## PHYSICS AT $DA\Phi NE^*$

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An update of the main physics items to be studied at the  $\phi$ -factory DA $\Phi$ NE under operation in Frascati is presented. Such studies include extraction of the CP-violating parameter  $\varepsilon'/\varepsilon$ , rare and very rare Kaon decays, tests of Chiral Perturbation Theory, scalar and pseudoscalar spectroscopy through radiative  $\phi$  decays, and measurement of the hadronic cross-section with aim to improve present limits on the accuracy of theoretical determination of  $\alpha_{\text{OED}}(M_Z)$  and  $(g-2)_{\mu}$ .

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#### 1. Introduction

In this talk I shall review the main physics items to be studied at the  $\phi$ -factory Double Accelerator For Nice Experiments (DA $\Phi$ NE), which has recently started operating in the INFN National Laboratories in Frascati. The design luminosity of DA $\Phi$ NE is  $\mathcal{L} = 5 \times 10^{32} \text{ cm}^{-2} \text{sec}^{-1}$ . With such luminosity, there are many interesting precision measurements expected from the three detectors which will operate at DA $\Phi$ NE. These are the K-Long Observation Experiment (KLOE) for particle physics, the nuclear physics detector FINUDA, and DEAR, DA $\Phi$ NE Exotic Atom Reasearch Detector, a detector for measuring the formation of Kaonic atoms.

From the beginning, the main scope of DA $\Phi$ NE has ben the study of direct CP-violation in the Kaon system through the copious number of  $\phi$ -decays into  $K\bar{K}$  pairs. But many other interesting and important measurements can be done [1], namely study of rare K-decays, scalar (S) and pseudoscalar (PS) meson spectroscopy through the radiative decays  $\phi \to \gamma$  S/PS, rare  $\rho$  and  $\omega$ -decays through the decays  $\phi \to \pi \rho/\omega$ , the measurement of

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the hadronic  $e^+e^-$  cross-section, and, through this, of the hadronic contribution to vacuum polarization in the evaluation of  $(g-2)_{\mu}$ , the scattering Kaon-nucleon at threshold, formation and decay properties of hypernuclei, and, as mentioned, formation of kaonic atoms.

Since the first proposal and operation of AdA in 1960 [2], Frascati has occupied a special place in the history of electron-positron storage rings. AdA had an orbit 408 cm long and an energy of 250 MeV per beam, and was soon followed by a bigger project, to be named ADONE, a storage ring with a 100 meter long orbit and an energy per beam of 1500 MeV. Radiative corrections to the experiments were known to be important and the special name of *Bond* factor was invented by Bruno Touschek for the value of the photon spectrum at this energy [3]. The first multihadronic production was observed at ADONE, which confirmed also the the discovery of the  $J/\Psi$  in 1974. ADONE was subsequently transformed into a synchtron light facility and again operated as an electron-positron facility in 1989, when it was used to study the nucleon form factor at threshold in the time-like region with the detector FENICE. Soon after, the DA $\Phi$ NE project for a  $\phi$ -factory with a c.of m. energy  $\sqrt{s} = 1.020$  GeV, was approved and construction began in 1993.

The Frascati  $\phi$ -factory project consists of a LINAC feeding an accumulator which injects positrons and electrons into two separate storage rings. The two separate rings cross at half angle  $\theta \approx 10 \div 15$  mrad in two interaction regions. DA $\Phi$ NE is a multibunch machine, which is presently approaching its first phase of operation with 30 bunches, each of which should reach the design luminosity of  $\mathcal{L}_{\text{single bunch}} \approx 4 \times 10^{30} \text{ cm}^{-2} \text{sec}^{-1}$ . In the second phase, the number of bunches will be increased to a maximum of 120, so as to reach the project luminosity of  $5 \times 10^{32} \text{ cm}^{-2} \text{sec}^{-1}$ . For such luminosity, we show in Table I the number of events in 1 year of running expected in the main channels of interest. In the next sections we shall describe some of inter-

TABLE I

_	Decay channel	Branching ratio	Events in $1 y$
	$K^+K^-$	49%	$1.1 \times 10^{10}$
	$K^0_{ m S}K^0_{ m L}$	34%	$7.5 \times 10^9$
	$\rho\pi+\pi^+\pi^-\pi^0$	16%	$3.4 \times 10^9$
	$\eta + \gamma$	1.3~%	$2.8 \times 10^8$
	$\eta' + \gamma$	$1.2 \times 10^{-4}$	
	$f_0\gamma$	$< 1 \times 10^{-4}$	

Main  $\phi$ -decay modes and Branching Ratio es at DA $\Phi$ NE

esting measurements to be performed and their physics content, starting in Sect. 2 with the measurement of direct CP-violation in the Kaon system. A discussion of rare K-decays, in particular of  $K_{l4}$ -decays will follow in Sect. 3, where there will also be mentioned the complementarity of such measurements with possible studies in the  $\gamma\gamma$  channel. Sect. 4 will be dedicated to proposed experiments to measure the hadronic cross-section. For the many other items of interest, we refer the reader to Ref. [1].

