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KINETICS OF THE DYNAMICAL INFORMATION SHANNON ENTROPY FOR COMPLEX SYSTEMS*

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Kinetic behaviour of dynamical information Shannon entropy is discussed for complex systems: physical systems with non-Markovian property and memory in correlation approximation, and biological and physiological systems with sequences of the Markovian and non-Markovian random noises. For the stochastic processes, a description of the information entropy in terms of normalized time correlation functions is given. The influence and important role of two mutually dependent channels of the entropy change, correlation (creation or generation of correlations) and anti-correlation (decay or annihilation of correlation) is discussed. The method developed here is also used in analysis of the density fluctuations in liquid cesium obtained from slow neutron scattering data, fractal kinetics of the long-range fluctuation in the short-time human memory and chaotic dynamics of R–R intervals of human ECG.

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1. Introduction

In this work we are concerned with the dynamical properties of Shannon entropy. This research has been strongly influenced by the book by Boris Kadomzev, *Dynamics and Information*, Moscow, 1997.

Complexity, nonlinearity and nonstationarity of physical, chemical, biological and physiological systems have been recently of profound interest. Complex systems are governed by numerous interacting variables and pose a high dimensional problem with drifting parameters of the influence and presence of many type of noises, internal and external perturbations. Such complexity may be due to stable scaling associated with "fractal" dynamics and peculiarities related to internal and external effects. The fractal

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mechanism of many systems has recently received much attention. Self-similarity in temporal dynamics as well as in the spatial structure has been reported for many physical, chemical, physiological and biological systems and in processes like ion channel kinetics, auditory nerve firings, lung inflation, muscular, cardiovascular and pulmonary systems, human cognition, walking, blood pressure and heart rate [1–15].

In this paper we are demonstrating a new concept in investigating the dynamics of the temporal evolution of complex system. The basic idea is that information (Shannon) entropy of random processes is holds both qualitative and quantitative data on the object under investigation. The paper is structured as follows. Section 2 contains a standard definition of an infinite chain of coupled nonlinear non-Markovian kinetic equations for the time-correlation functions of fluctuations of the physical system. The basic equation and the definition of the information (Shannon) entropy for the fractal dynamics of correlation processes are presented in Section 3. Section 4 gives some useful formulas for the correlation life time, anti-correlation decay time and for the parameters of correlation partition. In Section 5 we derive the pseudo-kinetic equations and the related memory functions for the time-dependent information entropy. We would like to stress that we introduce the correlation (creation of correlation) and anti-correlation (decay or annihilation of correlation) channels for the temporal changes of the information entropy. In Section 6 we consider the effect of scaling of the dynamical information entropy in liquid cesium. Section 7 incorporates the results of our analysis of fractal dynamics of the long-range correlations in the short-time human memory. Section 8 contains the analysis of chaotic dynamics of R–R intervals of human ECG. Section 9 contains some conclusions of the results obtained.

2. The basic non-Markov equations for the time correlation functions for the physical system

At first let us consider the time evolution of a dynamical variable $A(t)$, its statistical average[1] over a distribution $\langle \ldots \rangle \to A_{\mathrm{av}} = \langle A(t) \rangle$, and fluctuations $\delta A(t)$

$$A(t), A_{\mathrm{av}} = \langle A(t) \rangle, \delta A(t) = A(t) - \langle A(t) \rangle. \tag{1}$$

The variable $A(t)$ and fluctuations $\delta A(t)$ obey the Liouville equation of motion

$$\frac{d}{dt}\{\delta A(t)\} = i\hat{L}\delta A(t)\,, \tag{2}$$

[1] We consciously omit the important peculiarity connected with stationarity and ergodicity, which lies beyond the scope of the present paper and will be discussed separately elsewhere.

where we introduce the Liouville operator $\hat{L}$. Here we suppose that the variable $A(t)$, fluctuations $\delta A(t)$, operator $\hat{L}$ form a many-dimensional problem.

Successively applying the operator $\hat{L}$ to the dynamical variable $\delta A(t)$ we obtain an infinite set of dynamical functions

$$B_n(0) = (\hat{L})^n \delta A(0)\,. \tag{3}$$

Applying the Gram–Schmidt orthogonalization procedure [16, 17] to the set $B_n(0)$, we can obtain the following infinite set of dynamical variables W_n

$$\langle W_n^*(0), W_m(0)\rangle = \delta_{n,m}\langle |\ W_n(0)\ |^2\rangle\,,$$

where $\delta_{n,m}$ is the Kronecker symbol.

Now we may easily infer the recursive formulae in which the functions $W_n = W_n(t)$ are connected to the preceding ones with smaller indices:

$$\begin{aligned} W_0 &= \delta A(0), W_1 = \left(\hat{L} - \omega_0^{(0)}\right) W_0 \ldots\,, \\ W_n &= \left(\hat{L} - \omega_0^{(n-1)}\right) - \Omega_{n-1}^2 W_{n-2}, n > 1\,. \end{aligned} \tag{4}$$

Here we introduce the following notation

$$\omega_0^{(n)} = \frac{\langle W_n^* \hat{L} W_n\rangle}{\langle |\ W_n\ |^2\rangle}\,, \quad \Omega_n^2 = \frac{\langle |\ W_n\ |^2\rangle}{\langle |\ W_{n-1}\ |^2\rangle}\,, \tag{5}$$

where Ω_n are the general relaxation frequencies, and the frequencies $\omega_0^{(n)}$ describe the eigenspectrum of the Liouville operator $\hat{L}$.

The set of orthogonal functions (4) can be connected with the set of projectors which project an arbitrary dynamical variable Y on vectors belonging to the set

$$\begin{aligned} \Pi_n &= \frac{W_n\rangle\langle W_n^*}{\langle |W_n|^2\rangle}, \quad P_n = 1 - \Pi_n, \Pi_n \Pi_m = \delta_{n,m}\Pi_n\,, \\ P_n P_m &= \delta_{n,m} P_n, \Pi_n P_n = P_n \Pi_n = 0\,. \end{aligned} \tag{6}$$

Note that both sets (3) and (4) are infinite. If we execute the operations in the space of dynamical variables, then the formal expression (6) must be understood as following:

$$\Pi_n Y = W_n \frac{\langle W_n^* Y\rangle}{\langle |\ W_n\ |^2\rangle}\,, \quad Y \Pi_n = W_n^* \frac{\langle Y W_n\rangle}{\langle |\ W_n\ |^2\rangle}\,. \tag{7}$$

For the time correlation functions (TCF)

$$M_n(t) = \frac{\langle W_n^* \exp(i\hat{L}_{22}^{(n)} t) W_n \rangle}{\langle | W_n |^2 \rangle}\,, \tag{8}$$

applying successively projection operators P_n and Π_n to equation of motion (1) on the left and solving these system of equations, we obtain an infinite hierarchy of connected equations with indices $n \geq 0$

$$\frac{dM_n(t)}{dt} = i\omega_0^{(n)} M_n(t) - \Omega_{n+1}^2 \int_0^t d\tau M_{n+1}(\tau) M_n(t-\tau)\,. \tag{9}$$

