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STOCHASTIC RESONANCE IN SPIN-WAVE CHAOS: A SIMULATION*

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A model for noise-free stochastic resonance in chaotic nonlinear ferromagnetic resonance in coincidence regime is investigated numerically. In the model, interactions between the uniform mode and two pairs of parametric spin waves are taken into account. For certain values of the model parameters Pomeau–Maneville intermittency and on-off intermittency are observed. The case of slow periodic modulation of the rf field amplitude is considered and the signal analyzed reflects the sequence of laminar phases and bursts. Maximum of the signal-to-noise ratio is observed as the constant part of the rf field amplitude is varied in the neighbourhood of the intermittency threshold. The role of the thermal excitations of spin waves in the occurence of stochastic resonance is clarified. The results are in agreement with the recent experimental observations of stochastic resonance in spin-wave chaos.

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1. Introduction

Stochastic resonance (SR; for review see [1]) occurs in certain, mainly nonlinear systems driven by a combination of periodic and stochastic signal. An unexpected feature of SR is that when the input noise intensity is increased the degree of periodicity of the output signal goes through a maximum. The power spectrum density (PSD) of the output signal $S\,(f)$ in systems with SR consists of peaks at integer multiples of the periodic signal frequency f_{s} superimposed on a broad noise background $S_{\mathrm{N}}\,(f)$.

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A good measure of the periodicity of the output signal is the signal-to-noise ratio (SNR) in dB defined as SNR$=10\log\left[S_{\mathrm{P}}\left(f_{\mathrm{s}}\right)/S_{\mathrm{N}}\left(f_{\mathrm{s}}\right)\right]$, where $S_{\mathrm{P}}\left(f_{\mathrm{s}}\right)=S\left(f_{\mathrm{s}}\right)-S_{\mathrm{N}}\left(f_{\mathrm{s}}\right)$ is the first peak height. In system with SR, SNR has a maximum for non-zero input noise power. SR was observed in various systems, *e.g.* in bistable [2] and monostable [3] systems, in dynamical [4] and non-dynamical [5,6] threshold-crossing systems, in sensory neurons [4,7] and spatially extended systems [8].

In this paper we deal with noise-free SR [9]. By varying a control parameter of a chaotic system with periodic input, and thus by changing its internal dynamics, it is possible to maximize the periodic response of the system without external noise, *i.e.* to maximize SNR. Noise-free SR was observed in numerical simulations of chaotic oscillators [10-12], networks of chaotic neurons [13], and maps with various kinds of intermittency [9,14,15]. Recently, noise-free SR was observed in a physical experiment with intermittency in spin-wave chaotic dynamics in a ferromagnetic sphere [16]. This experiment was performed in perpendicular ferromagnetic resonance in coincidence regime, *i.e.* the uniform mode with frequency ω_0 was driven by the rf field with frequency $\omega=\omega_0$. As the rf field amplitude is increased beyond a certain threshold, in such a system the first-order Suhl instability occurs which consists in the decay of the uniform mode into pairs of parametric spin waves (SW) with opposite wave vectors and frequencies $\omega/2$. As a result of nonlinear interactions between SW and the uniform mode chaos in the time dependence of absorption in the sample may appear [18-21] (for review of various nonlinear effects due to SW interactions see *e.g.* [22]). In Ref. [16] the experimental conditions were chosen so that the time series of absorption showed Pomeau–Maneville intermittency (PMI) of type-III [23], *i.e.* the time dependence of absorption was a sequence of periodic (laminar) phases and chaotic bursts. When either the rf field amplitude or the dc field were varied the mean duration of laminar phases and bursts also changed. To observe SR, a weak and slow periodic component was added to the rf field amplitude The sequence of laminar phases and bursts was converted into a two-state signal and SNR was measured from the PSD of this signal. SNR showed a maximum *vs.* both the rf field amplitude and the dc field for f_{s} in the range between 1 kHz and 10 kHz.