## 2. CP-violation

DAΦNE Main Goal from the beginning has been the measurement of direct CP-violation in  $K_{\rm L}$  decay through the measurement of the ratio  $\frac{\varepsilon'}{\varepsilon}$  defined through the amplitude ratios

$$\bullet \eta_{+-} = \frac{A(K_{\rm L} \to \pi^+ \pi^-)}{A(K_{\rm S} \to \pi^+ \pi^-)} = \varepsilon + \varepsilon' ; \quad \eta_{00} = \frac{A(K_{\rm L} \to \pi^0 \pi^0)}{A(K_{\rm S} \to \pi^0 \pi^0)} = \varepsilon - 2\varepsilon' \quad (2.1)$$

The recent mesurement of this parameter at FermiLab [4] and the existence of previous measurements [5,6] not quite in agreement with each other, render particularly interesting the measurement to be done at DA $\Phi$ NE, where the experimental conditions are quite different. Indeed, the study of CPviolation is usually done at hadron accelerators, where the cross-section is large, *i.e.* of order  $\approx mb$ , whereas at an  $e^+e^-$  collider it is always an order  $\alpha^2$  smaller. However DA $\Phi$ NE special conditions, namely

- large project luminosity  $\rightarrow 5 \times 10^{32} \text{ cm}^{-2} \text{sec}^{-1}$
- low background at resonance
- tagged  $K_{\rm S}, K_{\rm L}, K^0, \bar{K}^0$  because of the special quantum state through the reaction

$$e^+e^- \rightarrow \phi \rightarrow K_{\rm S}K_{\rm L}$$
  
 $\rightarrow K^0\bar{K}^0$ 

can make the measurement competitive.

In the neutral Kaon-system the classification of CP-violation distinguishes between violation in the mixing matrix, or indirect CP-violation, *i.e.*  $(i - v) W + (i - v) \overline{W}$ 

$$K_{\rm S,L} \approx \frac{(1+\varepsilon)K \pm (1-\varepsilon)\bar{K}}{\sqrt{2}}$$
 (2.2)

with  $\varepsilon$  the amount of mixing in the mass matrix which is revealed through the ratio

$$\frac{\Gamma(K_{\rm L} \to \pi^{-}l^{+}\nu) - \Gamma(K_{\rm L} \to \pi^{+}l^{-}\bar{\nu})}{\Gamma(K_{\rm L} \to \pi^{-}l^{+}\nu) + \Gamma(\bar{K}_{\rm L} \to \pi^{+}l^{-}\bar{\nu})} \approx 2\Re\varepsilon$$
(2.3)

and which was measured to be of the order of  $10^{-3}$ , and CP-violation in the decay amplitude, also called direct CP-violation, which could in principle be measured through the ratio

$$\frac{\Gamma(K \to \pi^+ \pi^-) - \Gamma(\bar{K} \to \pi^+ \pi^-)}{\Gamma(K \to \pi^+ \pi^-) + \Gamma(\bar{K} \to \pi^+ \pi^-)} \approx 2\Re\varepsilon'$$
(2.4)

Actually direct CP-violation is measured via interference of mixing and decay, through the ratioes  $\eta_{+-}$  and  $\eta_{00}$ : direct CP implies  $\rightarrow \eta_{+-} \neq \eta_{00}$  i.e.  $\varepsilon' \neq 0$ . In Table II we show the present status of our knowledge on  $\frac{\varepsilon'}{\varepsilon}$ .

TABLE II

Theoretical and experimental status of $\varepsilon'/\varepsilon$			
Group	Theory	Experiment	
	$\Re \varepsilon'/\varepsilon \times 10^{-4}$	$\Re \varepsilon'/\varepsilon \times 10^{-4}$	
A.J.Buras et al. [7]			
	$0.26 \div 22.0$		
(S)			
(G)	$7.7^6_{-3.5}$ (NDR)		
	$5.2^{4.6}_{-2.7}(\mathrm{HV})$		
M. Ciuchini et al. [8]	$4.6\pm3.0\pm0.4$		
Bertolini et al. [9]	$17^{+14}_{-10}$		
E. Paschos et al. [10]	$9.9 \pm 4.1$		
NA31 $[5]$		$23\pm7$	
E731 [6]		$7.4\pm5.9$	
E832 [4]		$28 \pm 4.1$	