The function $M_0(t)$

$$M_0(t) = a(t) = \frac{\langle \delta A^*(0) \delta A(t) \rangle}{\langle | \delta A(0) |^2 \rangle} \tag{10}$$

is usually considered [17–19] as the function characterizing the statistical memory of the system. We adopt the following notation for the diagonal matrix elements of the Liouvillian ($n \geq 1$)

$$\hat{L}_{22}^{(0)} = \hat{L}, \hat{L}_{22}^{(n)} = P_{n-1} P_{n-2} \ldots P_0 \hat{L} P_0 \ldots P_{n-2} P_{n-1}\,. \tag{11}$$

TCF $a(t)$ in Eq. (10) and set of memory functions $M_n(t)$ (Eq. (8)) is of profound importance for our further considerations. It is convenient to rewrite the set of equations (9) as an infinite chain of coupled nonlinear non-Markovian kinetic equations for TCF $a(t)$

$$\begin{aligned}
\frac{da(t)}{dt} &= -\Omega_1^2 \int_0^t d\tau M_1(\tau) a(t-\tau) + i\omega_0^{(0)} a(t)\,, \\
\frac{dM_1(t)}{dt} &= -\Omega_2^2 \int_0^t d\tau M_2(\tau) M_1(t-\tau) + i\omega_0^{(1)} M_1(t)\,, \\
\frac{dM_2(t)}{dt} &= -\Omega_3^2 \int_0^t dt M_3(\tau) M_2(t-\tau) + i\omega_0^{(2)} M_2(t)\,.
\end{aligned} \tag{12}$$

3. Time-dependent information entropy and entropy memory function for the correlations of fluctuations in complex systems

For the initial TCF $a(t)$ and memory functions of the n–th order $M_n(t)$ in (11), (14) it is convenient to introduce microscopic relaxation (correlation) times as [20]

$$\tau = \Re\tilde{a}(0)\,, \qquad \tilde{a}(s) = \int_0^\infty dt\,\mathrm{e}^{-st}a(t)\,, \tag{13}$$

$$\tau_{m_1} = \Re\tilde{M}_1(0)\,, \quad \tilde{M}_1(s) = \int_0^\infty dt\,\mathrm{e}^{-st}M_1(t), \tag{14}$$

$$\tau_{m_n} = \Re\tilde{M}_n(0)\,, \quad \tilde{M}_n(s) = \int_0^\infty dt\,\mathrm{e}^{-st}M_n(t)\,. \tag{15}$$

In has been demonstrated in [20] that relaxation (correlation) and memory times can also be defined as

$$\tau_{l_\alpha} = \left\{\int_0^\infty dt\,t^k W_\alpha(t)\right\}^{1/k}, \quad \alpha = c, m\,, \tag{16}$$

where k is integer and $W_\alpha(t)$ is the time-dependent probability density connected with the TCF $a(t)$ and $M_1(t)$. $W_\alpha(t)$ is normalized:

$$\int_0^\infty dt W_\alpha(t) = 1\,. \tag{17}$$

The following choice corresponds to the most general case

$$\begin{aligned} W_\alpha(t) &= \mid F_\alpha(t)\mid^2 \left\{\int_0^\infty dt\mid F_\alpha(t)\mid^2\right\}^{-1}, \\ F_c(t) &= a(t)\,, \\ F_m(t) &= M_1(t)\,. \end{aligned} \tag{18}$$

The situation with $n = 1$ and $W_\alpha(t) = |F_\alpha(t)|\left\{\int_0^\infty dt|F_\alpha(t)|\right\}^{-1}$ could be considered as a special case. The definition (16) with $n = 1$ and

$$W_\alpha(t) = F_\alpha(t)\left\{\int_0^\infty dt F_\alpha(t)\right\}^{-1} \tag{19}$$

was used by Egelstaff (*Phys. Rev.* **A31**, 3802 (1985); *Z. Phys. Chem.* **156**, 311 (1988)) for the analysis of the slow neutron scattering data in condensed matter. However this definition is insufficient since the time integrals contain regions with negative time values. Moreover, in the general case, the TCF $a(t)$ and memory functions $M_n(t)$ themselves are complex functions. Let us note that the case with $n = 2$ in equations (16) and (18) has a striking analogy with the definition of the coherence time in optics [21, 22].

Now we can state that

$$P_n(t) =| M_n(t) |^2 , \quad n \geq 0 \tag{20}$$

is the probability of the correlation of fluctuations (or memory) for the nth level of relaxation (see [17, 23] for details). That is to say that we have two following probabilities of correlation creation and memory creation

$$P_{cc}(t) =| a(t) |^2, P_{cm}(t) =| M_1(t) |^2 \ . \tag{21}$$

Because of the fact that the total probability is bound to be normalized to unity $(\alpha = c, m)$

$$\sum_i P^i_{c\alpha}(t) = 1, i = c, a \, , \tag{22}$$

we can introduce the other probabilities

$$P_{ac}(t) = 1 - P_{cc}(t) = 1- | a(t) |^2; P_{am}(t) = 1 - P_{cm}(t) = 1- | M_1(t) |^2 \ . \tag{23}$$

$P_{ac}(t)$ represents the probability of decay or annihilation of correlations, and $P_{am}(t)$ in (23) would constitute the probability of annihilation of memory.

Two probabilities $P_{c\alpha}$ and $P_{a\alpha}(t)(\alpha = c, m)$ make feasible the presentation of the two statistical channels of fluctuations: creation of correlations (the first channel), and annihilation (decay) of correlations (the second channel). To assess quantitatively the differences between such two states (creation and annihilation of correlations), we can calculate the information entropy of correlation at time t

$$\begin{aligned} S_c(t) &= S_{cc}(t) + S_{ac}(t) \, , \\ S_m(t) &= S_{cm}(t) + S_{am}(t) \, , \end{aligned} \tag{24}$$

where $S_{\beta\alpha}(t)$ are the partial information (Shannon) entropy for $\beta = c, a$; $\alpha = c, m$

$$\begin{aligned} S_{cc}(t) &= -|a(t)|^2 \ln |a(t)|^2 \, , \\ S_{ac}(t) &= -\{1 - |a|^2\} \ln\{1 - |a|^2\} \, , \\ S_{cm}(t) &= -|M_1(t)|^2 \ln |M_1(t)|^2 \, , \\ S_{am}(t) &= -\{1 - |M_1|^2\} \ln\{1 - |M_1|^2\} \, . \end{aligned} \tag{25}$$

Introducing two different channels (creation and annihilation of correlations) will allow us to understand the hidden role of the existence of the correlations in the complicated behaviour of systems considered.