In Ref. [16] the occurence of SR in a system with intermittency was explained with the help of a model map. It was shown both analytically and numerically that in systems with intermittency the laminar and chaotic phases can play a role of the two states of an asymmetric bistable system which enables the appearance of SR when periodic signal is added. However, a direct numerical simulation based on the equations of motion for SW amplitudes was not performed, as accurate simulations of chaotic absorption require the inclusion of many interacting SW in the model, thus making the simulations extremely time-consuming [18, 21]. On the other hand in SW chaos low-dimensional models with only few interacting SW pairs can lead to qualitative agreement with experiment (see *e.g.* [24–27]). For coincidence regime such a model was considered in Ref. [27]. In the present paper simulations of SR in two kinds of intermittency in SW chaos in coincidence regime are performed on the basis of a simple model: PMI and on-off intermittency (OOI) [28, 29] (OOI was also observed in coincidence regime [27, 30]). In the model only two SW pairs interact with the uniform mode. With the rf field amplitude periodically modulated, SNR obtained from the sequence of laminar phases and bursts shows a maximum as a function of the constant part of the rf field amplitude. The estimated frequency range of the modulation for which SR may be observed is in agreement with the above-mentioned experiment. The role of thermal excitations of SW in the occurence of SR is discussed, too.

2. Model and methods of analysis

The model for chaos and SR in coincidence regime analyzed in this paper is a modification of that in Ref. [27]. The transverse rf field with frequency ω is assumed in the form $h_T(t)\cos\omega t$, where $h_T(t)$ is a periodically modulated amplitude of the rf field, with the modulation frequency $2\pi f_\mathrm{s} \ll \omega$. This field excites the uniform mode with frequency $\omega_0 \approx \omega$. In the first-order Suhl instability process, this mode decays into pairs of SW with opposite wave vectors $\pm k$ and frequencies $\omega_k \approx \omega/2$. Small detunings from the resonant frequencies $\Delta\omega_0 = \omega_0 - \omega$, $\Delta\omega_k = \omega_k - \omega/2$ are allowed; they are necessary for the occurence of chaos in the model, and may be attributed to finite sample dimensions or finite resonance linewidths [27]. The equations of motion for the complex uniform mode and SW amplitudes (c_0, $c_{\pm k}$, respectively) may be then obtained from the Hamiltonian $\mathcal{H}$ written in terms of creation and annihilation operators

$$\mathcal{H} = h_T(t)\cos\omega t\,(I_0 c_0^\star + \mathrm{c.c.}) + \sum_k \omega_k c_k^\star c_k + \sum_k V_{0,k} c_0^\star c_k c_{-k} + c.c. \qquad (1)$$

Replacing the operators with classical SW amplitudes yields the equations of motion in the form

$$\frac{\partial c_{0,k}}{\partial t} = -\eta_{0,k} c_{0,k} + i\frac{\partial\mathcal{H}}{\partial c_{0,k}^\star}\,, \qquad \frac{\partial c_{0,k}^\star}{\partial t} = -\eta_{0,k} c_{0,k}^\star - i\frac{\partial\mathcal{H}}{\partial c_{0,k}}\,. \qquad (2)$$

Here, I_0 is the coupling coefficient between the rf field and the uniform mode, $V_{0,k}$ are coupling coefficients between the uniform mode and parametric SW pairs, $\eta_{0,k}$ is a phenomenological damping and the star denotes the complex conjugate. It may be shown that the amplitudes of SW belonging to one pair differ only by a constant phase factor q_k [22], *i.e.* $c_{-k} = c_k \exp(iq_k)$. If the rf field amplitude is small only the uniform mode is excited and all SW amplitudes decay to zero. The threshold for the excitation of SW pair with wave vector k is $h_{T,\mathrm{crit}}^{(k)} = 2\,|\delta_0|\,|\delta_k| \,/\, |I_0|\,|V_{0,k}|$, where $\delta_{0,k} = \eta_{0,k} + i\Delta\omega_{0,k}$. In fact, as the rf field amplitude is increased, first the SW pair with the lowest threshold (denoted as 1 and called the critical pair) is excited at the first-order instability threshold $h_{T,\mathrm{crit}} = h_{T,\mathrm{crit}}^{(1)}$, and for higher h_T other pairs may but need not be excited; also periodic and chaotic states of the mode amplitudes may appear.

