STOCHASTIC RESONANCE IN A SYSTEM OF COUPLED CHAOTIC OSCILLATORS*

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Noise-free stochastic resonance is investigated numerically in a system of two coupled chaotic Rössler oscillators. Periodic signal is applied either additively or multiplicatively to the coupling term. When the coupling constant is varied the oscillators lose synchronization via attractor bubbling or on–off intermittency. Properly chosen signals are analyzed which reflect the sequence of synchronized (laminar) phases and non-synchronized bursts in the time evolution of the oscillators. Maximum of the signal-to-noise ratio as a function of the coupling constant is observed. Dependence of the signal-to-noise ratio on the frequency of the periodic signal and parameter mismatch between the oscillators is investigated. Possible applications of stochastic resonance in the recovery of signals in secure communication systems based on chaotic synchronization are briefly discussed.

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1. Introduction

Stochastic Resonance (SR; for recent review see [1]) occurs in certain, mainly nonlinear, systems driven by a combination of periodic and stochastic signal. In such systems the input noise intensity may be chosen so that a periodic component of the output signal is maximized against the output noise. The Power Spectrum Density (PSD) $S(f)$ of the output signal in systems with SR consists of peaks at the multiples of the input periodic signal frequency f_s, superimposed on a broad noise background $S_N(f)$. A good measure of SR is the signal-to-noise ratio (SNR) in dB at frequency f_s which is defined as SNR$= 10\log\left[S_P(f_s)/S_N(f_s)\right]$, where $S_P(f_s) = S(f_s) - S_N(f_s)$ is the first peak height. As a function of the input noise power SNR has a maximum in systems with SR. This phenomenon has been observed in

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various systems, *e.g.* in bistable [2] and monostable systems [3], in dynamical [4] and non-dynamical [5,6] threshold-crossing systems, in spatially extended systems [7], spiking neurons [4, 8].

A similar phenomenon is noise-free SR [9]. In this case, the changes in the internal chaotic dynamics of the system are used to increase the periodicity of the output signal and the external noise is not necessary. SNR shows a maximum as a function of the system control parameter. Noise-free SR was observed in chaotic maps with crises and intermittency [9–11], in numerical experiments with chaotic oscillators [12, 13], in chaotic neurons [14] and in a physical experiment with spin-wave chaos [15]. In a previous paper, preliminary results on noise-free SR in a system of coupled chaotic oscillators at the edge of synchronization were reported [16]. This is a special case of SR in on–off intermittency and attractor bubbling [11]. In the present paper these results are extended and detailed, and the applicability of SR for signal detection in secure communication based on synchronization of chaotic oscillators [17] is briefly discussed.

2. Model and methods of analysis

The system under study is a set of two chaotic Rössler oscillators which are mutually coupled via the y variable (coupling strength k). A small periodic signal $s(t)$ is either added to the coupling term in one of the oscillators (additive periodic forcing with amplitude δ) or it modulates the coupling strength of both oscillators (multiplicative periodic forcing with amplitude ε). The equations of motion are

$$\begin{aligned}
\dot{x}_1 &= -(y_1+z_1) , \qquad \dot{x}_2 = -(y_2+z_2) , \\
\dot{y}_1 &= x_1 + a_1 y_1 + k\left[1+\varepsilon s(t)\right](y_2-y_1) , \\
\dot{y}_2 &= x_2 + a_2 y_2 + k\left[1+\varepsilon s(t)\right]\left[y_1-y_2+\delta s(t)\right] , \\
\dot{z}_1 &= b + z_1(x_1-c) , \quad \dot{z}_2 = b + z_2(x_2-c) . \qquad (1)
\end{aligned}$$

The parameters are $b=0.2$, $c=10$, and usually $a_1=a_2=0.2$, but small deviations between a_1 and a_2 are allowed to model mismatch of parameters in an experimental system. The cases $\delta \neq 0, \varepsilon = 0$ and $\delta = 0, \varepsilon \neq 0$ are discussed separately.