4. Correlation life-times, annihilation of correlation times and nondimensional parameter of correlation partition

Now we can introduce a series of dimensional and nondimensional parameters for the description of subtle details of the complex processes. At first let us consider a simple example of the exponential relaxation for the normalized TCF

$$a(t) = a(0)\exp(-t/\tau_R)\,, \quad a(0) = 1\,. \tag{26}$$

In addition to the definitions (15), (16) let us introduce life-times of creation of correlations (τ_{cc}) and of annihilation of correlations (τ_{ac}):

$$\begin{aligned} \tau_{cc} &= \int_0^\infty dt S_{cc}(t) = -\int_0^\infty dt|a(t)|^2 \ln|a(t)|^2\,, \\ \tau_{ac} &= \int_0^\infty dt S_{ac}(t) = -\int_0^\infty dt\{1-|a(t)|^2\}\ln\{1-|a(t)|^2\}\,. \end{aligned} \tag{27}$$

We can also use the total correlation life-time and the memory life-time

$$\tau_s = \int_0^\infty S_c(t) = \int_0^\infty \{S_{ac}(t) + S_{cc}(t)\}\,, \tag{28}$$

$$\tau_m = \int_0^\infty dt S_m(t) = \int_0^\infty dt\{S_{cm}(t) + S_{am}(t)\}\,. \tag{29}$$

We propose a new nondimensional parameter of correlation (creation and annihilation) partition

$$\xi = \frac{\tau_{ac}}{\tau_{cc}}\,. \tag{30}$$

Using the integral [24]

$$\int_0^1 dx \frac{x\ln x}{1-x} = 1 - \frac{\pi^2}{6}$$

for the exponential relaxation function (26) we have

$$\begin{aligned}\tau_{cc} &= \frac{1}{2}\tau_{\mathrm{R}}\,, \quad \tau_{ac} = \left(\frac{\pi^2}{12} - \frac{1}{2}\right)\tau_{\mathrm{R}}\,,\\ \tau_s &= \frac{\pi^2}{12}\tau_{\mathrm{R}}\,, \quad \xi = \frac{\pi^2}{6} - 1 \cong 0,645.\end{aligned} \tag{31}$$

Note that the partition parameter ξ points to the relative splitting of notions of the creation of correlations and annihilation of correlations. The fact that life-times τ_{cc} and τ_{ac} show explicitly the importance of use of smoothed time dependent entropies $S_{cc}(t)$ and $S_{ac}(t)$ for an arbitrary value t is particularly convenient. On the other hand, the fact TCF $a(t)$ itself in general case assumes either complex or negative values may pose a problem for the calculation of correlation times. By definition, the entropies $S_{\alpha c}(t), \alpha = c, a$ can have only positive values, which clearly demonstrates their essentially statistical character.

5. Pseudokinetic equations for the time dependent channels of correlation entropies

The entropies defined in the previous section are subject to the following boundary conditions

$$\begin{aligned}\lim_{t\to 0} S_{cc}(t) &= 0\,, \quad \lim_{t\to\infty} S_{cc}(t) = 0\,,\\ \lim_{t\to 0} S_{ac}(t) &= 0\,, \quad \lim_{t\to\infty} S_{ac}(t) = 0\,,\\ \lim_{t\to 0} S_{c}(t) &= 0\,, \quad \lim_{t\to\infty} S_{c}(t) = 0\,.\end{aligned} \tag{32}$$

Along with the exact kinetic equation (14 a) for TCF, the pseudokinetic equation can be obtained from the relationship (14), (24), (25). For example, in the case of the creation of correlations channel we have

$$\frac{dS_{cc}(t)}{dt} = -\Omega_1^2 \int_0^t dt M_{cc}^{(1)}(t,\tau) S_{cc}(\tau)\,, \tag{33}$$

where $M_{cc}^{(1)}(t,\tau)$ are the relevant first order memory function

$$M_{cc}^{(1)}(t,\tau) = \frac{\ln \mid a(\tau) \mid^2}{1 + \ln \mid a(t) \mid^2} \left\{ \frac{a^*(t)}{a^*(\tau)} M_1(t-\tau) + \frac{a(t)}{a(\tau)} M_1^*(t-\tau) \right\}\,, \tag{34}$$

and for the annihilation of correlations channel we get

$$\frac{dS_{ac}(t)}{dt} = -\Omega_1^2 \int_0^t d\tau M_{ac}^{(1)}(t,\tau) S_{ac}(\tau) + N_{ac}(t)\,, \tag{35}$$

$$M_{ac}^{(1)}(t,\tau) = \frac{\frac{a^*(t)}{a^*(\tau)} M_1(t-\tau) + \frac{a(t)}{a(\tau)} M_1^*(t-\tau)}{\{1+\ln[1-|a(t)|^2]\}\ln[1-|a(\tau)|^2]}\,, \tag{36}$$

$$\begin{aligned} N_{ac}(t) &= -\Omega_1^2 \int_0^t d\tau \frac{1}{1+\ln[1-|a(t)|^2]} \\ &\quad \times \left\{ \frac{a^*(t)}{a^*(\tau)} M_1(t-\tau) + \frac{a(t)}{a(\tau)} M_1^*(t-\tau) \right\}, \end{aligned} \tag{37}$$

where $M_{ac}^{(1)}(t,\tau)$ is the relevant first order memory function and $N_{ac}(t)$ is the non-homogeneous part.

For the total time dependent correlation entropy we get the following pseudokinetic equation:

$$\frac{dS_c(t)}{dt} = -\Omega_1^2 \int_0^t d\tau M_c^{(1)}(t,\tau) S_c(\tau) + N_c(t)\ , \tag{38}$$

$$\begin{aligned} M_c^{(1)}(t,\tau) &= \frac{\frac{a^*(t)}{a^*(\tau)} M_1(t-\tau) + \frac{a(t)}{a(\tau)} M_1^*(t-\tau)}{\{\ln[1- \mid a(t) \mid^2] - \ln \mid a(t) \mid^2\}} \\ &\quad \times \left\{\ln[1- \mid a(\tau) \mid^2] - \ln \mid a(\tau) \mid^2\right\}^{-1}\ , \end{aligned} \tag{39}$$

$$\begin{aligned} N_c(t) &= -\Omega_1^2 \int_0^t d\tau \frac{[\frac{a^*(t)}{a^*(\tau)} M_1(t-\tau) + \frac{a(t)}{a(\tau)} M_1^*(t-\tau)]}{\{\ln[1- \mid a(t) \mid^2] - \ln \mid a(t) \mid^2\}} \\ &\quad \times \frac{\ln[1- \mid a(\tau) \mid^2]}{\{\ln[1 - a(\tau) \mid^2] - \ln \mid a(\tau) \mid^2\}}\,, \end{aligned} \tag{40}$$

where $M_c^{(1)}(t,\tau)$ is the first order memory function and $N_c(t)$ is the non-homogeneous part for the total correlation entropy.