In the following a simplified model will be considered, with only two SW pairs interacting with the uniform mode. Such low-dimensional models are widely used in modelling SW chaos [24–27] though there is no good theoretical justification for such a simplification. Eq. (2) may be rewritten in dimensionless variables with separated fast time dependence, $a_0(t) = |V_{0,1}|\, c_0(t) \exp(i\omega t)/\eta_1,\ a_k(t) = |V_{0,1}|\, c_k(t) \exp(iq_k/2) \exp(i\omega t/2)/\eta_1$. SW pair 2 has a higher instability threshold and is called a weak pair. After introducing the rescaled time $t' = \eta_1 t$ the final form of the equations is

$$
\begin{aligned}
\dot{a}_0 &= |\delta_0/\eta_1|\,|\delta_1/\eta_1|\,\varepsilon(t) - (\eta_0/\eta_1 + i\Delta\omega_0/\eta_1)\,a_0 - ia_1^2 \\
&\quad -i\left(|V_{0,2}| \,/\, |V_{0,1}|\right) a_2^2 + \xi_{\mathrm{th}}\,, \\
\dot{a}_1 &= -\left(1 + i\Delta\omega_1/\eta_1\right) a_1 - ia_1^\star a_0 + \xi_{\mathrm{th}}\,, \\
\dot{a}_2 &= -\left(\eta_2/\eta_1 + i\Delta\omega_2/\eta_1\right) a_2 - i\left(|V_{0,2}| \,/\, |V_{0,1}|\right) a_2^\star a_0 + \xi_{\mathrm{th}}\,. \qquad (3)
\end{aligned}
$$

In Eq. (3) the dot denotes the time derivative with respect to t' and $\varepsilon(t) = h_T(t)\,/h_{T,\,\mathrm{crit}}$. In addition, the thermal excitation level of SW $\xi_{\mathrm{th}} \ll 1$ is phenomenologically introduced. Absorption in the sample is proportional to $n_0 = |a_0|$.

For the two sets of parameters listed in Table I and constant rf field amplitude ε two kinds of intermittency were observed in Eq. (3). First, OOI was observed when ε was decreased below $\varepsilon_c \approx 3.02$ [27]. In the time series of $n_2 = |a_2|$ a sequence of laminar phases, during which the weak pair amplitude decreases almost to zero, and chaotic bursts, during which the level of excitation of the weak SW pair is comparable with that of the critical pair, occured; such behaviour is typical of OOI. Second, PMI with periodic laminar phases and chaotic bursts was observed above $\varepsilon_c = 7.96$. In order to observe SR the rf field amplitude was assumed as $\varepsilon(t) = \varepsilon + A\cos 2\pi f_{\mathrm{s}} t$. With small A, varying ε in the neighbourhood of ε_c influences the mean duration of laminar phases. Moreover, with small f_{s} it is clear that the sequence of laminar phases and bursts should have a strong periodic component, which suggests the possibility of the occurence of SR when ε is varied. It should be pointed out that intermittency in Eq. (3) is not atypical and occurs in a broad neighbourhood of the parameters from Table I in the parameter space.

TABLE I

Numerical parameters for Eq. (3).

	η_0/η_1	$\Delta\omega_0/\eta_1$	$\Delta\omega_1/\eta_1$	η_2/η_1	$\Delta\omega_2/\eta_1$	$\lvert V_{0,2}\rvert / \lvert V_{0,1}\rvert$	A	ε_c
OOI	1.25	-1.5	3.0	0.8	2.62	0.754	0.1	3.02
PMI	1.67	-1.67	3.33	1.0	-4.67	1.048	0.2	7.96

Eq. (3) was solved numerically using the fourth-and-fifth order Runge–Kutta method with continuous error control. Two examples of the time series of n_2 for the case of OOI and PMI in the presence of periodic component of the rf field amplitude are shown in Fig. 1(a), (b). For further analysis only the sequence of laminar phases and bursts is important. To distinguish between them the signal $Y(t) = \Theta(n_2(t) - \vartheta)$ was analyzed, where $\Theta(\cdot)$ is the Heaviside unit step function and the threshold ϑ was assumed 0.1 for OOI and 3.7 for PMI. Such distinction between the phases is not perfect, in particular in the case of PMI in which n_2 may cross the threshold many times during a chaotic burst (*cf.* Fig. 1(b)). However, more sophisticated methods in which the height of the local maxima in the signal $n_2(t)$ was analzyed yield results qualitatively similar to the ones presented below for $Y(t)$. A second factor which influences the quantitative results is the choice of ϑ. Besides, one should pay attention to the difference between the method used here to distinguish between the phases, and the experimental methods which can only be based on the measurement of absorption, *i.e.* n_0. In the present case, n_2 was chosen for further analysis simply because the difference between the phases was most visible.

