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FLUX TUBE CONSTANT DETERMINED FROM PROTON FORM FACTOR ANALYSES

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The dipole fit to the proton form factor is extended impirically in a three quark analysis of the electromagnetic form factors $G_{\rm E}$ and $G_{\rm M}$. A relativisitic three quark Dirac shell model wave function is used to describe the quarks in the proton rest frame. Fits beyond the dipole to the electromagnetic form factors lead to the upper and lower components of the shell model wave function, and eventually to a model determination of the potentials acting on the quarks. Asymptotically the scalar potential is a confining linear potential. Its slope allows one to estimate the flux tube constant. A best fit of the electromagnetic form factors using the composite three quark wave function yields similiar values for the flux tube constant. The flux tube constant found is about one half the 0.9 GeV/fm found from meson studies.

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1. Introduction

The proton charge form factor and its relation to the quark quark interaction has been developed [1] in a three constituent quark one component wave function, Schroedinger equation approach. There an analytic dipole shape for the charge form factor was inverted to yield a one component hyperradial composite wave function for the three quarks. This could be further inverted to yield the hypercentral potential acting on the quarks. The proton is modeled here as three low mass quarks bound into a $(1/2^+)^3$ configuration coupled to a J of $1/2$. The Dirac equation is used to describe the dynamics of such relativistic bound quarks, including both the upper component, F, and lower component G of the wave function.

The Dirac magnetic moment of a bound quark depends on both components of the wave function in a relativistic approach. In the $k=-1$, $1/2^+$

state, the electric and magnetic form factors for such a single particle bound in a $(1/2^+)$ level, are:

$$G_{\mathrm{E}} = \int j_0(qr)[F^2 + G^2]dr \tag{1}$$

and

$$G_{\mathrm{M}} = [-4M/q] \int j_1(qr) FGdr\,. \tag{2}$$

With q the lab frame momentum transfer, the magnetic moment is the q going to zero limit of G_{M}. In the constituent quark Schroedinger aproach, the lower component G was neglected in determining the charge form factor G_{E}. The main point of this paper is to see how the lower component of the Dirac wave function for bound quarks can be determined from analyzing jointly the electric form factor, G_{E} and the magnetic form factor, G_{M}. If the upper and lower components can both be determined, then the potentials acting on the quarks in the Dirac equation dynamics can be determined. From these components, the potential, and the flux tube constant can then be estimated. Miller[2] has shown that the external potentials possible in the Dirac equation that conserve parity, total angular momentum, and are time reversal invariant are a radial component of a tensor, a scalar , and a zeroth component of a vector potential. The tensor potential is assumed to be zero.

It is possible to invert the G_{M} form factor in a manner very similiar to the inversion[1] of G_{E}. The difficulty is that there are four independent hyperradial components[3] to determine using the composite three quark wave function in the three body Dirac equation approach, but only two relations, G_{E} and G_{M} to determine them with. Hyperspherical coordinates were used in references [3,4], and the three body Dirac equation was solved in hypercentral approximation. The hypercentral approximation limits the composite three quark wave function to a single configuration, the $(1/2^+)^3$ for the proton. The composite three quark wave function has eight hyperradial components to be determined. For three identical particles, and with each particle with the same set of quantum numbers, one expects the components R_2, R_3, and R_5 to be equal, and also for the components R_4, R_6, and R_7 to equal each other. Then the wave function has only four unknown independent components, R_1, R_2, R_4, and R_8. R_1 is the component that survives in the nonrelativistic limit. These components correspond to having 0, 1, 2, and 3 quark lower components present in the composite three quark wave function. These composite components are determined by solving the three body Dirac equation. They depend on the potentials acting, and do not exhibit constant ratios.

