# MEMORY PROPERTIES OF ARTIFICIAL NEURAL NETWORKS WITH DIFFERENT TYPES OF DILUTIONS AND DAMAGES*

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Memory properties of the Hopfield type neural networks with four different types of dilution of synaptic connections (dilution inside blocks, dilution outside blocks and dilution of excitory/inhibitory synapses) as well as damaging of a part of neurons, are numerically investigated. Number of stored bits per neuron an stored bits per synapse for these networks were calculated and compared. Influence of the type of dilution on the memory properties of the network is discussed.

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## 1. Introduction

One of the main properties of neural networks are their memory properties. Human brain is extremely effective in this case — it can store, depending on the approximation, from $10^{21}$ to $10^{15}$ patterns containing $10^{11}$ bits (for review of results see Ref. [1]). Memory properties of artificial neural networks were also extensively studied, due to the different applications of such networks. Important measure of the memory properties of neural networks is a number of stored bits per neuron or a number of stored bits per synapse [1,2]. In human brain neural network is not fully connected — synaptic connections are diluted: for $10^{11}$ neurons the number of synaptic connections per neuron is of the order of $10^4$ . Maximal storage capacity of artificial neural network $\alpha_c$ was analysed for fully connected networks as well as networks with diluted synaptic connections. For fully connected Hopfield

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network it was found analytically [1,3] that in the case of thermodynamic limit (*i.e.* for the number of neurons $N \longrightarrow \infty$) $p = \alpha_c N = 0.138N$ patterns, each consisting of $N$ bits, can be stored effectively.

The important reason for the investigations of memory properties of diluted artificial neural networks is the fact that in their hardware implementations it is easier to increase the number of neurons than the number of connections between them (*e.g.* in the form of conducting paths). High density of these paths leads to the problems with their topology. One of the types of dilution, rather frequently analysed in the literature, is a random dilution of synaptic connections. For this case, it was found that the maximal storage capacity (proportional to the number of bits stored per neuron) decreases with the increase of dilution of synaptic connections; at the same time the number of bits stored per synapse increases and this parameter is greater than for the case of fully connected network (see *e.g.* [1,4–7])

In the present work four types of dilutions of synaptic connections or modifications of the states of neurons introduced to the Hopfield type neural network (which may refer to some properties of living neural networks), are numerically examined. The storage capacity of such networks is discussed and compared.

## 2. The model of neural network

In our work neural network consists of $N$ neurons $S_i$ which may have two states: $+1$ (firing) or $-1$ (rest). All neurons are connected with synaptic connections $J_{ij} \in [-1, +1]$, which are constructed according to the standard Hebb's rule [8]. In the network $p$ random patterns were stored. In the process of time evolution of the network the effective retrieval of stored patterns was controlled, *i.e.* the value of overlap $m^\mu = (1/N) \sum_i \xi_i^\mu S_i$ was calculated in the computations (where $\mu = 1, 2, \ldots p$). It was assumed that the pattern is retrieved correctly if $m^\mu \geq 0.97$. As the starting configuration for the case of $\mu$-th pattern, this pattern with a 1 or 10% of flipped neurons was used. Synchronous dynamics of the network was used in computations.

Maximal storage capacity of the network $\alpha_c$ may have different definitions. Let us assume, that when in the network $p = p_+$ patterns can be correctly retrieved, then maximal storage capacity is given by $\alpha_c = p_+/N$. In the case of infinite Hopfield network, when an additional $p_+ + 1$ pattern is stored in the network, blackout catastrophe is observed, *i.e.* no one pattern can be retrieved succesfully. In Fig. 1(a) the dependence between maximal storage capacity $\alpha_c$ and $\alpha = p/N$ for $N \longrightarrow \infty$ is shown. On the other hand, for the networks with definite size considered here, typical relation (calculated for $N = 1024$ neurons) is shown in Fig. 1(b). As we see blackout catastrophe is not observed and the first maximum of this curve will be treated as a value of maximal storage capacity $\alpha_c$.

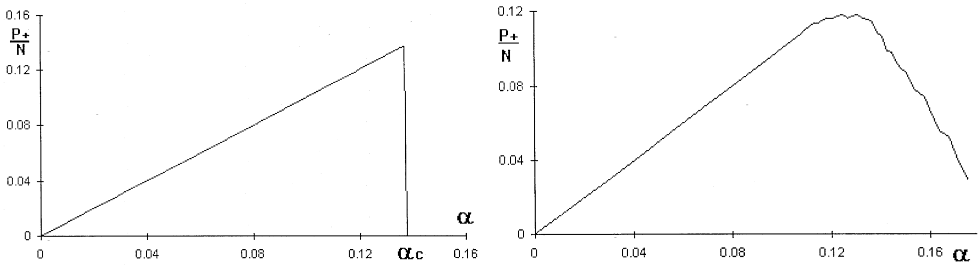


Fig. 1. Maximal storage capacity $\alpha_c$ for the infinite Hopfield's network — (a). Typical storage capacity for the network with definite size calculated for $N = 1024$ neurons — (b).

