauge-theory revolution, and I confidently expect that nature will give us clues that will lead us to still greater understanding.

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