For $\varepsilon=\delta=0$, $a_1=a_2$ and $k>k_{\mathrm{c}}\approx 0.12$ it was observed that the two oscillators show synchronized chaotic behaviour [18] (for review of chaotic synchronization see [19]), *i.e.* if $\boldsymbol{x}_1=[x_1,y_1,z_1]$ and $\boldsymbol{x}_2=[x_2,y_2,z_2]$ are the state vectors of the oscillators 1 and 2, respectively, then, after all transients die out, $\boldsymbol{x}_1(t)=\boldsymbol{x}_2(t)$. This equality defines a three-dimensional manifold in a six dimensional phase space to which the motion of the system (1) is constrained. If $k<k_{\mathrm{c}}$ the oscillators lose synchronization. For k just below

k_c the periods of synchronized behaviour are interrupted by chaotic bursts during which $\boldsymbol{x}_1 \neq \boldsymbol{x}_2$, so if the distance between trajectories $d = ||\boldsymbol{x}_1 - \boldsymbol{x}_2||$ is measured, this results in a sequence of laminar phases, during which $d \approx 0$, and bursts. This is an example of on–off intermittency [20, 21]. If a small perturbation is added in the direction transverse to the synchronization manifold, or if there is a small mismatch between the parameters of the coupled system, chaotic bursts occur even for $k > k_c$. This phenomenon is called attractor bubbling [22] and it is caused by the transverse instability of periodic orbits embedded within the synchronized attractor [23, 24]. The bursts occur more frequently for increasing $k_c - k$ and for stronger transverse perturbation or mismatch [20, 22].

Let us start with the case $\varepsilon = 0$, $\delta \neq 0$. In Eq. (1) the transverse perturbation is $\delta s(t)$. If $k \gg k_c$ and $\delta \ll 1$ the oscillators are almost perfectly synchronized and the variable $\Delta y(t) = y_1(t) - y_2(t) + \delta s(t)$ fulfils the equality $\Delta y(t) \approx \delta s(t)$. Thus, if the transmitted signal from the oscillator 1 (transmitter) is $y_1(t) + \delta s(t)$, its periodic component $\delta s(t)$ may be recovered almost without distortion as equal to $\Delta y(t)$ at the location of the oscillator 2 (receiver). This is the idea of chaotic masking technique used for secure communication [17].

In this paper the case when k is close to k_c is investigated. Then the quality of recovering $\delta s(t)$ as $\Delta y(t)$ decreases, as chaotic bursts are superimposed on the information signal. As the perturbation $\delta s(t)$ is periodic, the sequence of laminar phases and bursts has a periodic component. The bursts and laminar phases can be distinguished by passing $\Delta y(t)$ through a threshold device. It was shown that signals from systems with on–off intermittency and attractor bubbling, passed through a threshold, show SR, i.e. SNR measured from such signals shows maximum as a function of the control parameter (coupling strength k) [11, 16], like in the case of SR in other threshold devices [4, 5]. This idea is applied to the signal $\Delta y(t)$. Thus we try to use SR to improve the quality of recovery of the transmitted signal if the transmitter and receiver are not perfectly synchronized, instead of increasing k in order to synchronize them and thus to recover $\delta s(t)$ without distortion.

The following signals were considered. First, for $s(t) = 1 + \cos 2\pi f_s t$ we define $Y^{(1)}(t) = \Theta(|\Delta y(t)| - \vartheta)$, where $\Theta(\cdot)$ is the Heaviside unit step function, and ϑ is a threshold. Typically $2\delta < \vartheta$ is set, so a typical condition for SR is fulfilled. The signal $Y^{(1)}(t)$ is a sequence of 0 and 1 which distinguishes only laminar phases from bursts; this is in agreement with the character of $0 \leq s(t) \leq 2$. Second, for $s(t) = \cos 2\pi f_s t$ we define $Y^{(2)}(t) = \mathrm{sign}[\Delta y(t)]\,\Theta(|\Delta y(t)| - \vartheta)$, $\delta < \vartheta$. The signal $Y^{(2)}(t)$ is a sequence of -1, 0 and 1 which distinguishes not only laminar phases from bursts, but also takes into account the agreement between the sign of $s(t)$

and $\Delta y\,(t)$ (if only bursts and laminar phases were distinguished, the fundamental peak would appear at $f_s/2$ in PSD of $Y^{(2)}\,(t)$ in this case). Below it is shown that SR is observed as k is varied close to $k_{\rm c}$ if the PSD is evaluated from $Y^{(1,2)}\,(t)$, for a wide range of frequencies f_s.