The recent KTeV [4] result is based on analysis of  $\approx 20\%$  of the collected data, and further statistics will be presented in the near future. The CERN experiment NA48 is also expected to present the analysis very shortly. Results from KLOE should be available within one year. At DA $\Phi$ NE the method of the Double Ratio

$$\frac{N_{\rm L}^{\pm}/N_{\rm S}^{\pm}}{N_{\rm L}^{0}/N_{\rm S}^{0}} = |\frac{A(K_{\rm L} \to \pi^{+}\pi^{)}/A(K_{\rm S} \to \pi^{+}\pi^{-})}{A(K_{\rm L} \to \pi^{0}\pi^{0})/A(K_{\rm S} \to \pi^{0}\pi^{0})}|^{2} \approx 1 + 6\Re\varepsilon'/\varepsilon$$

is expected to bring a sensitivity of  $\delta (\varepsilon'/\varepsilon) = 1 \times 10^{-4} @L = 5 \times 10^{32} \text{cm}^{-2} \text{sec}^{-1}$ . The special conditions in which DA $\Phi$ NE will operate,

make possible another type of measurements of the ratio  $\varepsilon'/\varepsilon$ , *i.e.* through interferometry, to be sketched below. Although the method of the double ratio leads to a to a higher sensitivity than in the interferometry case, the latter method can bring information not just on  $\Re \varepsilon'/\varepsilon$  but also on  $\Im \varepsilon'/\varepsilon$ . This type of experiments measures the number of events for which there is interference between the final states in which the  $\{K_{\rm L}, K_{\rm S}\}$  pair decays. Consider in fact the decay

$$\phi \to K\bar{K} \to (f_1, t_1) + (f_2, t_2),$$

where  $f_{1,2}$  are two possible final states of the  $K_{L,S}$  decays and  $t_{1,2}$  the two decay times, measured through the distance [11] travelled by the Kaons before decaying. The number of events observed to decay into a given set of final states  $\{f_1, f_2\}$  with the time set  $\{t_1, t_2\}$  is a function of the various parameter of the Kaon system and of the CP-violation ratio  $\varepsilon'/\varepsilon$ , *i.e.* one has [12]

$$I(f_1, f_2, \Delta t) = \frac{1}{2\Gamma} |\langle f_1 | K_{\rm S} \rangle \langle f_2 | K_{\rm S} \rangle|^2 \times \mathcal{G},$$
  
$$\mathcal{G} = |\eta_1|^2 e^{-\Gamma_{\rm L} \Delta t} + |\eta_2|^2 e^{-\Gamma_{\rm S} \Delta t}$$
  
$$-2|\eta_1||\eta_2|e^{-\Gamma \Delta t/2} \cos\left(\Delta m \Delta t + \Delta \phi\right)$$

with  $\eta_i = \langle f_i | K_{\rm L} \rangle / \langle f_i | K_{\rm S} \rangle$  and the time difference  $\Delta t = t_1 - t_2$ . For  $f_1 = \pi^+ \pi^-$  and  $f_2 = \pi^0 \pi^0$  the inteference pattern is shown in Fig. 1, which we reproduce from Ref. [12]



Fig. 1. The interference pattern shown corresponds to  $\delta(\Re \varepsilon' / \varepsilon) = 1.8 \times 10^{-4}$  and  $\delta(\Im \varepsilon' / \varepsilon) = 3.4 \times 10^{-3}$ .

### 3. Rare K-decays

With the expected luminosity rare and very rare K-decays up to branching ratioes of  $10^{-9}$ , can be expected to be measured at DA $\Phi$ NE. Within Standard model expectations, rare decays of interest and measurable at DA $\Phi$ NE include radiative Kaon decays, measurement of the scalar form factor in  $K_{l3}$ -decays, checks of ChPT in  $K_{l4}$ -decays. Radiative Kaon-decays have been studied in great detail within Chiral Perturbation Theory and we list in Table III some of the expected [13] branching ratioes and the actual statistical significance which can be reached at DA $\Phi$ NE.

