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TWO GAMMA DECAY WIDTH OF D MESON IN BOUND STATE MODEL

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We have estimated the two gamma decay width of D meson by using the bound state model of Holdom and Sutherland. Here we have derived an effective quark level Lagrangian for $c \to u\gamma$ and $c \to u\gamma\gamma$ and hence we have calculated the decay width of $D \to \gamma\gamma$. We have obtained the branching ratio for the above decay mode as: Br $(D^0 \to 2\gamma) = 8.63 \times 10^{-6}$.

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1. Introduction

In our earlier paper [1] we have estimated the photonic decay width of heavy D meson in the single loop constituent quark model. The contribution to the two gamma decay of heavy pseudoscalar D meson is computed in the standard model as a function of the quark mass and in [2] we have calculated the two gamma decay width of heavy D meson by pole dominace model. The quark model takes into account only the short distance contribution and the pole dominance model is essential for taking into account the long distance contribution. Recently Holdom and Sutherland [3,4] have invented a model for heavy-light system of quarks [*e.g.* $B(b\overline{d})$ and D $(c\overline{u})$] mesons. In this model, the matrix elements of quark operators are obtained in terms of loop integrals. This model involves vertices where quarks couple to mesons, essentially as in the chiral quark model [5]. The difference is that, in the present case, the meson-quark vertex includes a form factor such that the effect of high momenta following into a light quark is damped. Holdom and Sutherland calculated $B \to K^*\gamma$ within this model with a resonable result. In this paper, we want to study the $D \to \gamma\gamma$ decay in the same model. So

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here we are studying the exotic flavour changing two gamma decay of heavy D meson by using constituent quark model and these are both second order in both the weak and electromagnetic interactions.

In the process $c \to u\gamma$ and its hadronic $D \to \rho\gamma$ counterpart, the light quark is merely a spectator. However, additional effects from the motion of the light quark might be expected in flavour changing two photon decays, such as $c \to u\gamma\gamma$ and its hadronic $D \to \gamma\gamma$ version, where the light and heavy quarks fuse together.

Our paper is organised as follows: in Section 2 we have derived an effective Lagrangian for the $c \to u\gamma$ transition at the quark level and in Section 3 we have estimated the parameters of the bound state model corresponding to the two gamma decay of D meson. In Section 4 we have estimated the $D \to 2\gamma$ decay width by using bound state model and Section 5 contains results and discussion.

2. Effective Lagrangian for radiative $c \to u\gamma$ transition

The flavour changing vertex $c \to u\gamma$ proceeds in one loop through the exchange of d, s and b quark and W boson, is given by:

$$V_\mu = G_1 \overline{u}(\gamma_\mu q^2 - q_\mu q)(1-\gamma_5)c + iG_2[\overline{u}\sigma_{\mu\nu}q^\nu(1+\gamma_5)cm_c + \overline{u}\sigma_{\mu\nu}q^\nu(1-\gamma_5)cm_u]\,, \tag{1}$$

where $q_\mu = p^c_\mu - p^u_\mu$ is the momentum transfer between c and u quarks and p^c_μ and p^u_μ are the four momenta of the c and u quarks, respectively.

For a real photon emission, the first term vanishes identically due to electromagnetic gauge condition and the dominant contribution to G_2 is from s and b quarks.

Now the one-loop level electroweak transitions $c \to u\gamma$ and $c \to u\gamma\gamma$, related by Ward identities, can be combined into effective Lagrangian

$$L(c \to u\gamma) = A\varepsilon^{\mu\nu\lambda\rho}F_{\mu\nu}(u_L iD_\lambda \gamma_\rho c_L)\,, \tag{2}$$

where the covariant derivative D_λ acts on both the u and c quark fields. Actually, the covariant derivative contains both the gluon and photon fields. Also the coefficients $A \sim eG_F\lambda_{KM}$ contains the result of the relevant loop diagrams, and λ_{KM} is the relevant KM factors for the flavour changing $c \to u\gamma$ transitions.