The case $\delta = 0, \varepsilon \neq 0$ has no direct analogy in secure communication. It is included in order to model SR with multiplicative periodic forcing. The periodic signal is $s\,(t) = 1 + \cos 2\pi f_s t$, the measured quantity is $\Delta y\,(t) = y_1\,(t) - y_2\,(t)$ and the signal analyzed is $Y^{(1)}\,(t)$ defined as above.

Eq. (1) for different k and f_s was solved using a fourth-and-fifth order Runge–Kutta method with permanent error control. The time series Δy, and $Y^{(1,2)}\,(t)$ were sampled; the sampling time varied with f_s and with the integration step. FFT with a square window was calculated from 4096 sampled points of the time series and typically 100 transforms were averaged to obtain PSD for a single time series. Further, SNR was calculated and normalized to a standard bandwidth $\Delta f = 1/8$Hz [2]. To obtain final results, SNR was then averaged typically over time series with 20 randomly chosen initial conditions and initial phases of $s\,(t)$ in Eq. (1).

3. Results and discussion

First the case of $\delta \neq 0$, $\varepsilon = 0$ is discussed. For the periodic signal $s\,(t) = 1 + \cos 2\pi f_s t$ we set $\delta = 0.03$, and for $s\,(t) = \cos 2\pi f_s t$ $\delta = 0.04$, and in both cases $\vartheta = 0.1$. Examples of the time series $\Delta y\,(t)$ are shown in Fig. 1(a)–(c) for three different frequencies f_s and for k below, but close to $k_{\rm c}$, and an example of the measured PSD is shown in Fig. 1(d). For $f_s = 1$Hz (Fig. 1(a)) periodic oscillations are much faster than the chaotic ones and the former are simply superimposed on the latter. The sequence of bursts and laminar phases has no periodic component, as the bursts are initiated at random times by the averaged influence of $\delta s\,(t)$. However, a periodic component of $Y^{(1)}\,(t)$ exists: every time when the chaotic component of $\Delta y\,(t)$ slowly approaches the threshold a sequence of threshold-crossing events occurs which is caused by the fast periodic component of the signal. Thus high SNR in PSD of $Y^{(1)}\,(t)$ is expected. For $f_s = 1/1024$Hz (Fig. 1(c)) the chaotic oscillations are much faster than the periodic ones. It can be seen that chaotic bursts occur mostly when $s\,(t)$ is at a maximum. Thus the sequence of bursts has a strong periodic component and again high SNR in PSD of $Y^{(1)}\,(t)$ is expected. For $f_s = 1/16$Hz (Fig. 1(b)) the frequency of periodic oscillations is comparable with the characteristic time scale of chaotic oscillations, and none of the above mechanisms works. Thus rather small SNR in PSD of $Y^{(1)}\,(t)$ is expected. The above suggestions are in general confirmed by a direct evaluation of SNR (Fig. 2(a)–(b)). In both cases when SNR is evaluated from $Y^{(1)}\,(t)$ and $Y^{(2)}\,(t)$ SNR has a maximum as a

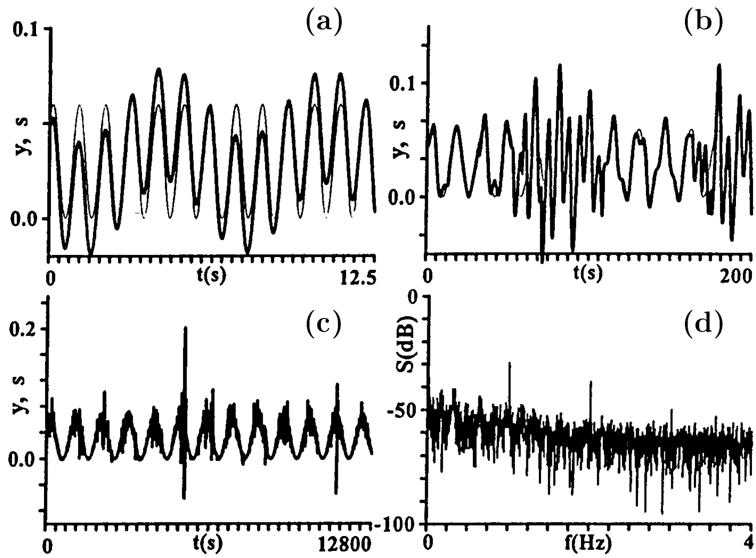

Fig. 1. The case of additive periodic signal $s(t) = 1 + \cos 2\pi f_s t$, $\delta = 0.03$. (a)–(c) Time series $\Delta y(t)$ (bold) and periodic signal $\delta s(t)$ (thin) for $f_s = 1$Hz (a), $f_s = 1/16$Hz (b) and $f_s = 1/1024$Hz (c), and for k such that SNR from $Y^{(1)}(t)$ is close to maximum; (d) PSD of the signal $Y^{(1)}(t)$ for $f_s = 1$Hz and $k = 0.105$.