TABLE III

Channel		$BR_{exp}$	$BR_{theor}$	#  events/yr			
Two photons in the final state							
$K_{\rm S} \to \gamma \gamma$		$(2.4 \pm 1.2) \cdot 10^{-6}$	$2.1 \cdot 10^{-6}$	$3.6 \cdot 10^3$			
$K_{\rm L} \to \gamma \gamma$		$(5.73 \pm 0.27) \cdot 10^{-4}$	$\sim 5 \cdot 10^{-4}$	$6.3\cdot 10^5$			
$K_{\rm L} \rightarrow \pi^0 \gamma \gamma$		$(1.70 \pm 0.28) \cdot 10^{-6}$	$\sim$ 10 $^{-6}$	$1.9\cdot 10^3$			
$K_{ m S}  ightarrow \pi^0 \gamma \gamma$	$(M_{\gamma\gamma} > 220 \text{ MeV})$	—	$3.8\cdot10^{-8}$	65			
$K^+ \rightarrow \pi^+ \gamma \gamma$		$< 1.5 \cdot 10^{-4}$	$\sim 5 \cdot 10^{-7}$	$\sim 4.5 \cdot 10^3$			
$K_{\rm L} \rightarrow \pi^0 \pi^0 \gamma \gamma$	$( M_{\gamma\gamma} - M_{\pi}  > 20 \text{ MeV})$	—	$3 \cdot 10^{-8}$	33			
$K_{\rm S} \to \pi^0 \pi^0 \gamma \gamma$	$( M_{\gamma\gamma} - M_{\pi}  > 20 \text{ MeV})$	—	$5 \cdot 10^{-9}$	8			
	One photon in the fina	al state, internal brems	sstrahlung				
$K_{\rm S}  o \pi^+ \pi^- \gamma$	$(E_{\gamma}^{*} > 50 \text{ MeV})$	$(1.78 \pm 0.05) \cdot 10^{-3}$	$1.75 \cdot 10^{-3}$	$3\cdot 10^{6}$			
$K_{\rm L} \rightarrow \pi^+ \pi^- \gamma$	$(E_{\gamma}^{*}>20 \text{ MeV})$	$(1.49 \pm 0.08) \cdot 10^{-5}$	$1.42\cdot 10^{-5}$	$1.5\cdot 10^4$			
$K^+ \rightarrow \pi^+ \pi^0 \gamma$	$(T_c^* = (55 - 90) \text{ MeV})$	$(2.57 \pm 0.16) \cdot 10^{-4}$	$2.61 \cdot 10^{-4}$	$2.3\cdot 10^6$			
	One photon in th	e final state, direct em	ission				
$K_{\rm S}  o \pi^+ \pi^- \gamma$	$(E_{\gamma}^{*} > 50 \text{ MeV})$	$< 9 \cdot 10^{-5}$	$\sim$ 10 <sup>-6</sup>	$\sim$ 1.7 $\cdot$ 10 <sup>3</sup>			
$K_{\rm L}  ightarrow \pi^+ \pi^- \gamma$	$(E_{\gamma}^*>20 \text{ MeV})$	$(3.19 \pm 0.16) \cdot 10^{-5}$	$\sim10^{-5}$	$3.5\cdot 10^4$			
$K^+ \rightarrow \pi^+ \pi^0 \gamma$	$(T_c^* = (55 - 90) \text{ MeV})$	$(1.8 \pm 0.4) \cdot 10^{-5}$	$\sim 10^{-5}$	$1.6 \cdot 10^{5}$			
	Lepton pair without pions						
$K_{\rm L} \rightarrow \mu^+ \mu^-$		$(7.4 \pm 0.4) \cdot 10^{-9}$	$\sim 7 \cdot 10^{-9}$	8			
$K_{\rm L} \rightarrow \gamma e^+ e^-$		$(9.1 \pm 0.5) \cdot 10^{-6}$	$9 \cdot 10^{-6}$	$1.0\cdot 10^4$			
$K_{\rm L} \to \gamma \mu^+ \mu^-$		$(2.8 \pm 2.8) \cdot 10^{-7}$	$3.6 \cdot 10^{-7}$	$4.0 \cdot 10^2$			
$K_{\rm S} \rightarrow \gamma e^+ e^-$		—	$3.4 \cdot 10^{-8}$	58			
$K_{\rm L} \rightarrow e^+ e^- e^+ e^-$		$(3.9 \pm 0.7) \cdot 10^{-8}$	—	43			
Lepton pair with pions							
$K_{\rm S} \rightarrow \pi^0 e^+ e^-$		$< 1.1 \cdot 10^{-6}$	$> 5 \cdot 10^{-10}$	> 1			
$K_{\rm S} \to \pi^0 \mu^+ \mu^-$			$> 10^{-10}$				
$K^+ \rightarrow \pi^+ e^+ e^-$		$(2.74 \pm 0.23) \cdot 10^{-7}$	$\sim 3 \cdot 10^{-7}$	$2.5 \cdot 10^{3}$			
$K^+ \to \pi^+ \mu^+ \mu^-$		$< 2.3 \cdot 10^{-7}$	$6 \cdot 10^{-8}$	$5.4 \cdot 10^{2}$			
$K_{\rm L} \rightarrow \pi^+ \pi^- e^+ e^-$	-	$< 2.5 \cdot 10^{-6}$	$2.8 \cdot 10^{-7}$	$3.1 \cdot 10^{2}$			

Radiative kaon decays of interest for  $DA\Phi NE$  [13]