The pseudokinetic equations (33), (34), and (37) are very useful for the careful analysis of memory and non-Markovian effects in the time evolution of information entropies of correlations of the physical systems.

6. The effect of dynamical scaling on the dynamical information entropy in liquid cesium

The Shannon entropy is defined as

$$S = -\sum_{i=1}^{n} P_i(t) \ln P_i(t), \tag{41}$$

where i numbers states of the system and the probability distribution $P_i(t)$ is normalized

$$\sum_{i=1}^{n} P_i(t) = 1. \tag{42}$$

In formaulas (41), (42) no explicit dependence on the number n of discrete states is present. However, a similar dependence is still available. For the entropy $S(t)$ we have

$$n = 1\,, \quad \text{for} \quad P = 0\,, \quad \text{we have} \quad S = 0\,, \tag{43}$$

$$n \neq 1\,, \quad \text{for} \quad P_i = \frac{1}{n}\,, \quad \text{we have} \quad S = \ln n\,. \tag{44}$$

The entropy increases from 0 to $\ln n$ as a system passes from full order ($n = 1$) to full disorder ($n \neq 1$). So, increasing the number of levels enhances the information content of Shannon entropy. Therefore, considering the processes of correlations and memory by (20)–(24), one can find

$$\begin{aligned}
&P_{cc}(t) + P_{ac}(t) = 1\,,\\
&P_{cm}(t) + P_{am}(t) = 1\,,\\
&P_1(t) + P_2(t) + P_3(t) + P_4(t) = 1\,,\\
&P_1 = P_{cc}(t)P_{cm}(t)\,,\\
&P_2(t) = P_{cc}(t)P_{am}(t)\,,\\
&P_3(t) = P_{ac}(t)P_{cm}(t)\,,\\
&P_4 = P_{ac}(t)P_{am}(t)\,.
\end{aligned} \tag{45}$$

In line Eq. (41) we are dealing here with 4th channels entropy

$$\begin{aligned}
S \;=\; & -P_{cc}(t)P_{cm}(t) \ln P_{cc}(t)P_{cm}(t)\\
& -P_{cc}(t)P_{am}(t) \ln P_{cc}(t)P_{am}(t)\\
& -P_{ac}(t)P_{cm}(t) \ln P_{ac}(t)P_{cm}(t)\\
& -P_{ac}(t)P_{am}(t) \ln P_{ac}(t)P_{am}(t)\,.
\end{aligned} \tag{46}$$

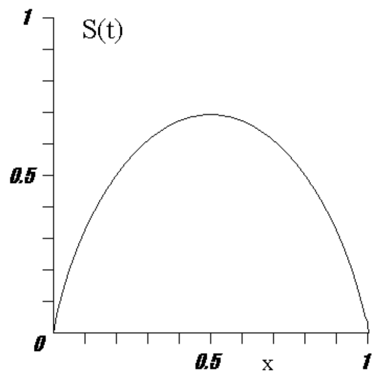


Fig. 1. Evolution in time of the dynamical information entropy $S(t) = -x \ln x - (1-x) \ln(1-x)$, where $x = | a(t) |^2, a(t)$ is the time correlation function.

The formulas (45), (46) are very convenient for taking into account non-Markovian effects and statistical memory in dynamical entropy. Figure 1 shows the TCF $x = | a(t) |^2$-dependence of the entropy with maximum $S = \ln 2$ at the value $| a(t) | = 1/\sqrt{2}$.

Fig. 2 gives the temporal dependence $S(t)$ calculated with formula (46) for the some values of scaling parameter $\alpha = 1,0; 1,5; 2,0797; 2,9005$ and $\alpha < 1$. Using the scaling parameter and Zwanzig–Mori memory function formalism Sharma *et al.*, *Phys. Rev.* **E54**, 3652 (1996); **55**, 564 (1997) calculated the dynamical structure factor $S(q, \omega)$ of liquid cesium near its melting point (see Fig. 2). Subsituting $M_3(q,t) = M_2(q, \alpha t)$ R.K. Sharma with coauthors have shown that this approach predicts the collective density excitation peak in $S(q, \omega)$ for wave vector $q < 1.2\,\text{Å}^{-1}$ at a frequency that is in agreement with experimental results. From these curves it is obvious that

(1) non-Markovian effects in kinetics of initial TCF give rise to increasing of the informativity of Shannon entropy (41);

(2) the dynamical scaling change significantly entropy itself and its parameters;

(3) the character of dynamics of the entropy is extremely sensitive to the collective excitations in the experimental systems

in accordance with Eqs. (43), (44).

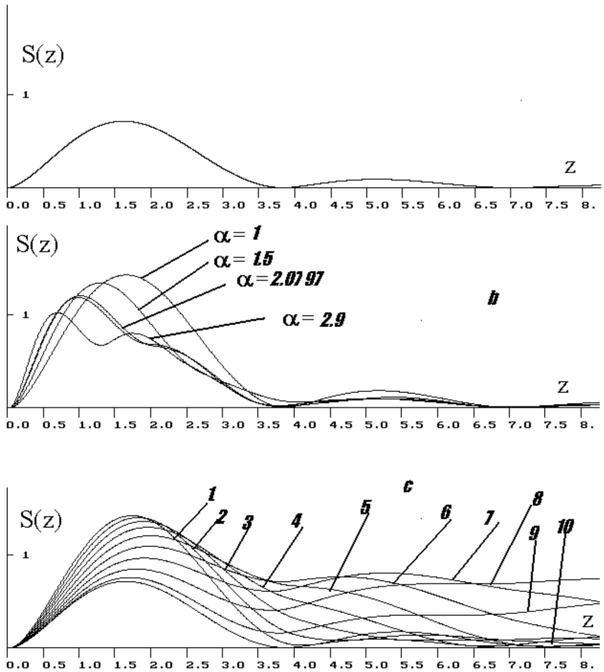
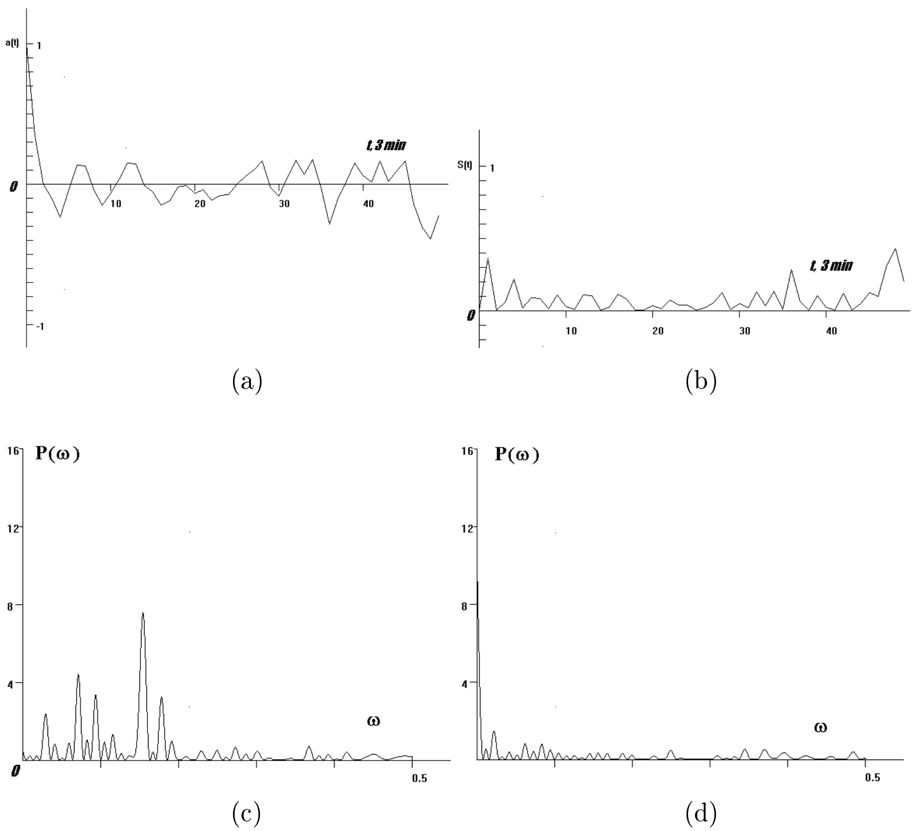