Alternatively, a shell model approach can be employed, which reduces the number of independent wave function components to two. The shell model wave function is then a product wave function,

$$\Phi = \psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\psi_3(\vec{r}_3)\,. \tag{3}$$

The one quark wave function,ψ_1 is to be a solution of the one body Dirac equation, with scalar and vector potentials S and V:

$$(\vec{\alpha}\cdot\vec{p_1} + [S+m]\beta)\psi_1 = [E-V]\psi_1\,, \tag{4}$$

where $\vec{\alpha}$, β are the Dirac matrices. After angular integration, for the $k=-1$, $(1/2^+)$ state, the equations for F and G are:

$$dF/dr - F/r = [m+S-V+E]G \tag{5}$$

and

$$dG/dr + G/r = [m+S+V-E]F\,. \tag{6}$$

Here each one quark wave function has the two radial components F, and G. These components are normalized as

$$\int [F^2+G^2]dr = 1\,. \tag{7}$$

In this case, the two proton form factors lead to determining both the upper and lower components of the one quark wave function.

A linear confining [5] two body potential, βr_{ij} was used by Ref. [1] to describe the quark confinment in the proton. Instead of a two body potential, here a three body flux tube potential is sought from the form factor analysis, based on the ideas of QCD [6]. The flux tube potential [7] is $V_{\rm flux} = bL$, where L is the sum of the lengths from each of the quarks to a central location. The central location is determined by minimizing L, for fixed quark locations. b is called the flux tube constant. In a shell model approach, the central location for the flux tube is taken as the origin, and the flux tube potential for the shell model is $V_{\rm flux} = b(r_1+r_2+r_3)$. The asymptotic slope of any such determined scalar potential will provide an estimate for the flux tube constant.

These form factors were viewed as rest frame form factors in Ref. [1], as no distinction was made between the lab, rest, or Breit frames. Mitra and Kumari [8] rewrote the relativistic form factor proposed by Licht and Pagnamenta [9] in the form:

$$F_{AB}(\vec{q}^2) = \int \Pi_1^{n-1} d\vec{x}_{Ai} d\vec{x}_{Bi} \delta(\vec{x}_{Ai}-\vec{x}_{Bi})\psi_B^*(\vec{x}_{Bi}) {\rm e}^{ip(z_{A1}+z_{B1})}\psi_A(\vec{x}_{Ai})\,, \tag{8}$$

p is the Breit frame momentum which equals $q/2$ in the equal mass case. This relativistic form factor exhibits an acceptable asymptotic behavior [8,10–13] of q^{2-2n} as q^2 tends to infinity. The relativistic form factor takes into acount the Lorentz contraction along the direction of momentum transfer, taken as the z direction. Thus the spherical in the rest frame wave functions, become flattened toward pancake shapes in the Breit frame. The relativistic form factor is written in this way to emphasize the symmetric treatment of the initial and final state wave functions and coordinates. For the elastic scattering appropiate for the electromagnetic form factors, particle A equals B. The number of quarks in the system, n, is three. The delta function is regarded as a function of the mixed coordinates, $(\vec{x}_A, \vec{x}_B)$, which is also subject to the Lorentz transformation. This is the transformation from the rest frame into the Breit frame. Taking the common direction of motion as the z direction, the Lorentz transformations at time $t = 0$ are expressed by:

$$z_{Ai} = \frac{z_i E_A}{M_A}, \qquad z_{Bi} = \frac{z_i E_B}{M_B}\,, \tag{9}$$

where

$$E^2_{A,B} = p^2 + M^2_{A,B}, \quad \text{and} \quad 2\gamma_p = \frac{M_A}{E_A} + \frac{M_B}{E_B}\,. \tag{10}$$

For elastic scattering, $M_A = M_B$, but for inelastic reactions, the coordinates are symmetically handled. Also, the relativistic form factor becomes:

$$F_{AB} \to \left(\frac{M_A M_B}{E_A E_B}\right)^{n-1} F_{\rm rest}(4p^2\gamma_p^2)\,. \tag{11}$$

$F_{\rm rest}$ is just the usual non-relativistic form factor, but evaluated at a reduced momentum transfer. For the elastic scattering case, the $4p^2\gamma_p^2$ becomes k^2 with

$$k^2 = \frac{q^2}{1 + q^2/4M_a^2}\,, \tag{12}$$

k is the reduced momentum transfer in the rest frame, and q is the momentum transfer in the lab frame.