Other interesting Kaon decays to be studied concern the scalar form factor in  $K_{\mu3}$  decay and the measurement of  $\pi\pi$  phaseshift at threshold in  $K_{l4}$ -decays. For the first of these, the high statistics expected at DA $\Phi$ NE should definitely clarify the theoretical situation [14]. Here, we shall describe in some detail the more complex situation situation concerning  $K_{l4}$  decays [14]. These decays constitute the only available source of clear information on  $\pi\pi$  scattering at threshold. From the decays  $K \Rightarrow \pi\pi l\nu$  one can in fact learn about  $\pi\pi \to \pi\pi$  and hence about the pion-pion scattering length in the isospin I = 0 channels,  $a_0^0$ . There are various theoretical predictions for this number, starting from the current algebra value  $a_0^0 = 0.16 \ m_{\pi}^{-1}$  [15], to the more recent predictions in Chiral Perturbation Theory to leading [16] and next to leading order [17],  $a_0^0 = 0.2 \pm 0.01$  in units of  $m_{\pi}^{-1}$ , and in G(eneralized)Chiral Perturbation Theory [18] where  $a_0^0 = 0.263 \pm 0.052$ . These number can be compared with the experimental information from the CERN-Saclay experiment [19] which has been fitted with  $a_0^0 = 0.26 \pm 0.05$ . A fit to the data with different values of the scattering length, is shown in Fig. 2 from [20].



Fig. 2. I = 0 S-wave  $\pi\pi$  phase shifts,  $\delta_0^0$ , below 1 GeV: experimental values compared to BFP's fits [21] using the Roy [22] equations, fitting  $\delta_0^0$  above 500 MeV to the phases. Curves a, b and c correspond to alternative values for the I = 0 S-wave scattering length  $a_0^0 = 0.17, 0.30$  and 0.50.



Fig. 3. Results for the  $\pi\pi$  phase shift difference  $\delta_0^0 - \delta_1^1$  inferred from the Geneva-Saclay  $K_{e4}$  experiment and different theoretical calculations.



Fig. 4. We show the data for  $\gamma \gamma \to \pi^0 \pi^0$  from Crystal Ball Collaboration [24] in comparison with results from Chiral Perturbation Theory one loop result [25]  $O(p^4)$  (dashes), Chiral Perturbation Theory two loop result [26]  $O(p^6)$  (full) and those from a dispersive analysis [27] with  $a_0^0 = 0.2$  (band).

Recent theoretical estimates [17, 18] are shown in Fig. 3.

Complementary information on  $a_0^0$  can be extracted from the process  $\gamma \gamma \rightarrow \pi^0 \pi^0$  and  $\gamma \gamma \rightarrow \pi^+ \pi^-$ . Phase space is limited for  $\gamma \gamma$  physics at DA $\Phi$ NE, but detector capabilities (KLOE) and full phase space utilization make it a possibility worth pursuing. The physics items of interest are mainly



Fig. 5. Integrated cross-section data for  $\gamma \gamma \rightarrow \pi^0 \pi^0$  as a function of the  $\pi \pi$  invariant mass.

- study of  $\pi\pi$  system at threshold, where one can study the loop structure of Chiral Perturbation Theory [23], pion polarizability and azymuthal correlations both in the  $\pi^+\pi^-$  and  $\pi^0\pi^0$  system;
- study of the decay constant and transition form factors for  $\pi^0, \eta$

The present theoretical and experimental status of  $\gamma \gamma \rightarrow \pi^0 \pi^0$ , which is the most interesting process from the Chiral Perturbation Theory point of view, is shown in Fig. 4.

It is worth noticing the strong dependence of the process  $\gamma \gamma \rightarrow \pi^0 \pi^0$  from the value of  $a_0^0$ . This is shown in Fig. 5 reproduced from [28].

In Fig. 5 data are from Crystal Ball scaled to the full angular range by a factor of 1.25 [28] and the lines, labelled by the value of the  $I = 0 \pi \pi S$ -wave scattering length in steps of 0.05 from 0.1 to 0.3, illustrate the effect on the dispersive predictions of different extrapolations of the  $\pi \pi$  phases above 520 MeV down to threshold. The bands above 500 MeV on the  $a_0^0 = 0.1$  and 0.3 curves mark the range generated by different asymptotics for the vector exchanges.

Finally, in view of the observed  $K^+ \to \pi^+ \nu \bar{\nu}$  event [29] at Brookhaven, it has been suggested [30] that DA $\Phi$ NE and KLOE may have a chance to improve present experimental limits for the decay  $K \to \pi^0 \nu \bar{\nu}$ . Although there is no chance to positively observe this decay, whose B.R. in the Standard Model is a very small  $3 \times 10^{-11}$  [31], the special experimental conditions at DA $\Phi$ NE and the characteristics of the detector may be able to exclude possible deviations from the Standard Model expectations.