Fig. 2. Time dependence of the dynamical information entropy $S(z), z = 2\Omega t$, $M_3(t) = M_2(\alpha t)$: (a) two-channel evolution without memory; (b) four-channel evolution with $\alpha \geq 1$; (c) four-channel evolution with scale parameter α: 1 — $\alpha = 0,9$; 2 — $\alpha = 0,8$; 3 — $\alpha = 0,7$; 4 — $\alpha = 0,6$; 5 — $\alpha = 0,5$; 6 — $\alpha = 0,4$; 7 — $\alpha = 0,3$; 8 — $\alpha = 0,2$; 9 — $\alpha = 0,1$; 10 — $\alpha = 0,01$.

7. Fractal dynamics of the long-range correlations in short-time human memory

It should be pointed out that results presented in Sections 2–5 have a wide area of practical implementation for complex systems in physics, chemistry, biology and living systems. All results obtained hold for the physical systems, and this allows to use exact kinetic equations (14) together with (26)–(31). In the case of complex systems of the nonphysical nature, the exact kinetic equations (14) do not exist. However, results (26)–(31) still stand and they are very useful in describing random dynamics of complex system.

Some results of the research on the temporal correlations of the short-time human memory are presented here. An experiment has been performed on the free recollection in the 2 group of 84 volunteers: 56 students of the senior courses of the Physics Department of the University and 18 schoolboys. With the purposes of decreasing the influence of the semantic content of various objects, lists involving only three-digit or two-digit numbers have been used. Each list included thirty or fifteen numbers. Each of these lists was read out aloud to the subjects, the subjects recorded a number, and so that operation was repeated down to the end of the list. After that, the subjects were supposed to note all the numbers they remembered. That procedure was carried out repeatedly. Next list was offered after the first one and so on. Only up to 100 measurements were available. The delay time between two successive experiments was 5 min or 3 min. The ratio of the number of the properly reconstructed objects to the number of all objects was used as the numerical value of the experiment. Thus, we have the series of values, each determined by the ratio of the actual number of proper responses to the number of all possible proper responses.



(a) (b)

(c) (d)

Fig. 3. Example of time and frequency behaviour of TCF and dynamical information entropy for the short-time human memory for subject L.K., $\langle n\rangle = 9,71$; $\delta = 18,091\%$; $\tau_c = 9,193$ min; $\tau_{ac} = 3,0177$ min; $\xi = 0,32826$): (a) TCF; (b) information entropy; (c) power spectrum of TCF and (d) power frequency spectrum of dynamical entropy in units 9 min^2.

We analyzed the following data obtained from the subjects: time correlation function (TCF), probability of creation of the correlations, probability of annihilation of the correlations, time dependent channels of entropies of creation and annihilation of the correlations and the total entropy, the correlation life-time and annihilation of the correlation time, the total time of correlations, and the parameter of correlation partition.

In figures 3, 4 the time dependence of TCF and entropy, and the corresponding power spectra are presented. The following conclusions can be made from these examples:

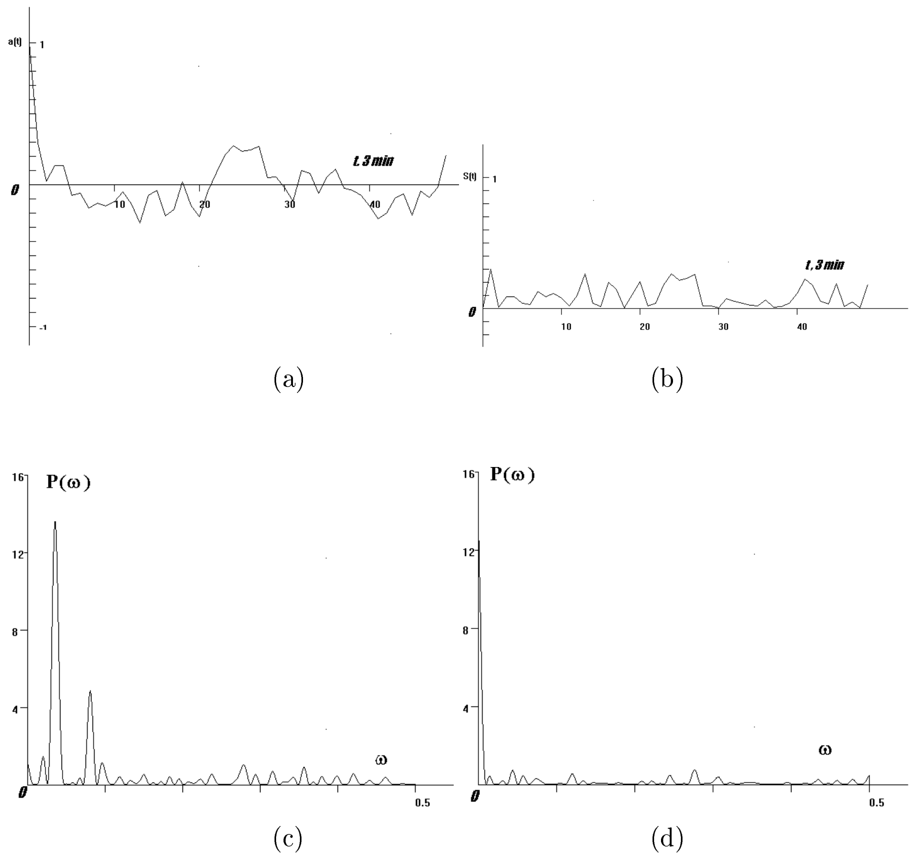


Fig. 4. Example of time and frequency behaviour of TCF and entropy for the short-time human memory for subject R.L., $\langle n\rangle = 8,44; \delta = 18,38\%, \tau_{cc} = 11,53\,\mathrm{min}; \tau_{ac} = 3,532\,\mathrm{min}; \xi = 0,30632$: a) TCF; b) information entropy; c) power spectrum of TCF and d) power spectrum of entropy in units 9 min^2.