2. Dirac three quark shell model

The three quark Dirac shell model for the proton assumes the quarks move independently about a common center of mass, located at the origin in the rest frame of the proton. The model uses both the upper and lower components of the Dirac equation wave function. The dipole fits to the proton electric and magnetic form factors, $G_{\rm E}$ and $G_{\rm M}$ suggest the F and G, the upper and lower componet radial functions are r times an exponential and r squared times an exponential. The dipole fit to the form factor is:

$$D = \frac{1}{[1 + q^2/\alpha^2]^2}\,, \tag{13}$$

where $\alpha^2 = 0.71 GeV^2$ and q^2 is the squared momentum transfer to the proton in $(\mathrm{GeV})^2$. This form factor to a first approximation, describes all the momentum dependencies of both G_{M}, and G_{E}, the current and charge form factors [14,15] of the proton. An analytic expression for the electric form factor is necessary for ease in the numerical evaluation of the bessel inversions to determine F and G. The electric form factor was fit by the expression:

$$\frac{G_{\mathrm{E}}(q)}{D} = [1 + q^2/\alpha^2]^2 \;\; \frac{a^2/(1 + k^2/\alpha_0^2)^2 + b^2(1 - k^2/\alpha_0^2)^4}{[1 + q^2/4M_a^2]^2}\,, \tag{14}$$

where $a^2 + b^2 = 1$. A best fit was obtained for a=0.75, α_0^2=2.052 GeV^2, and for M_a=0.632 GeV. The magnetic form factor was paramaterized as:

$$\frac{G_{\mathrm{M}}}{2.793D} = C + [1 + q^2/\alpha^2]\,\frac{1 + q^2/4M_a^2}{[1 + q^2/\alpha_1^2]^3}\,, \tag{15}$$

where

$$\frac{1}{\alpha_1^2} = \frac{1}{4M_a^2} + \frac{1}{\alpha_0^2}\,. \tag{16}$$

Added to $G_{\mathrm{M}}/2.793D$ is a correction term, C, beyond the dipole fit of $[1 + q^2/\alpha^2]^2 C_5/[1 + q^2/4M_a^2]$, where

$$C_5 = 0.24\frac{(2k^2/\beta^2) - (k^4/\beta^4)}{[1 + k^2/\beta^2]^2} \tag{17}$$

and

$$\beta^2 = \frac{15\mathrm{GeV}^2}{1 + (10/4M_a^2)}\,. \tag{18}$$

This relativistic approach [8–10] taking into account Lorentz contraction of the wave functions in the Breit frame, along the direction of momentum transfer, is to take the electric or magnetic form factors as

$$G(q) = \frac{1}{[1 + q^2/4M_a^2]^2} G_{\mathrm{rest}}(k)\,. \tag{19}$$

It is possible to fit the magnetic moment in this shell model by using the Dirac magnetic moment from the bound quarks, but not simultaneously with fitting the electric and magnetic form factors for large momentum transfer. Shown in Fig. 1 is a best fit in a chi squared sense for $G_{\rm E}/D$ and for $G_{\rm M}/2.793D$ while varying M_a and α_0 as parameters. They are determined to be 0.632 GeV and 1.42 GeV respectively. For comparison, the experimental proton form factors of Bosted *et al.* [16] are shown. The form factors well reproduce the experimental data. The electric form factor drops below the dipole fit by about 5 to 10 percent. The magnetic form factor rises above the dipole fit by about 5 percent and then gradually falls below the dipole for larger momentum transfer.

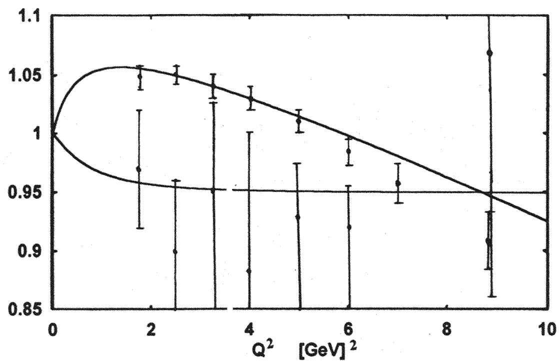

Fig. 1. Best fit to $G_{\rm E}/D$ and $G_{\rm M}/2.793\,D$ for the proton. The upper curve is the magnetic form factor, the lower is the electric. D is the dipole fit to the form factors.