# 4. The hadronic cross-section and the contribution to $(g-2)_{\mu}$

One of the most important precision measurements in particle physics is the measurement of the anomalous magnetic moment of the muon. The present experimental status of this measurement is summarized in Table IV

#### TABLE IV

Status of measurement of anomalous magnetic moment of the muon

Laboratory	$a_{\mu}^{\mathrm{EXP}} \simeq (\frac{g-2)}{2})_{\mu}$	Precision	Year
	$(1\ 162\pm5)\times10^{-6}$	4300  ppm	1960
CERN [32]	$(116\ 616\pm 31) \times 10^{-8}$	$270 \mathrm{~ppm}$	1970
	$(1\ 165\ 924\pm 8.5)\times 10^{-9}$	$7.3~\mathrm{ppm}$	1979
BNL [33]	$(1\ 165\ 925\pm 15)\times 10^{-9}$	$12.9 \mathrm{~ppm}$	1998
BNL [34]	$(\dots \pm 0.40) \times 10^{-9}$	< 1  ppm	$\geq 1999$

Theoretically [35] this quantity is given as

$$a_{\mu}^{th} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{Weak}} + a_{\mu}^{\text{had}}$$

$$\tag{4.1}$$

where, in units of  $10^{-11}$ ,

$$\begin{split} a^{QED}_{\mu} &= 116584705.6(2.9) \text{ has been calculated [36] up to order } (\alpha/\pi)^5 \\ a^{Weak}_{\mu} &= 195(\text{one loop}) - 41(4)(\text{two loops}) \text{ [37,38]} \\ a^{had}_{\mu} &= a^{\text{vac-pol}}_{\mu} + a^{\text{light-light}}_{\mu} \end{split}$$

with  $a_{\mu}^{had}$ , to leading order, calculated through the first of the four graphs shown in Fig. 6.



Fig. 6. Graphs contributing to  $a_{\mu}^{\text{had}}$ 

In Ref. [36], the complete standard model prediction for  $a_{\mu}$  is given as

$$a_{\mu} = 116591596(67) \times 10^{-11} \tag{4.2}$$

with a theoretical error already close to the goal of the E821 experiment at BNL [34]. Such a small error was recently obtained through a number of theoretical improvements in the calculation of  $a_{\mu}^{had}$  [39]. However, other estimates differ on the size of the error, namely, the authors of Ref. [40] attribute a larger uncertainty to the quantity  $a_{\mu}^{had}$ . Their updated evaluation for the leading order contribution from vacuum polarization alone, in units of  $10^{-11}$  reads

$$a_{\mu}^{\text{had}} = 6967 \pm 119 \text{ from vacuum polarization to fourth order in } \alpha$$
. (4.3)

To clarify the origin of the error and of the difference in evaluation, let us examine this term in more detail. Up to sixth order in  $\alpha$ , the contribution to  $a_{\mu}^{\text{had}}$  is written as

$$a_{\mu}^{\text{had}} = a_{\mu}^{\text{vac-pol}}(4\text{th order}) + a_{\mu}^{\text{vac-pol}}(6\text{th order}) + a_{\mu}^{l-l}, \qquad (4.4)$$

where [41]

$$a_{\mu}^{\text{vac-pol}}(6\text{th order}) = (-100 \pm 6) \times 10^{-11}$$

The other three graphs shown in Fig. 6 contribute to the light-by-light scattering term  $a_{\mu}^{l-l}$  for which [42, 43] one can use the average estimate [36]  $a_{\mu}^{l-l} = (-85 \pm 25) \times 10^{-11}$ . For the charged pion loop, the pointlike case has been considered, as well as the case where  $\rho$ -dominated form factors are associated with the  $\gamma \to \pi\pi$  vertex. The error here is probably larger than what is quoted [44]. The uncertainties can be reduced to both the treatment of the  $\rho$ -dominated form factors and the masses circulating in the loop. These terms are of the same order as the higher order contributions to the vacuum polarization graphs (not shown) as well as of the two loops contribution to  $a_{\mu}^{\text{Weak}}$ . Notice also that it is the vacuum polarization graphs which drive the error on  $a_{\mu}$ . These hadronic contributions to the anomaly from vacuum polararization are not computed from firts principles with sufficient accuracy [38] and it is necessary to use an experimental input through dispersion relations. Indeed theoretical calculations for leading order are so far based upon

$$a_{\mu}^{\text{vac-pol}} = \frac{1}{4\pi^3} \int_{4m_{\pi}^2}^{\infty} \sigma_{e^+e^- \to \text{hadrons}}(s) \hat{K}(s) \frac{ds}{s}, \qquad (4.5)$$

where the function  $\hat{K}(s)$  has a monotonic behaviour.  $a_{\mu}^{\text{vac-pol}}$  is then evaluated using measured hadronic  $e^+e^-$  cross-sections and their error and, since the integral is strongly peaked for small s values, most of the error on  $a_{\mu}^{\text{vac-pol}}$  comes from the region  $\leq 1$  GeV, basically around the  $\rho - \omega$  resonances.

We show in the following table the latest evaluation of these contributions [40] and the accuracy which is requested in order to obtain  $a_{\mu}^{\text{had}}$  with an error comparable or below the one expected from E821.

## TABLE V

Contributions to  $a_{\mu}^{\text{vac-had}}$  in units  $10^{-10}$  from a number of exclusive channels and required accuracy [40].