1) all frequency spectra are characterized by the availability of some distinctive frequency peaks;

2) dynamical entropy is a nonlinear transformation of a signal. Its frequency spectrum is different from the TCF spectrum — the high frequency peaks are suppressed and shifted into the domain of the low frequencies, and vice versa, entropy amplifies the low frequency peaks. Because of this joint treatment of TCF and entropy, we were able to investigate all the areas of frequency spectra more carefully;

3) the most talented students show noticeable peaks in the low frequency area between $0.1\omega_0$ and $0.01\omega_0$, where $\omega_0 = 0.0349\text{s}^{-1}$. This is consistent with the oscillation period $T = 30 \div 300$ min. Probably, these low frequency peaks are associated with the superslow electric potentials of the cortex.

8. The chaotic dynamics of R–R intervals in human ECG

Here we consider application the method of the information entropy to the analysis of the temporal changes in chaotic parameters of human ECG. Figure 5 shows a schematic representation of human ECG. Figures 6–8 present data obtained from individual patients. In Figs. 6, 7 the dynamical functions and power spectra of TCF and entropy for healthy woman (patient C3) and patient with sinus arrythmia (D3) by the short-time ECG-data (200 and 400 heart beats, respectively) are shown. Comparing static (mean value of the heart beat, absolute and relative dispersion) and kinetic parameters (correlation life-time τ_c, annihilation of correlation time τ_{ac}, parameter ξ of correlation partition), one can see the following:

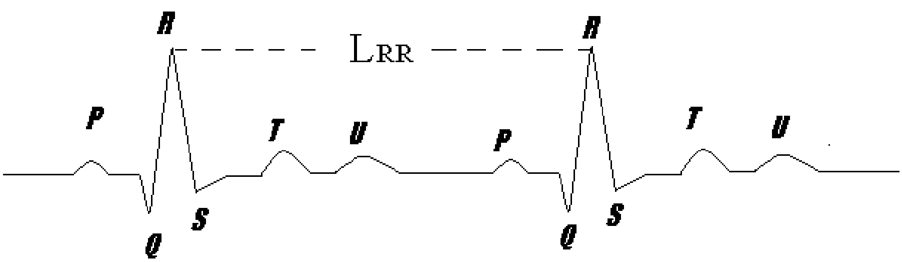


Fig. 5. Definition of the characteristic points and of intervals of the human ECG (sketch).

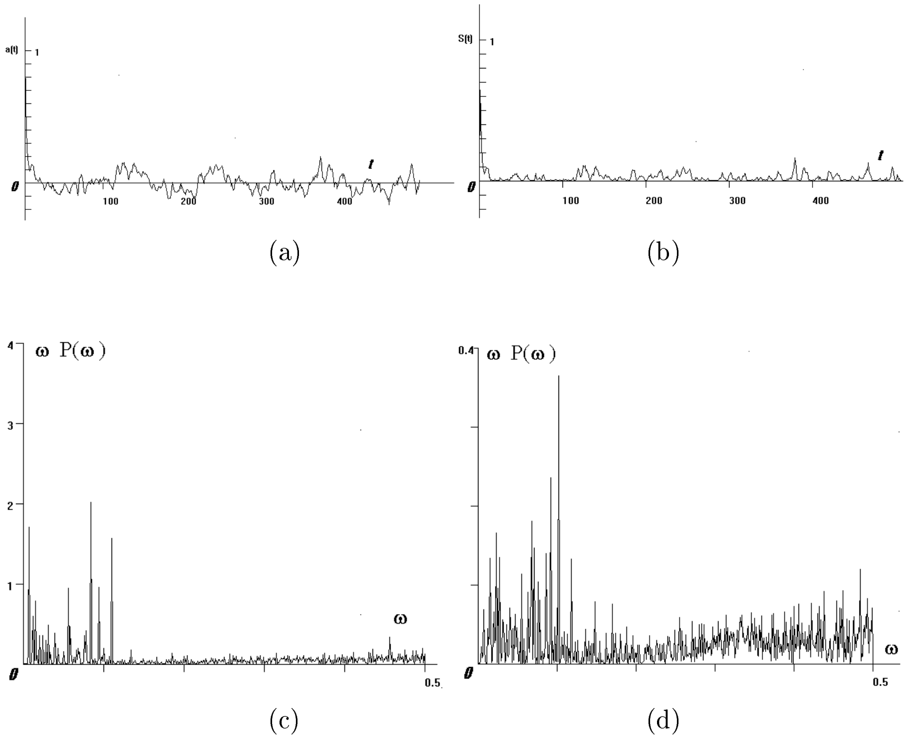


Fig. 6. Examples of time behaviour and power spectra of TCF and dynamical information entropy for the short-time dynamics of R–R intervals human ECG: (a) (healthy Akhm., 27 y, 1 time unit $= \langle l_{\mathrm{RR}} \rangle = 995,51\mathrm{ms}; \delta = 5,208\%; \tau_{cc} = 10,61226\mathrm{t.u.}; \tau_{ac} = 2,51\mathrm{t.u.}; \xi = 0,23652$): (a) TCF; (b) dynamical information entropy; (c) power spectrum of TCF and (d) power spectrum of dynamical entropy in normalized form $\omega P(\omega), s^2$ Hz.

1) the static fluctuations of patient D3 are stronger than in patient C3;
2) the kinetic parameters of patient C3 are approximately 10 times higher than in D3;
3) the low frequency peak (approx.,0.06–0.2 s^{-1}) of healthy man disappear at sinus arrythmia.