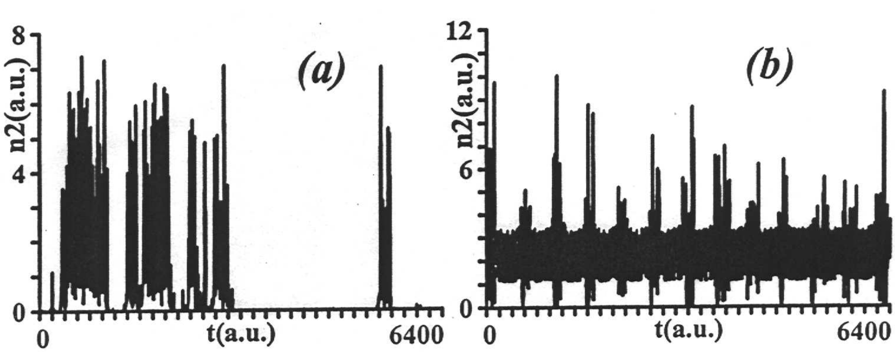

Fig. 1. Time series of $n_2(t) = |a_2(t)|$ for the case of (a) OOI, (b) PMI.

From $Y(t)$ PSD and SNR were evaluated as functions of ε. Typically 100 series of 4096 points sampled with the time step $\Delta t' = 1.0$ or 4.0 (in renormalized time units) were averaged to obtain PSD, and then the result was further averaged over five different initial conditions in Eq. (3) and initial phases of the periodic signal. PSD was obtained from FFT of $Y(t)$ with square window. From PSD, SNR was evaluated and normalized to a standard bandwidth [2] $\Delta f' = 1/4096$ Hz in rescaled time units (*i.e.* c.a. $\Delta f \approx 1/4096$ MHz$= 0.24$ kHz in real time units).

3. Results

The results of numerical modelling of SR in spin-wave chaos were obtained for a range of $f_{\rm s}$ and $\xi_{\rm th}$. Typical values of SW damping $\eta_{0,k}$ are on the order of $10^6\,{\rm s}^{-1}$, thus the investigated frequencies $f_{\rm s}$ ranged from 1/512MHz $\approx$ 1.95kHz to 1/32MHz $\approx$ 31.25kHz in real time units; this is the same frequency range as in the experiment of Ref. [16] (henceforth, all frequencies are given in real time units). The order of magnitude of $\xi_{\rm th}$ is 10^{-10} in liquid helium temperature and 10^{-9} in room temperature [31]; for comparison, the cases with $\xi_{\rm th} = 0$ (zero temperature) and $\xi_{\rm th} = 10^{-4}$ (unphysically large thermal excitations) were also considered. The amplitude A was assumed as 0.1 in the case of OOI and 0.2 in the case of PMI, thus A was small enough to remain inside the intermittency regime when ε was varied near ε_c.

SNR obtained from PSD of $Y(t)$ in the case of OOI and $\xi_{\rm th} = 10^{-10}$ is shown in Fig. 2(a) *vs* ε for different $f_{\rm s}$. For $f_{\rm s} \leq 1/32$MHz the curves with maxima were observed indicating the occurence of noise-free SR. For decreasing frequencies SNR increases and saturates in the adiabatic limit $f_{\rm s} \longrightarrow 0$. Such behaviour is typical of SR in *e.g.* bistable systems [2]; as mentioned in Sec. 1, intermittent systems from the point of view of the theory of SR are equivalent to asymmetric bistable systems [16]. The role of thermal excitations is shown in Fig. 2(b) for $f_{\rm s} = 1/128$MHz. In the limit $\xi_{\rm th} = 0$ the rising part of the curve SNR *vs* ε (looking in the direction of decreasing ε) is very steep. In fact, it is difficult to observe non-zero SNR in this range of the rf field amplitude. The addition of even small thermal background noise $\xi_{\rm th}$ smoothes out the curve in which a distinct maximum occurs. This situation is typical of systems with OOI in which addition of noise leads to qualitative changes in the dynamics and occurence of attractor bubbling below the OOI threshold [29]. For $\xi = 10^{-4}$ bursts and significant values of SNR are observed for ε much above $\varepsilon_c \approx 3.02$, out of the intermittency regime, and the location of the maximum shifts also out of the intermittency regime.