Final state	value	acc.	Final state	value	acc.
$ ho, \omega  ightarrow \pi^+ \pi^-$	506	0.3%	$3\pi$	4	10%
$\omega \to 3\pi$	47	$\sim 1\%$	$K^+K^-$	4	$\downarrow$
$\phi$	40	$\downarrow$	$K_{ m S}K_{ m L}$	1	
$\pi^+\pi^-\pi^0\pi^0$	24		$\pi^+\pi^-\pi^+\pi^-\pi^0$	1.8	
$\pi^+\pi^-\pi^+\pi^-$	14		$\pi^{+}\pi^{-}\pi^{+}\pi^{-}\pi^{+}\pi^{-}$	0.5	
$\pi^{+}\pi^{-}\pi^{+}\pi^{-}\pi^{0}\pi^{0}$	5	10%	$par{p}$	0.2	
$2 \text{ GeV} \le E \le M_{J/\Psi}$	22				
$M_{J/\Psi} \leq M_{\Upsilon}$	20				
$\vec{E} \ge M_{\Upsilon}$	$\leq 5$				

This table reflects the fact that in the regione above  $\simeq 2$  GeV, remarkable improvements have been obtained using PQCD [39, 45], but it also shows the importance of the region below the  $\phi$  resonance. Here the present error is still in the range from 5% to 20% [40], with estimates which change according as to whether one uses or not input from  $\tau$  decay data, with results obtained through theoretical improvements like use of Conserved Vector Current (CVC) and  $\tau$  spectral functions applied to ALEPH and OPAL data, Perturbative Quantum ChromoDynamics (PQCD) and QCD Sum Rules [39].

As a result while the region above  $\approx 1.5$  GeV can be now safely evaluated theoretically, there still remains the region below the  $\phi$  where the main portion of the present remaining error lies. An energy scan around the  $\rho$  at DA $\Phi$ NE could reduce the error. Presently there is ongoing work at Novosibirsk where the goal is a 2 percent systematic uncertainty. The requirements for energy scan at DA $\Phi$ NE to reduce the error can be estimated from the fact that the contribution from  $e^+e^- \rightarrow \rho \rightarrow \pi^+\pi^-$  represents  $\approx 70\%$  of the lowest order estimate of  $a_{\mu}^{\rm vac-had}$ . Thus the  $\rho$  contribution should be measured with an accuracy of at least 10% of  $a_{\mu}^{\rm Weak}$ , *i.e.*  $\pm 15 \times 10^{-11}$ , which implies a measurement of the integral to an accuracy of  $\approx 0.3\%$ . An integrated luminosity of  $\approx 10$  nb<sup>-1</sup> is required for measurements around  $\rho$ mass. For the complete measurement (at lower energies more statistics for the same accuracy is required) the (modest) requirement is  $\approx 380$  nb<sup>-1</sup> at DA $\Phi$ NE luminosity [47]. It has been realized however that DA $\Phi$ NE will probably not go out of the  $\phi$  peak for the next 5 years and alternative proposals [48–50] have been put forward and are presently under scrutiny. These proposals are based on the use of Initial State Bremsstrahlung to tune the energy of the hadronic system and access c.m. energies below the  $\phi$ , *i.e.* through the process [51]



Fig. 7. ISR and the Hadronic Contribution to  $a_{\mu}^{\text{vac-pol}}$ 

 $e^+(p_1)e^-(p_2) \rightarrow \gamma(k) + \gamma * \rightarrow \gamma(k) + \text{hadrons}$  $(p_1 + p_2)^2 = M_{\phi}^2$ 

To obtain an estimate of this cross-section for the DA $\Phi$ NE setup, consider the simple case of soft photon approximation <sup>1</sup>, where, in leading order, the process shown in Fig. 7 has cross-section

$$\frac{d\sigma(k,\theta_{\gamma})}{dkd\cos\theta_{\gamma}} \approx 2\sigma_h \frac{\alpha}{\pi} \frac{dk}{k} \frac{\sin^2 \theta_{\gamma}}{(1-\beta^2\cos^2 \theta_{\gamma})^2} \,,$$

where  $\theta_{\gamma}$  is the photon angle with respect to the beam. For photons emitted at an angle greater than  $\theta_{\min}$  the differential cross-section as a function of the photon energy is shown in Fig. 8, from where one can see the reduction of the the cross-section for two different values of the angle  $\theta_{\min}$ .

To leading order, the photon spectrum is factorizable, and therefore the contribution to  $a_{\mu}^{\text{vac-pol}}$  in a region from threshold to just below the  $\phi$  is now obtained as an integral sum over the photon energy, rather than over the  $e^+e^-$  c.m. energy, *i.e.* one can write [48]

$$a^h_{\mu} = \frac{M^2_{\phi}}{\pi} \int\limits_{k_{\min}}^{k_{\max}} dk F(k, M^2_{\phi}, \theta_{\min}) \frac{d\sigma(k)}{dk} , \qquad (4.6)$$

where

<sup>&</sup>lt;sup>1</sup> for an expression of arbitrary photon energy, see [51]



Fig. 8. From [48]: differential cross-section for ISR process

- $d\sigma(k)/dk$  is the cross-section for Initial State Radiation processes (ISR) integrated over  $\theta_{\gamma}$ ,
- $F(k, M_{\phi}^2, \theta_{\min})$  is inversely proportional to the radiative factor for emission of a photon of energy k at all angles larger than  $\theta_{\min}$  relative to the beam direction.