Fig. 8 gives the data obtained by the long-time ECG spectra (approximately, 4000 heart beats) for a patient with syndrome of sinus knot weakness (Golub, 43 y). The special peculiarities have engaged our attention at the comparison studies of different patients:

1) Difference in phase density is very significant;
2) static fluctuations (value of relative dispersion δ) contain no information value;

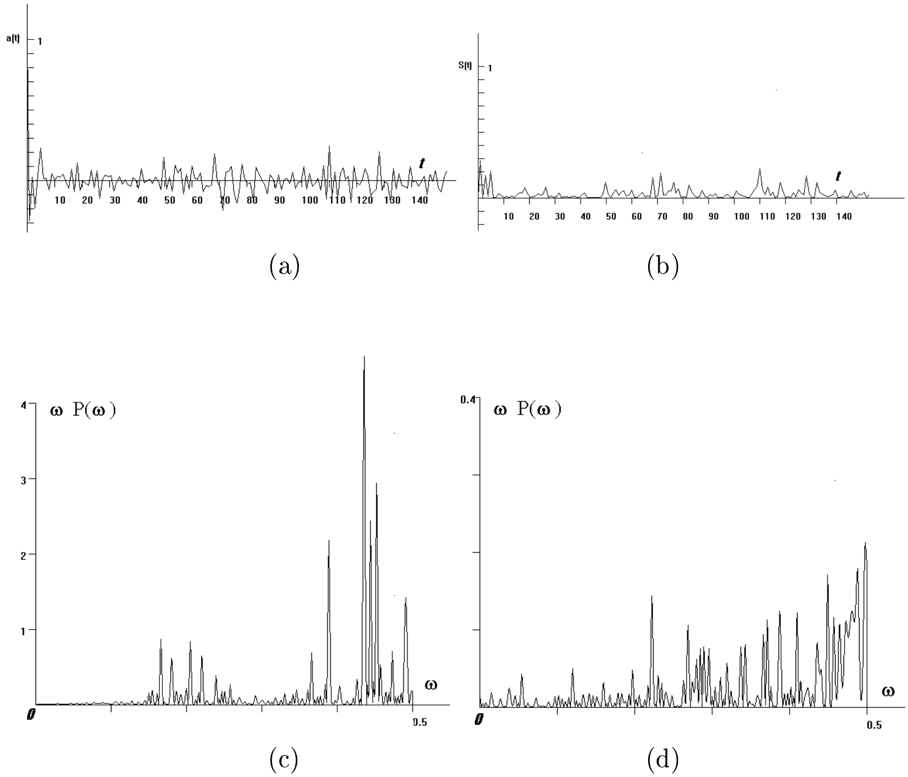


Fig. 7. Examples of time behaviour and frequency spectrum of TCF and dynamical information entropy for the short-time dynamics of R–R intervals of human ECG: (patient Nekh., acute stage of miocardial infarction, 47 y, 1 time unit$=\langle l_{RR}\rangle =$ $11,92,76$ms; $\delta = 20,917\%; \tau_{cc} = 3,87533$t.u.; $\tau_{ac} = 0,92,868$t.u.; $\xi = 0,23964$): (a) TCF; (b) dynamical information entropy; (c) power frequency spectrum of TCF and (d) power frequency spectrum of dynamical entropy in normalized form $\omega P(\omega), s^2$ Hz.

3) correlation times τ_c and τ_{ac} of a healthy man is much longer than in other patients;
4) parameter of correlation partition ξ of a healthy man is much larger than in other patients;
5) appreciable difference of low-frequency spectra of correlation and entropy exist for the healthy man;
6) TCF anf entropy power spectra differ considerably, especially in the middle to low frequencies.

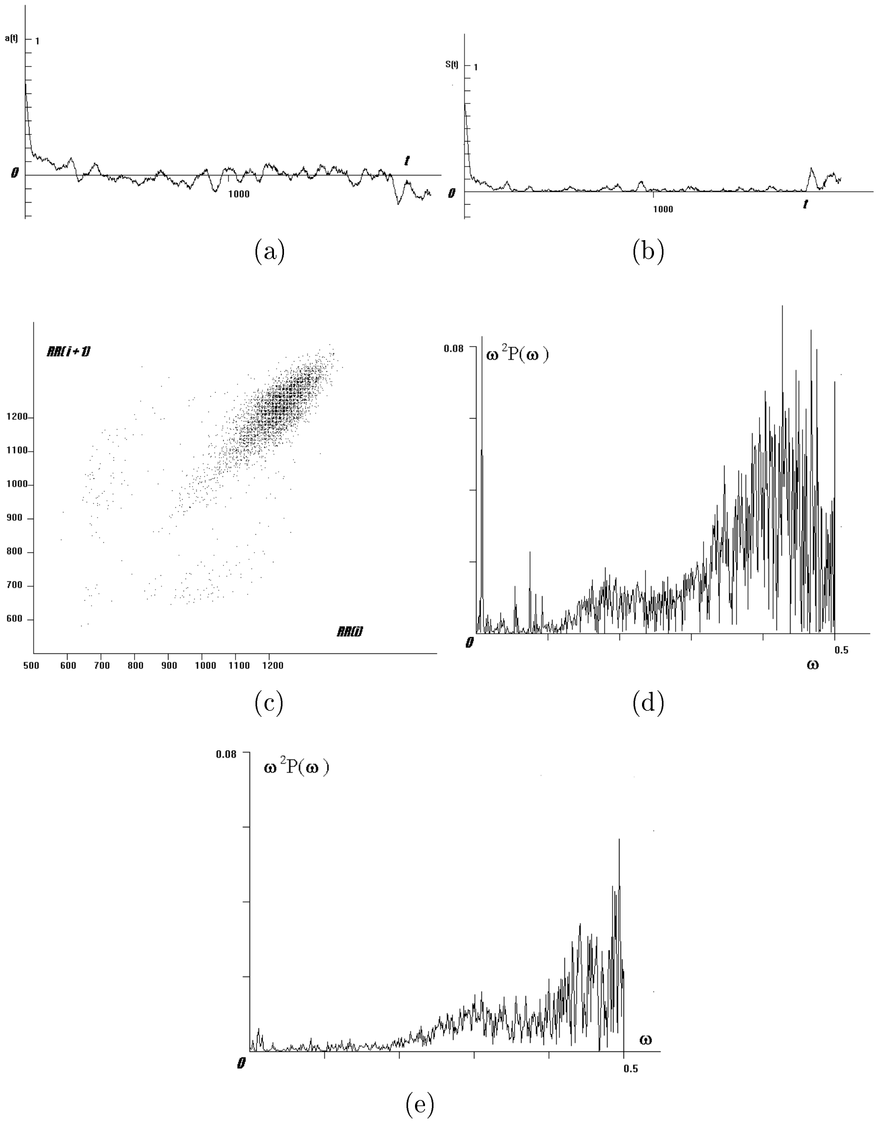


Fig. 8. Example of time evolution and frequency spectrum of the long-time dynamics of R–R intervals in human ECG for a patient with syndrome of sinus knot weaknees (G., 43 y; $\langle l_{\mathrm{R--R}} = 1204\rangle$ms; $\tau_{cc} = 57,04$s; $\tau_{ac} = 16,74$; $\xi = 0,29352$; $\delta = 9,492\%$): (a) time behaviour of TFC; (b) time evolution of the dynamical entropy; (c) two-dimensional image of R–R intervals return map with $i = 1$; (d) power spectrum of time correlations, $\omega^2 P(\omega), s^2(\mathrm{Hz})^2$; (e) power spectrum of dynamical information entropy, $\omega^2 P(\omega), s^2(\mathrm{Hz})^2$.