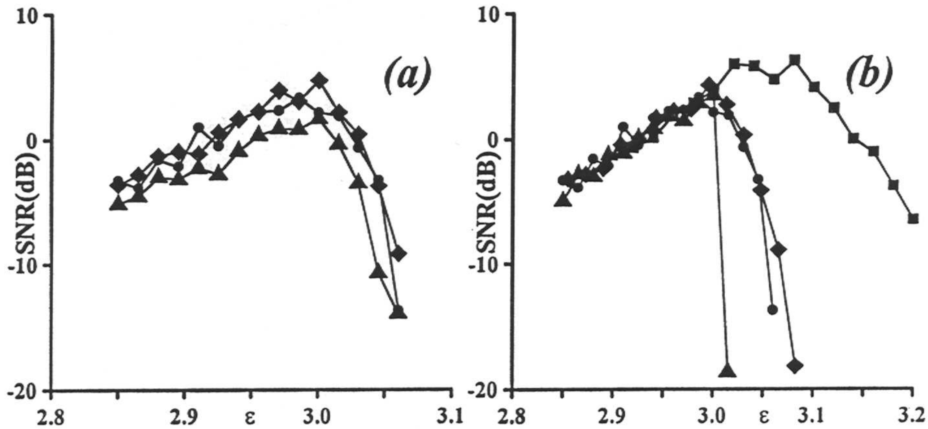

Fig. 2. SNR *vs* ε for the case of OOI, (a) effect of periodic signal frequency, $f_s = 1/32$MHz (triangles), $f_s = 1/128$MHz (circles), $f_s = 1/512$MHz (diamonds), in all cases $\xi_{th} = 1.0 \cdot 10^{-10}$, (b) effect of thermal SW excitations, $\xi_{th} = 0$ (triangles), $\xi_{th} = 10^{-10}$ (circles), $\xi_{th} = 10^{-9}$ (diamonds), $\xi_{th} = 10^{-4}$ (squares), in all cases $f_s = 1/128$MHz.

It is interesting to note that in the case of OOI SR may be observed also when PSD of $n_2(t)$ is analyzed (Fig. 3). The time series of n_2 resembles the sequence of 0 and 1 in $Y(t)$, thus leading to similar behaviour of SNR. SNR obtained after passing n_2 through a threshold exceeds that evaluated directly from n_2; the mechanism of this phenomenon is probably similar to the one described in threshold devices by Loerincz *et al.* [6].

SNR obtained from PSD of $Y(t)$ in the case of PMI and $\xi_{th} = 10^{-10}$ is shown as a function of ε in Fig. 4(a). The results are similar to the ones in the case of OOI. Fig. 4(a) shows that it is possible to model noise-free SR in PMI in SW chaos using a simple model Eq. (3). The role of thermal excitations of SW seems to be less visible in the case of PMI than in OOI (Fig. 4(b)), at least for the parameters A, ϑ used in this paper. In particular, there is no significant difference between the SNR *vs* ε curves for $\xi_{th} = 0$ and $\xi_{th} = 10^{-10}$ and 10^{-9}. However, large ξ_{th} causes evident changes in SNR. In the case of PMI, SR was not obtained when PSD of $n_2(t)$ was analyzed: SNR decreases monotonously with increasing ε. This is caused by the fact that periodic oscillations of the rf field amplitude lead to a periodic modulation (with frequency f_s) of the amplitude of periodic oscillations of n_2 below the intermittency threshold, for $\varepsilon < \varepsilon_c = 7.96$. Thus a periodic response of n_2 may be stronger below the intermittency threshold than the periodicity of chaotic bursts which can be seen in Fig. 1(b) above the intermittency threshold. The signal $Y(t)$ simply ignores the periodic

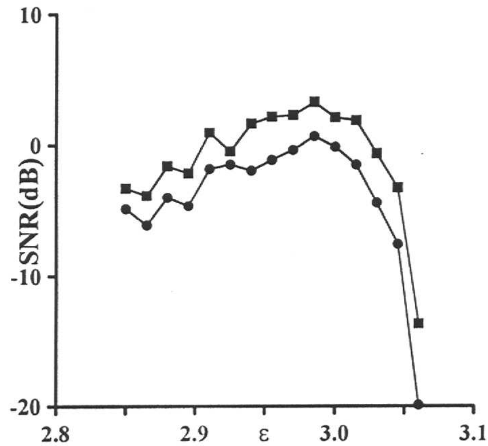

Fig. 3. SNR *vs* ε evaluated from $Y(t)$ (squares) and $n_2(t)$ (circles) for the case of OOI, $f_s = 1/128$MHz, $\xi_{th} = 10^{-10}$.