Preliminary findings by Spagnolo [48], using the photon resolution of the KLOE detector, for 3 months running at full luminosity, indicated that the error on  $a_{\mu}^{\text{vac-pol}}$  from the region below the the  $\phi$  could be as small as  $10^{-3}$ . The errors come from two distinct sources, *i.e.* 

- statistics, which indicate  $\delta_{\mu}^{\text{vac-pol,stat}} = 1.7 \times 10^{-11}$  for one year of run at full luminosity and pessimistic efficiencies for photon detection
- systematics, which give  $\delta_{\mu}^{\text{vac-pol,syst}} = 9.2 \times 10^{-11}$  from approximating the integral with a finite sum and because of the finite energy resolution of the Electromagnetic Calorimeter.

This method needs calculation of radiative corrections and full detector simulations. Particular attention will be needed to estimate the contribution of hard collinear radiation from the initial state which can affect the determination of the available energy to the hadronic system [49]. This proposed measurements relies uniquely on the photon detection and may be affected by errors due to misidentification of the photon, in particular the possible sources of brackground are

- $\phi \to \pi^+ \pi^- \pi^0$  which appears as  $\pi^+ \pi^- \gamma$  if one of the photons from the  $\pi^0$  goes undetected ( as it is the case if  $E_{\gamma} \leq 20$  MeV). This process has a large cross-section and it should not be undeestimated.
- $\phi \to \pi^+ \pi^- \gamma$ , with the photon emitted from the final state pions, *i.e.* due to FSR.

The background due to FSR is particularly dangerous for low photon energies, as one see from Fig. 9 from Ref. [48]. In such case, a cut on the photon energy is mandatory to reduce the uncertainty on the determination of the energy of the hadronic system. An interesting possibility [50] is to measure both the photon energy, using the photon as a trigger to select the energy of the hadronic system, as well as the  $Q^2$  of the final state. This method takes advantage of the high precision of the hadronic calorimeter in KLOE. In particular, present studies are concentrating on the determination of the  $\rho$  excitation curve and the two pion final state. Final State Radiation is still a dangerous source of background when the energy of the photon is a few ten MeV's. Simulation and detailed suggestion for cuts both on the photon and the pions are proposed in Ref. [50].



Fig. 9. Cross-section for the process  $e^+e^- \to \pi^+\pi^-\gamma$  at  $\sqrt{s} = M_{\phi}^2$ , with the photon emitted from the initial electrons (heavy line) or the final pions (thin line).

#### 5. Conclusion

Some of the highlights of the DA $\Phi$ NE physics program have been been presented, namely the measurement of direct CP-violation, rare K-decays and the measurement of the hadronic  $e^+e^-$  cross-section. Many other interesting measurements, in addition to the ones discussed here, will be performed and we present in Table VI a summary of what can be expected at DA $\Phi$ NE in the future years of operation 5.

## TABLE VI

The physics spectrum that can be explored by the  $e^+e^-\text{DA}\Phi\text{NE}$  collider in the next 5 years and physics results which can be expected. Estimates are based on a design luminosity of  $5 \times 10^{32} \text{ cm}^{-2} \text{sec}^{-1}$ .

$\sqrt{s}$ in MeV	$500 \div 1000$	1020	$1100 \div 1500$
CP/CPT		arepsilon'/arepsilon	
K-decays precision		up to $10^{-8} \div 10^{-9}$	
$\eta,\eta' \ { m physics}$		$\phi \to \eta, \eta' \gamma$	
$\pi\pi$ phase-shifts		$\begin{array}{c} \phi \to K\bar{K} \\ K \to e\nu\pi\pi \end{array}$	
Scalar and Pseudoscalar meson structure		$\phi  ightarrow a_0/f_0\gamma$ $ ightarrow \eta'\gamma  ightarrow ( ho,\omega)\gamma\gamma$	$ \begin{array}{c} \rho, \omega (1450) \\ \rightarrow (\eta, \eta', \eta^*) \gamma \end{array} $
Light Vector mesons	VMD and ChPT refined	the $q\bar{q}$ content of $\phi, \omega$	
Higher Vector Mesons			decay width mass parameters
Scalar Glueballs		$\phi  ightarrow a_0/f_0\gamma$	
$\sigma_{\text{total}} \rightarrow e^+ e^-$ and hadronic contribution to $(g-2)_{\mu}$	presently most of the error (60%) comes from here	5% of the error	15%

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