function of k, and the location of the maximum is shifted towards greater k for lower f_s; this is typical of many dynamical systems showing SR [2]. SR is most visible (SNR values are highest) both for very high and very low f_s, and for f_s comparable with the characteristic time scale of chaotic oscillations of $\Delta y(t)$ SR is weak. Moreover, SR is much stronger for very high f_s than for very low f_s. Such dependence of SNR on f_s has not been observed in other systems with SR: *e.g.* in dynamical bistable systems SNR increases with decreasing f_s [2], while in non-dynamical threshold-crossing systems SNR is independent of f_s [5]. However, some analogies can be found. In the limit $f_s \longrightarrow 0$ the bursts are not single spikes, as in typical threshold devices, but they have a certain distribution of lengths. Thus *e.g.* the signal $Y^{(1)}(t)$ resembles the one obtained from an asymmetric bistable system (with two-state encoding 0, 1) rather than from a threshold-crossing system. This explains the increase of SNR for low frequencies. In the opposite limit $f_s \longrightarrow \infty$ the intervals $Y^{(1)}(t) = 1$ are often much shorter, like in threshold devices, as they appear when the fast periodic component crosses ϑ. It was shown by Jung [25] that in threshold devices SNR increases with f_s if the period $1/f_s$ is substantially lower than the correlation time of the input noise. Clearly, this is the case in Fig. 1(a), where the chaotic component of the signal is smooth in the time scale of $1/f_s$. In Ref. [16] it was suggested

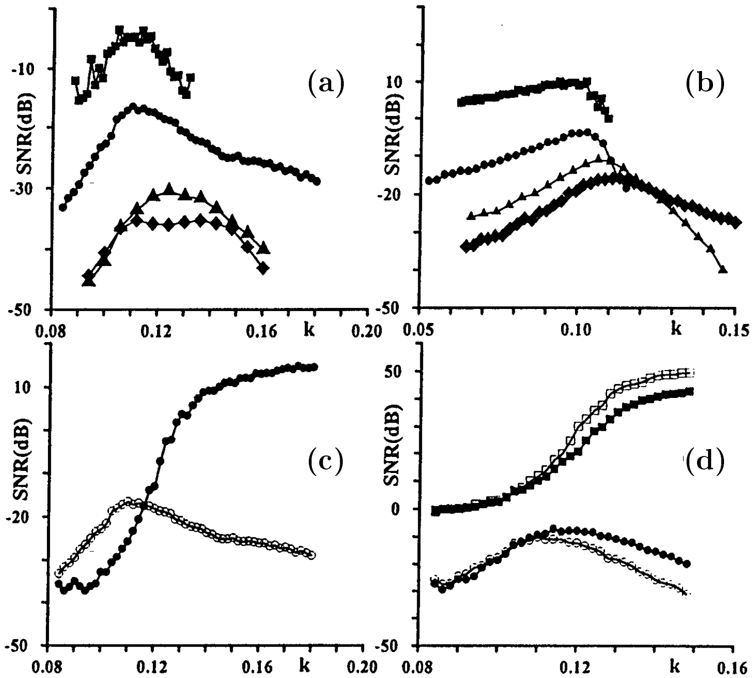

Fig. 2. (a),(c)–(d) The case of additive periodic signal $s(t) = 1+\cos 2\pi f_s t$, $\delta = 0.03$, (b) The case of additive periodic signal $s(t) = \cos 2\pi f_s t$, $\delta = 0.04$. (a) SNR *vs.* k for $f_s = 16$Hz (squares), $f_s = 1/16$Hz (circles), $f_s = 1/256$Hz (diamonds), $f_s = 1/1024$Hz (triangles); (b) SNR *vs.* k for $f_s = 16$Hz (squares), $f_s = 1$Hz (circles), $f_s = 1/16$Hz (diamonds), $f_s = 1/256$Hz (triangles); (c) SNR *vs.* k for $f_s = 1/16$Hz and the signal $Y^{(1)}(t)$ (empty circles) and $\Delta y(t)$ (filled circles). (d) Effect of the parameter mismatch on SNR *vs.* k, $f_s = 1$Hz, empty symbols for $a_2 = a_1$, filled symbols for $a_2 = a_1 + 10^{-4}$, circles for the signal $Y^{(1)}(t)$, squares for the signal $\Delta y(t)$.