The above presented data demonstrate a high diagnostic value of the dynamical information entropy in cardiovascular research as a whole.

9. Discussion

In the present paper we have considered a new concept of stochastic dynamics based on successive use of the information entropy for the correlation of fluctuations of variables used to describe a given system. It is a specific feature of our method that it extracts the information entropy of the time dependent state by the probability and the time correlation function of the random fluctuations of a complex system. That is especially true in regard to two time dependent channels of the information entropy: the creation of correlations and the annihilation of correlations. The application of the time dependent information entropy permits us to use a set of time dependent stochastic functions (TCF, probabilities of creation and annihilation of correlations, total correlation entropy and its two channels) and correlation parameters (correlation life-time, annihilation of correlation time, the total correlation time, the parameter of correlation partition). This set gives us the detailed information of the characteristics of stochastic dynamics of the complex system.

Our preliminary investigation of the short-time human memory leads us to the conclusion that the fluctuations in the values of numbers of recollection display fractal dynamics and long-range stable correlations in the young subjects.

Our findings indicate that the parameter of short-time human memory exhibits long-range time correlations. Fluctuations in the memory parameter are statistically correlated with variations in the numerous values of parameters earlier, and this influence decays in a scale-invariant, fractal manner. This behaviour appears to be intrinsic to the human memory.

From neurophysiological control viewpoint, this behaviour is of interest because it signifies the presence of long-term dependence. The mechanism(s) responsible for these parameters of memory correlations are largely unknown. The unexpected observations of long-range correlations in short-time memory raises important questions concerning neuron networks dynamics and the origins of fluctuations in parameters of memory. Many natural phenomena are characterized by short-term correlations with a characteristic time scale and an autocorrelation function that decays exponentially. In contrast, long-range correlations have only been observed under vary specific conditions, for example when a system is near its critical point. In that case there exists no well-defined correlation length and autocorrelation function decays according to a power law. The present value is statistically correlated not only with its most recent value, but also with its long-term history in a scale-invariant fractal manner. The establishment of long-range correlations in short-time memory therefore raises the possibility of exsitence of a yet unidentified mechanism underlying neural control of short-time memory. Hopefully, future studies will help to determine the origin of this fractal scaling and find out whether (and to what extent) a stochastic or deterministic mechanism lies behind this property of neural control.

Our treatment of the chaotic dynamics of R–R intervals in human ECG shows both the extremely complex character of such dynamics and big information content of the dynamical information entropy. Our investigation had a preliminary character. However, it shows that there are big number of static, dynamical and frequency parameters of the human heart dynamics. It can be predicted with confidence that the dynamical entropy will let us establish fundamentally new diagnostic techniques of assessment of the state of the human cardiovascular system.

The dynamical information entropy for realistic complex system allows an adequate description of the temporal behaviour with the many-scale correlations of fluctuations. Moreover, using the results of experimental investigation of different object, it is possible to calculate their static, correlation and information functions and parameters.

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REFERENCES

[1] C.M. Viswanathan, C.K. Peng, H.E. Stanley, A.L. Goldberger, *Phys. Rev.* **E55**, 845 (1997).

[2] P.Ch. Ivanov, M.G. Rosenblum, C.K. Peng, J. Mietus, S. Havlin, H.E. Goldberger, *Nature* **383**, 327 (1996).

[3] C. Webber, T.P. Zbilut: *Recurrent Structuring of Dynamical and Spatial Systems*, in *Complexity in the Living : A Modelistic Approach, Interdisciplinary Science Reviews*, ed. by A. Colosimo, and A. Lesk, Oxford University Press, New York 1997.

[4] C.K. Peng, S. Havlin, H.E. Stanley, A.L. Goldberger, *Chaos* **5**, 82 (1995).

[5] J.M. Hausdorff, S.K. Peng, *Phys. Rev.* **E54**, 2154 (1996).

[6] J.M. Hausdorff, P.L. Purdon, C.K. Peng, Z. Ladin, J.Y. Wei, A.L. Goldberger, *J. Appl. Physiol.* **80** (5), 1448 (1996).

[7] J.M. Hausdorff, S.L. Mitchell, R. Firton, C.K. Peng, M.E. Cudkowicz, J.Y. Wei, A.L. Goldberger, *J. Appl. Physiol.* **82** (1), 262 (1997).

[8] M. Paluš, *Physica* **D93**, 64 (1996).

[9] M. Paluš, *Phys. Lett.* **A227**, 301 (1997).

[10] D. Hoyer, K. Schmidt, U. Zwiener, R. Baner, *Cardiovascular Research* **31**, 434 (1996).

[11] J.J. Zebrowski, W. Poplawska, R. Baranowski, *Phys. Rev.* **E50**, 4187 (1994).

[12] A.S. Kelso, S.L. Bressler, S. Buchanan, G.C. DeGuzman, M. Ding, A. Fuchs, T. Holroyd, *Phys. Lett.* **A169**, 134 (1992).

[13] J.A.S. Kelso, J.J. Buchanan, G.C. DeGuzman, M. Ding, *Phys. Lett.* **A179**, 364 (1993).

[14] C.L. Webber, J.P. Zbilut, *Neural Network Estimation of Cardiac Nondeterminism* in *Intelligent Engineering Through Artificial Neural Networks*, Vol. 4, ed. by C.H. Dagli, B.R. Fernandez, J. Ghosh, R.T.S. Kumara, ASME Press, 1994, p.695.

[15] L.L. Trulla, A. Giuliani, J.P. Zbilut, C.L. Webber, *Phys. Lett.* **A223**, 255 (1996).

[16] M. Reed, B. Simon, Academic Press, New York 1972.

[17] R.M. Yulmetyev, N.R. Khusnutdinov, *J. Phys. A : Math. Gen.* **27**, 5363 (1994).

[18] R. Zwanzig, *Phys. Rev.*, **124**, 983 (1961).

[19] H. Mori, *Prog. Theor. Phys.* **34**, 765 (1965).

[20] V.Yu. Shurygin, R.M. Yulmetyev, V.V. Vorobjev, *Phys. Lett.* **A148**, 199 (1990).

[21] M. Born, E. Wolf, *Principles of Optics*, Pergamon Press, Oxford 1964.

[22] J. Perina, *Coherence of Light*, Van Nostrand/Reinhold Publ., New York 1972.

[23] R.M. Yulmetyev, V.Yu. Shurygin, T.R. Yulmetyev, *Physica* **A242**, 509 (1997).

[24] I.S. Gradstein, I.M. Ryzik, *Tables of Integrals, Summs Series and Products*, GIFMZ Publ., Moscow 1962.