The FG product is numerically determined by bessel inversion transformation of $G_{\rm M}/2.793\,D$. The sum of the squared components, F^2+G^2, is determined from the bessel inversion of $G_{\rm E}/D$. The F^2 and G^2 functions determined in this way are shown in Fig. 2. The large component peaks at a radius of about 2.5/GeV and the lower component is largest near the origin. This behavior is explained by the attractive potentials acting in the Dirac equation dynamics. The asymptotic phase of F/G is assumed negative, as is usual. The scalar $(S+m)$, and Vector parts of the Dirac equation, $(V-E)$, are then numerically determined, and shown in Fig. 3. Both the scalar and vector potentials show an attractive coulombic shape for small radii. The scalar potential asymptotically rises linearly. The lower curve is the vector potential minus the energy E. If E is about 1 GeV, the vector potential found asymptotically goes to zero. We assume the quark mass m is of order 0.010 GeV and is neglected. The scalar potential, S, determined is indeed seen to asymptotically resemble a linear confining potential plus an attrac-

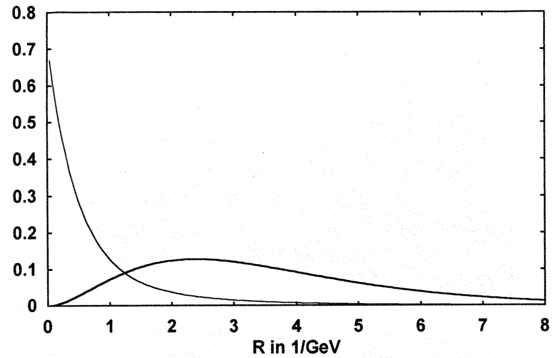

Fig. 2. F^2 and G^2, the upper and lower component of the quark shell model wave functions, determined by bessel transforms from the electromagnetic form factors.

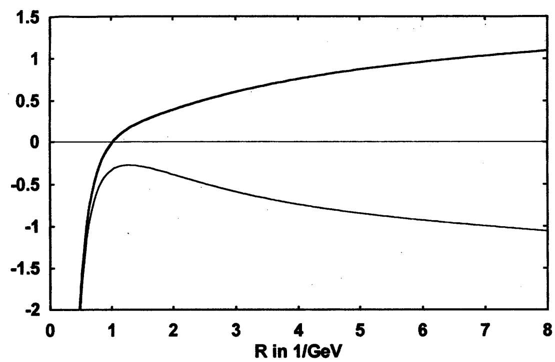

Fig. 3. Rest frame Scalar and Vector potentials deduced from the radial components F, and G. The Scalar potential(upper curve) asymptotically resembles the linear flux tube potential. The vertical scale is in GeV.

tive coulombic term. The slope of the asymptotically large part of the scalar potential is 0.079 GeV^2. This shape is consistent with that from a flux tube potential, allowing the identification of a flux tube constant b=0.40 GeV/fm.

3. Conclusions

Using the Mitra, Kumari, Stanley and Robson relativistic form factor approach, the electric and magnetic form factors of the proton have been fit. Using a Dirac shell model wave function to describe the three quarks in the proton, the form factors can be inverted to provide F and G, the upper and lower components of the quark radial wave function. From these, a scalar linear binding potential is deduced using the Dirac equation for a $(1/2^+)$

state. The asymptotic slope of this potential yields a flux tube constant of $b = 0.40$ GeV/fm. This analysis yields a confining potential slope about one half the 0.9 GeV/fm [5] inferred from meson studies. The relativistic approach including both the upper and lower components of the quark wave functions can be fit to reproduce the shape of the charge distribution of the proton, as evidenced by the reproduction of the form factors. The potentials inferred resemble a linear confining potential plus a short ranged coulombic attraction. These are comparable to a flux tube scalar confining potential plus a one gluon exchange coulombic potential commonly used in Dirac analyses of quark data.

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