that the increase of SNR for both $f_s \longrightarrow 0$ and $f_s \longrightarrow \infty$ distinguishes noise-free (chaotic) SR from SR in systems with input noise. The above analysis reveals that this is not necessarily true. In the limit of low f_s SR is a dynamical phenomenon. The influence of periodic signal modifies the system dynamics, and introduces periodicity in the time sequence of laminar phases and bursts. Thus the increase of SR with decreasing periodic signal frequency is expected both here and in other chaotic systems with SR. In the limit of high f_s, however, the effect is non-dynamical: $\Delta y(t)$ is a direct sum of the fast periodic and slow chaotic component. This is caused by the definition of $\Delta y(t)$ which contains both input periodic signal and system variables. In other chaotic systems such a separation of time scales need not be possible (in particular, when only dynamical variables are measured at the output) and SNR will decrease to zero with increasing f_s. Thus the increase of SNR in the limit of high frequencies of the periodic signal is rather a particular property of the system and signals under study.

In the case of $\delta \neq 0$, $\varepsilon = 0$ SR is obtained only if SNR is evaluated from the PSD of $Y^{(1,2)}(t)$, but not directly from $\Delta y(t)$. In the latter case SNR increases monotonically with k (Fig. 2(c)). Thus the source of SR here is passing the signal through a threshold. Moreover, SNR calculated from $\Delta y(t)$ usually exceeds the one obtained from $Y^{(1,2)}(t)$ for a whole range of k, independently of f_s. This is typical in SR. An exception is the case of periodic signal frequencies comparable with the characteristic time of chaotic oscillations of the system (1). As shown in Fig. 2(c) SNR from $Y^{(1)}(t)$ surpasses that from $\Delta y(t)$ for k in a narrow interval below k_c. It may be an interesting example of the increase of SNR by SR [6]. However, this is only a result of a numerical experiment, so further study is necessary to explain the origin of this result. Besides, there are two limitations for the application of SR for enhanced recovering of signals in secure communication, at least in the system (1). First, the increase of SNR by SR occurs in the frequency range where SNR from both $Y^{(1,2)}(t)$ and $\Delta y(t)$ is low. Second, the maximum value of SNR obtained from the signal passed through a threshold is for all investigated frequencies much lower than that obtained directly from the PSD of $\Delta y(t)$ in the limit $k \longrightarrow \infty$.

The effect of parameter mismatch on SNR evaluated from $\Delta y(t)$ and $Y^{(1,2)}(t)$ may be different. This is shown in Fig. 2(d) for $a_1 = 0.2$, $a_2 = a_1 + 10^{-4}$ in Eq. (1). In the former case small parameter mismatch decreases SNR, while in the latter case SNR is increased and the maximum of SNR is shifted towards larger k. This is true for a whole range of f_s investigated, if the mismatch is small in comparison with ϑ. However, in the system under study this rise of SNR is not big and in the limit of large k SNR from $\Delta y(t)$ is still much higher. Thus, although the rise of SNR with parameter mismatch between the transmitter and receiver may be useful from the point of view of signal detection in secure communication, there may be no real gain of this effect in practical applications.

For completeness the results for the system (1) with $\delta = 0$ and $\varepsilon \neq 0$ are now presented. The parameters $\varepsilon = 0.01$ and $\vartheta = 0.1$ were assumed. In the case of multiplicative periodic signal it was shown [11] that SR may be observed not only when the signal $Y^{(1)}(t)$, but also $\Delta y(t)$ is analyzed. In the system (1) SR in the latter case was not observed; probably the amplitude ε was too small. It is also known that the rising part of the curves SNR *vs.* k may be very steep if the periodic signal is applied multiplicatively [11]. In the present case the rise was so fast that this part of the curves was difficult