Now one can decompose the effective Lagrangian into two parts namely L_σ and L_F; where L_σ is the well known off-diagonal magnetic moment term and is given by:

$$L_\sigma = A_\sigma \overline{u}(m_c\sigma_{\mu\nu}F^{\mu\nu}(1+\gamma_5) + m_u\sigma_{\mu\nu}F^{\mu\nu}(1-\gamma_5))c \tag{3}$$

and it is associated with the on shell u and c quarks and L_F is given by:

$$L_F = A_F \overline{u}[(i\gamma.D - m_u)\sigma_{\mu\nu}F^{\mu\nu}(1-\gamma_5) + \sigma_{\mu\nu}F^{\mu\nu}(1+\gamma_5)(i\gamma.D - m_c)]c \quad (4)$$

which vanishes by applying the perturbative equation of motion *i.e.* $(i\gamma.D - m_{u,c}) \to 0$. The difference between A_F and A_σ is due to the different anomalous dimensions of the operators in Eqs. (3) and (4) and the A's have the form (6)–(8) (in our notation):

$$A_{\sigma,F} = \frac{4G_F}{\sqrt{2}}\lambda_{KM}\frac{e}{16\pi^2}C_7^{\sigma,F}\,, \tag{5}$$

where $C_7^\sigma = -0.28$ [6] and $C_7^F/C_7^\sigma \approx 4/3$ [8] at the scale $\mu = m_b$.

Now here to calculate the $D \to 2\gamma$ amplitude from the Lagrangian in equation (2) we use bound state model and the bound state model is associated with the constituent quark mass instead of the current quark mass. So in equations (3) and (4) m_u and m_c represent the constituent quark masses of up quark and charm quark, respectively.

Again we can split the off shell operator L_F into the one and the two photon pieces as $L_F = L_F^{1\gamma} + L_F^{2\gamma}$ where $L_F^{2\gamma}$ is related with the quark loop diagram Fig. 1(a) and $L_F^{1\gamma}$ is associated with Fig. 1(b) and Fig. 1(c). In [6] it is shown in detail that even if there is a cancellation between the contributions from $L_F^{2\gamma}$ and a part $L_F^{1\gamma}$, there is a finite remaining nonzero off-shell contribution which has been neglected in the literature by appealing to the perturbative equations of motion. The diagram for the magnetic moment term L_σ are the same as for $L_F^{1\gamma}$. Here one thing should be noted that the magnetic moment term is the most important numerically for $D \to 2\gamma$, in contrast with the $K \to 2\gamma$, where the off-shell contribution is most important [9].

3. Determination of the parameters of the bound state model for D meson

In this section, we want to determine the parameters of the model (*i.e.* Λ_D, Z_D and the decay constant f_D for the D meson. Here we take the effective quark masses of u and c quarks as : $m_u = 250$ MeV and $m_c = 1500$ MeV and the physical mass of D meson as $M_D = 1865$ MeV [10].

To find Λ_D, we first calculate the Feynman diagram representing the $D \to D$ amplitude *i.e.* meson self energy $\Sigma(k^2)$, given by:

$$i\Sigma(k^2) = N_c \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}[i\gamma_5 G_D i S_{F_u}(q) i\gamma_5 G_D i S_{F_c}(q+k)]\,, \tag{6}$$

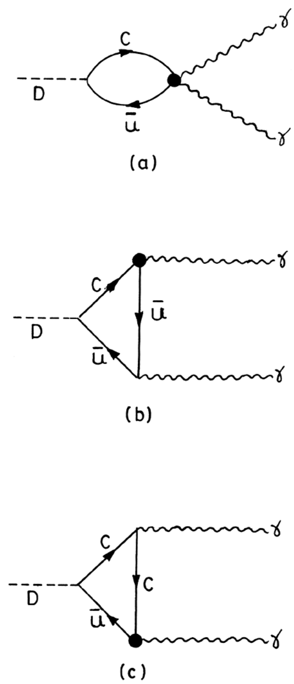

Fig. 1. The two-photon contribution and the one-photon contribution to $D \to \gamma\gamma$. The blacked shaded circle denotes the effective vertex corresponding to L_F or L_σ in the last two diagrams.

where $G_D = \frac{Z_D^2}{\Lambda_D^2 - q^2}$, N_c is the number of colour and q and k are the u quark and the D meson momentum respectively and $S_{F_q} = (\gamma.q - m)^{-1}$ is the quark propagator for constituent quark mass (m).

Now Λ_D can be obtained by plotting Re $[\Sigma(\Lambda_D)]$ vs Λ_D and noting the value of Λ_D for which Re $[\Sigma(\Lambda_D)] = 0$. Here we use the real Λ_D beacuse when we are plotting $\Sigma(\Lambda_D^2)$ vs Λ_D, we observe that there arises an imaginary part because $M_c + M_u \langle M_D$ and we neglect this imaginary part [3,4].

In Fig. 2, we plot Re $[\Sigma(\Lambda_D)]$ vs Λ_D. In this plot, we have used $k^2 \approx M_B^2$ and the effective quark masses of u and c quark. From the graph, we have observed that Re $[\Sigma(\Lambda_D)] = 0$ for $\Lambda_D = 545$ MeV.