## 3. The types of dilution of synaptic connections

The most known type of dilution of synaptic connections is random dilution, which means that synaptic connections (having the values resulting from the Hebb's learning rule) are cut with certain probability $C$. Synaptic connections in this case have the form

$$J_{ij} = \frac{c_{ij}}{Nc} \sum_{\mu=1}^{p} \xi_i^\mu \xi_j^\mu \,, \tag{1}$$

where

$$c_{ij} = \left\{ \begin{array}{ll} 0 & \text{with probability } C \\ 1 & \text{with probability } 1 - C \end{array} \right. . \tag{2}$$

For this case maximal storage capacity of the network decreases (almost linearly) with increasing values of dilution parameter $C$ [1].

In the first type of dilutions of synaptic connections investigated here, the network was divided into $n$ blocks, consisting of $V$ neurons each. All synaptic connections inside blocks were cut, while connections between blocks were unchanged in comparison to the values resulting from the Hebb's rule. Thus, synaptic connections are given by

$$J_{ij}^{ab} = \left\{ \begin{array}{ll} \frac{1}{(n-1)\cdot V} \sum_{\mu=1}^{p} \xi_{ia}^\mu \xi_{jb}^\mu & a \neq b \\ 0 & a = b \end{array} \right. , \tag{3}$$

where $i, j$ are indices numbering neurons inside the blocks; $a, b$ are indices localizing block in the network. The matrix $[\boldsymbol{J}]$ for the whole network (and the for $n = 4$ blocks) has the form

$$\left( \begin{array}{cccc} [0] & [J_{ij}] & [J_{ij}] & [J_{ij}] \\ [J_{ij}] & [0] & [J_{ij}] & [J_{ij}] \\ [J_{ij}] & [J_{ij}] & [0] & [J_{ij}] \\ [J_{ij}] & [J_{ij}] & [J_{ij}] & [0] \end{array} \right) , \tag{4}$$

where $[\mathbf{0}]$ is the matrix consisting of zeroes and $[\boldsymbol{J_{ij}}]$ contains values resulting from the Hebb's rule. Network consiting of $N = 1260$ neurons, which enable to divide it into $n = 2, 3, 4, 5, 6, 7$ and 9 blocks, was investigated. Below we call this type of dilution — dilution inside blocks.

In the second type of dilution of synaptic connections the network was divided into $n$ blocks, each consisting of $V$ neurons (as previously), but now synaptic connections between blocks were diluted in a following way. All connections between blocks were cut, except the connections between neurons with the same indices inside the blocks. Thus, the synaptic connections have the form

$$J_{ij}^{ab} = \begin{cases} \frac{1}{(n-1)\cdot V} \sum_{\mu=1}^{p} \xi_{ia}^{\mu} \xi_{jb}^{\mu} & a \neq b \;\&\; i = j \\ \frac{1}{(n-1)\cdot V} \sum_{\mu=1}^{p} \xi_{ia}^{\mu} \xi_{jb}^{\mu} & a = b \;\&\; i \neq j \\ 0 & \sim \end{cases} , \qquad (5)$$

where the meanings of $i, j, a$ and $b$ and the values of $n$ are the same as earlier. For the present case matrix of synaptic connections for the whole network has the form

$$\begin{pmatrix} [J_{ij}] & [0'] & [0'] & [0'] \\ [0'] & [J_{ij}] & [0'] & [0'] \\ [0'] & [0'] & [J_{ij}] & [0'] \\ [0'] & [0'] & [0'] & [J_{ij}] \end{pmatrix}, \qquad (6)$$

where

$$[J_{ij}] = \begin{pmatrix} {\scriptstyle 0} & & J_{ij} \\ & \ddots & \\ J_{ij} & & {\scriptstyle 0} \end{pmatrix} \quad \text{and} \quad [0'] = \begin{pmatrix} {\scriptstyle J_{ij}} & & 0 \\ & \ddots & \\ 0 & & {\scriptstyle J_{ij}} \end{pmatrix}. \qquad (7)$$

Such type of dilution of synaptic connections exists also in a human neural network. It was indicated in the physiological investigations that in the brain certain parts with very high density of synaptic connections occur. On the other hand, the number of connections between other, functionally differing pats of human brain, are rather sparse [9,10]. The simplest example are relatively sparse synaptic connections between two cerebral hemispheres. Below this type of dilution is called — dilution outside the blocks.