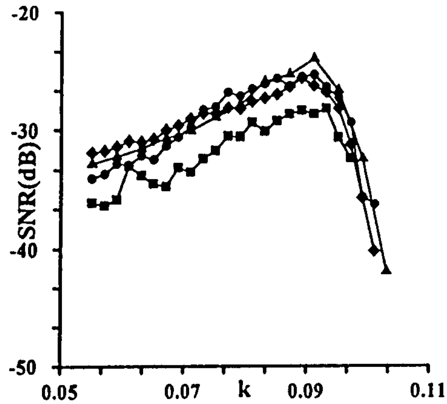

Fig. 3. The case of multiplicative periodic signal $s(t) = 1 + \cos 2\pi f_s t$, $\varepsilon = 0.01$. SNR *vs.* k for $f_s = 1/16$Hz (squares), $f_s = 1/32$Hz (circles), $f_s = 1/64$Hz (diamonds), $f_s = 1/1024$Hz (triangles).

to observe. In order to obtain distinct maximum the system with a small parameter mismatch $a_1 = 0.2$, $a_2 = a_1 + 10^{-8}$ was investigated. Even such a small mismatch resulted in smooth curves with maxima (Fig. 3). Such a strong influence of the perturbations perpendicular to the synchronization manifold on system dynamics is typical in systems with on–off intermittency; simply, attractor bubbling for non-zero perturbations occurs. In Fig. 3 it can be seen that SNR increases for decreasing f_s and in the adiabatic limit $f_s \longrightarrow 0$ it becomes frequency-independent. This is in agreement with the results obtained in a system with on–off intermittency and discrete time [11,16]. Such behaviour of SNR is also typical in dynamical systems: again, in the limit of low frequencies of the periodic signal the sequence of 0 and 1 in $Y^{(1)}(t)$ resembles that from an asymmetric bistable system.

4. Summary and conclusions

In this paper noise-free SR was studied in a system of two mutually coupled chaotic Rössler oscillators at the edge of their chaotic synchronization. Periodic signal was applied either additively or multiplicatively to the coupling term, and the measured signal was the difference between the coupled variables of the oscillators, with periodic signal added in the case of additive periodic forcing. This signal at $k \approx k_c$ showed a sequence of laminar phases and bursts characteristic of on–off intermittency and attractor bubbling. After passing this signal through a threshold SNR was measured from the PSD as a function of k, and curves with maxima vere obtained which indicates the occurrence of SR. In the case of additive periodic signal SR was most visible for high frequencies of the periodic signal (which was interpreted as a non-dynamical effect) and low frequencies (which is typical of dynamical asymmetric bistable systems showing SR). In the case of multiplicative periodic signal SNR increased with decreasing frequency.

For additive periodic signal the question if it is possible to use SR as a signal detection tool in secure communication was briefly discussed. There are two differences between the system (1) and typical systems used for secure communication [17]. First, here coupled oscillators are used instead of systems based on the complete replacement technique [17, 19]. The possibility to change the degree of synchronization by means of varying k offers greater flexibility and enables the occurrence of noise-free SR. Second, in this paper the case of mutual coupling of chaotic oscillators was considered. In secure communication, a more natural choice is to have one-way (from transmitter to receiver only) coupling. However, from the point of view of the chaotic synchronization problem a mutually coupled system (1) is equivalent to analogous system with one-way coupling and the coupling constant $k/2$ [19], thus the results of the present paper should be valid in the latter case, too. If the frequency of the periodic signal was comparable with the characteristic time scale of the chaotic oscillations in the measured signal, SNR obtained after passing the signal through a threshold exceeded that from the full measured signal in a narrow interval of k below the synchronization threshold. Also small parameter mismatch could lead to increase of SNR. However, these effects were weak and did not lead to improvement of SNR in comparison with that evaluated from $\Delta y\,(t)$ in the limit $k \longrightarrow \infty$. In general, the increase of SNR due to SR should not be expected.

Of course these results are not decisive and the problem of applicability of SR in secure communication requires more careful study. In particular, a broader range of signal amplitudes and thresholds, and other methods of communication (*e.g.* periodic modulation of the transmitter parameters [17]) should be investigated. At the present stage it may be concluded that SR occurs in coupled oscillators at the edge of synchronization and that it may be used to improve SNR in chaotic masking if the range of possible changes of k is constrained. In this case, particularly high SNR is obtained for high frequencies of periodic signal (but still lower than that obtained directly from $\Delta y\,(t)$), but one should remember that masking high-frequency signals is usually not effective as the PSD of chaotic systems at high frequencies decreases.

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