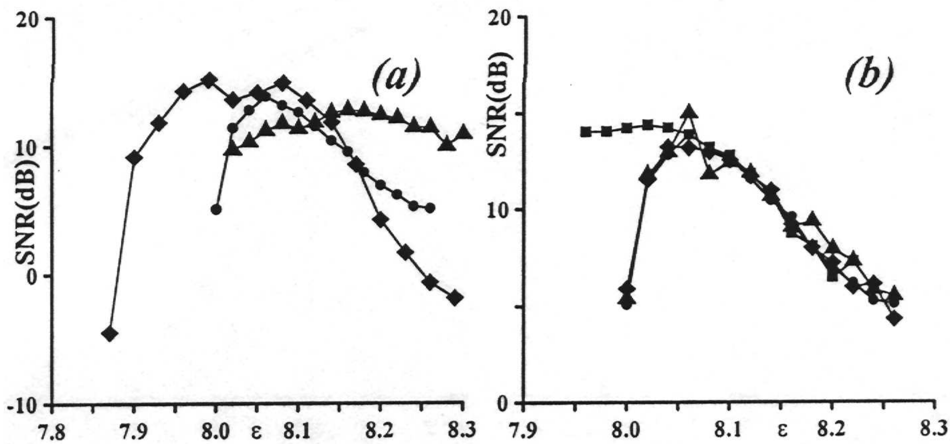

Fig. 4. SNR *vs* ε for the case of PMI, (a) effect of periodic signal frequency, $f_s = 1/32$MHz (triangles), $f_s = 1/128$MHz (circles), $f_s = 1/512$MHz (diamonds), in all cases $\xi_{th} = 1.0 \cdot 10^{-10}$, (b) effect of thermal SW excitations, $\xi_{th} = 0$ (triangles), $\xi_{th} = 10^{-10}$ (circles), $\xi_{th} = 10^{-9}$ (diamonds), $\xi_{th} = 10^{-4}$ (squares), in all cases $f_s = 1/128$MHz.

modulation of n_2 with frequency f_s during laminar phases, and thus also for $\varepsilon < \varepsilon_c$. This resembles the situation in bistable two-well systems in which SNR diverges in the limit of zero noise when the intrawell motion is included in the analysis [2].

4. Discussion and conclusions

In this paper, noise-free SR in chaotic nonlinear ferromagnetic resonance in coincidence regime was modelled numerically on the basis of a simple model of two SW pairs interacting with the uniform mode, Eq. (1). Two kinds of intermittent signals were analyzed: OOI and PMI. SNR was evaluated from a signal reflecting the sequence of laminar phases and bursts. When the rf field amplitude was slowly modulated with a weak periodic signal and the constant part of this amplitude was changed near the intermittency threshold, SNR showed a maximum, indicating the occurence of SR.

A quantitative comparison of the simulation results in the case of PMI with experimental results of Ref. [16] is not possible, however, because such details as the amplitude of periodic modulation and the range of investigated rf field amplitudes were not given. From other studies it is known that PMI in coincidence regime was observed for the rf field power c.a. 10 dB above the Suhl threshold, so for ε between c.a. 3.2 and 10, depending on the definition of the logarithmic scale used in Ref. [21]. In Eq. (3) PMI may be observed in this range of ε depending on other parameters; for our analysis, a typical case was chosen. The range of frequencies of periodic modulation of the rf field amplitude for which SR was observed is in agreement with that in the experiment of Ref. [16]. Experimentally, SR was also observed when the dc field was changed with constant ε. In Eq. (3) it would be equivalent to changing $\Delta\omega_{0,k}$ which can also lead to the occurence of SR and OOI, but this case has not been analyzed here. Another quantity measured in Ref. [16] and not investigated here was the probability distribution of the lengths of chaotic bursts.

In our simulations, the effect of thermal excitations of SW on SNR was analyzed. It seems that this influence is stronger in the case of OOI in which it is difficult to obtain SNR *vs* ε curves with smooth minima without thermal noise, and the "tail" of these curves is observed out of the intermittency range due to the occurence of attractor bubbling. It would be interesting to check this experimentally, the more that OOI in SW chaos was observed [30].

To summarize, we managed to simulate noise-free SR using a simple model in the chaotic system in which this phenomenon was observed experimentally. The occurence of SR in the model could be expected as it showed intermittent dynamics, *i.e.* the system posesses a clearly defined time scale (mean duration of laminar phases) which can be periodically modulated by appropriately varying the control parameter, the rf field amplitude. A more important outcome of this work is that the simulation results agree with the experimental ones at least qualitatively. This demonstartes the usefulness of a simple model of chaos in coincidence regime also in the case of SR, like in many other situations in SW chaos.

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