In the next type of modification of synaptic connections the network was divided into two equal parts. In the first part of the network all inhibitory synaptic connections (*e.g.* those with negative values) were cut, while the excitory (positive) connections (as result from the Hebb's rule) were unchanged. In the second part of the network all connections with positive values were cut, while the connections with negative values were unchanged. Synaptic connections between neurons belonging to both parts of the network were diluted also: in a part of them only positive and in the second part only negative connections were maintained. Full matrix of synaptic connections for this type of dilution has the form

$$\left(\begin{array}{ccc|ccc} 0 & & A^+ & & & \\ & \ddots & & & C^- & \\ A^+ & & 0 & & & \\ \hline & & & 0 & & B^- \\ & C^+ & & & \ddots & \\ & & & B^- & & 0 \end{array}\right), \tag{8}$$

where $\boldsymbol{A}^+$ denotes the matrix of positive Hebb's connections for the first $N/2$ neurons, $\boldsymbol{B}^-$ denotes matrix of negative Hebb's connections and matrices $\boldsymbol{C}^+$ and $\boldsymbol{C}^-$ denote the synaptic connections between the first and the second parts of the network. This type of dilution will be called — dilution of excitory / inhibitory synapses.

In the last type of modification of the network the influence of damaging of a part of neurons on the memory properties was investigated. It was assumed, that the states of certain number $K$ of the neurons in the network are blocked during the time evolution: $K/2$ of blocked neurons is in rest states and $K/2$ neurons in firing states. This kind of damaging of neurons may refer to some perturbations of the amount of neurotransmitters in a part of synapses. This modification is called below — a network with partial damaging.

## 4. Results and discussion

For the case of dilution inside blocks maximal storage capacity $\alpha_c$ is showed by the curve marked by triangles in Fig. 2. It decreases slightly slower with increasing dilution parameter $C$ than in the case of randomly diluted networks (*cf.* curve marked by triangles with the curve marked with squares in Fig. 2). In this case maximal value of $C$ parameter equals 0.5, which corresponds to the division of the network into $n = 2$ blocks.

For the case of dilution outside the blocks maximal storage capacity $\alpha_c$ is shown by a curve marked with circles in Fig. 2. This curve starts at $C = 0.5$ which corresponds to the minimal dilution of synaptic connections in this case which occurs for $n = 2$ blocks. For increasing number of blocks, which results in increasing values of dilution parameter $C$, maximal storage capacity $\alpha_c$ decreases faster than for the case of randomly diluted network (cf. curves marked with circles with the curve marked with squares in this

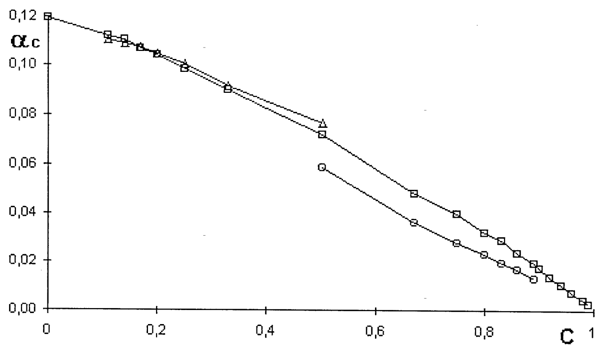


Fig. 2. Maximal storage capacity $\alpha_c$ as a function of dilution parameter $C$ for the network with $N = 1260$ neurons. Dilution inside blocks — triangles; random dilution — squares; dilution outside blocks — circles.

picture). Note that both curves for increasing number of blocks $n$ tend to $C = 1$, which results from the fact that dilution vanishes for increasing number of blocks ($n \longrightarrow N$) and the network become fully unconnected.

Comparison of storage capacity per synapse $\alpha_c/C$ for the networks with dilution inside the blocks with the case of dilution outside the blocks and with the case of random dilution is shown in Fig. 3 (symbols on the curves are the same as in Fig. 2). It is interesting that $\alpha_c/C$ for network with dilution outside the blocks is almost constant, irrespectively of the value of dilution parameter $C$.

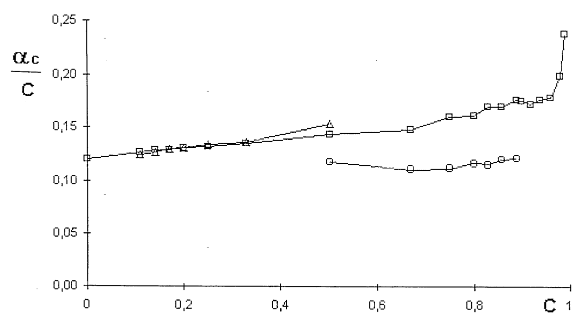


Fig. 3. Maximal Storage capacity per synapse $\alpha_c/C$ as a function of dilution parameter $C$. Symbols on the curves are the same as in Fig. 2.

Comparing two first types of dilution of synaptic connections we can notice that the dilution inside the blocks is more efective *i.e.* the influence of the increase of dilution $C$ for decrease of maximal storage capacity $\alpha_c$ is smaller than in the case of random dilution of network.

In the dilution of excitory/inhibitory synapses the number of effectively retrieved patterns $p_+$ *vs.* number of stored patterns $p$ is shown in Fig. 4. It can be seen that the curve has rather flat maximum for $p_+/p \approx 0.8$, which is much smaller value than for the case of previous types of dilution. On the other hand, memory properties decreases much slower with increasing $p$ than previously — in this kind of dilution overloading of memory has not such critical influence on memory properties as earlier.

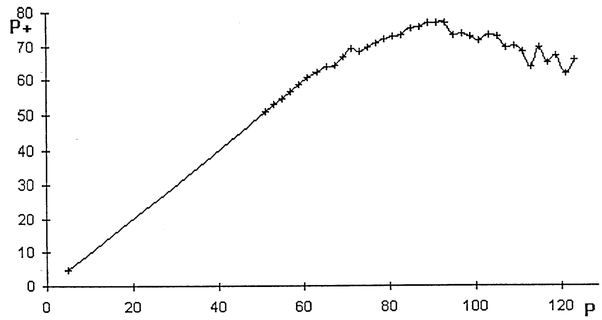


Fig. 4. Maximal number of effectively retrieved patters $p_+$ as a function of stored pattern $p$ for the network with dilution of excitory/inhibitory synapses.

For the case of network with the blocked neurons, during the control of similarity of the state of the network to the retrieved pattern, overlap $m^\mu$ was calculated only for not damaged neurons. Thus, it was possible to compare real similarity of current network's state to the $\mu$-th pattern (for the case of $K$ blocked neurons overlap is decreased as $m^\mu - K/N$). Influence of the number of blocked neurons $K$ for maximal critical capacity is shown in Fig. 5, where the standard maximal capacity is marked by (+), while the maximal capacity per functioning neuron — $\alpha_c = p/(N-K)$ — is marked by (x). As we see the maximal stoarge capacity decreases with the increasing number of blocked neurons. For the case of random, unbiased patterns, which were used in the present work, half of the blocked neurons do not agree with the pattern. For the proper work of such network cutting off these $K$ damaged neurons from the network would have less influence on the memory properties of the network than their autonomous behavior. Maximal critical capacity of such network without $K$ blocked neurons would have constant value $\alpha_c(K=0) = 0.12$ (with the size effect neglected).

In conclusions we can say that dilutions of synaptic connections influence significantly the memory properties of the network. This influence depends on the type of dilution — the dilution inside the blocks seems to be the most effective, *i.e.* gives the highest storage capacity per synapse. As was expected, the partial damaging of the network has not critical meaning for its memory — storage capacity uniformly decreases with increase of the

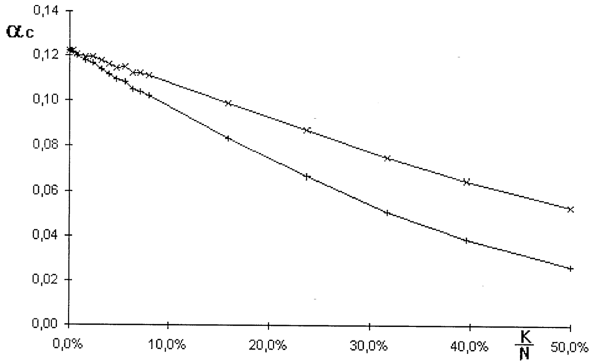


Fig. 5. Maximal critical capacity as a function of number $K$ of blocked neurons; (+) — standard capacity; (x) — capacity per functioning neuron.

number of damaged neurons. It results from our computations that cutting off the damaged neurons has less influence on the work of the whole network than their improper functioning.

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