Now we want to estimate the parameter Z_D and for the purpose we write the meson self-energy in equation (6) as

$$\Sigma(k^2) = \Sigma(k^2 = M_D^2) + (k^2 - M_D^2)\overline{\Sigma}(k^2) \tag{7}$$

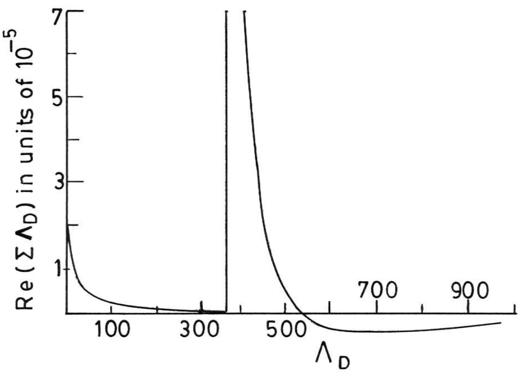

Fig. 2. Variation of Re $[\Sigma(\Lambda_D)]$ with Λ_D.

with the condition

$$\overline{\Sigma}(k^2 = M_B^2) = 1\,. \tag{8}$$

After numerical intregration (equation (6)) we have finally obtained (by using the requirement (8)) $Z_D = 639$ MeV.

Now we proceed to evaluate the decay constant f_D of D meson with the bound state model and it is given by the following expression:

$$if_D p^\mu = -N_c \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}\left[i\gamma_5 G_D i S_{F_u}(q)\gamma_\mu\gamma_5 i S_{F_c}(q+p)\right]. \tag{9}$$

After numerical integration we have obtained the value of $f_D \approx 74$ MeV, which is order of $\frac{1}{3}$ of the physical value *i.e.* $f_D = 208 \pm 35 \pm 12$ MeV [11]. The values of the parameters of the bound state model are summarized in Table I.

TABLE I

The parameters of the bound state model

$M_u = 250$ MeV

$M_c = 1500$ MeV

$\Lambda_D = 545$ MeV

$Z_D = 639$ MeV

$f_D = 74$ MeV

4. $D \to 2\gamma$ decay amplitude

We now want to estimate the decay amplitude of $D \to 2\gamma$ and it can be obtained from Fig. 1. The decay amplitude of the above decay process can be written in the form:

$$M(D \to 2\gamma) = M_{\mu\nu}\varepsilon_\mu(k_1)\varepsilon_\nu(k_2)\,, \tag{10}$$

where $\varepsilon' s$ are the photon polarisation vectors and

$$\begin{aligned} M^{\mu\nu} &= \frac{N_c Z_D^2 e}{8\pi^2}[Ci(g^{\mu\nu}k_1.k_2 - k_1^\nu k_2^\mu) \\ &\quad + D\varepsilon^{\mu\alpha\nu\beta}(k_1)_\alpha(k_2)_\beta]\,, \end{aligned} \tag{11}$$

where C and D are obtained from the diagram in Fig. 1 by numerical integration and are given by:

$$C = -\frac{0.8706(A_R - A_L)}{\Lambda_D^2} + \frac{A_F}{\Lambda_D^2}(0.897M_u - 0.02977M_c) \tag{12}$$

and

$$D = -\frac{0.8706(A_R + A_L)}{\Lambda_D^2} + \frac{A_F}{\Lambda_D^2}(0.897M_u + 0.02977M_c)\,, \tag{13}$$

where $A_L = A_F(M_u - m_u) + A_\sigma m_u$, $A_R = A_F(M_c - m_c) + A_\sigma m_c$. Here m_c and m_u represent the current quark masses of c and u quark respectively and M_c and M_u are the constituent quark masses of the quarks under consideration.

Therefore the two gamma decay width of D meson in the bound state model is given by

$$\begin{aligned} \Gamma(D \to 2\gamma) &= \frac{N_c^2 Z_D^4 \alpha M_D^3}{512\pi^4}(|C|^2 + |D|^2) \\ &= 13.53 \times 10^{-15}\ \text{MeV} \end{aligned} \tag{14}$$

Hence the branching ratio of $D \to 2\gamma$ decay is given by:

$$\text{Br}\,(D \to 2\gamma) = 8.63 \times 10^{-6}\,. \tag{15}$$

5. Results and discussion

We have estimated the branching ratio for the process $D \to 2\gamma$ within the bound state model and we have obtained the branching ratio as: Br $(D \to 2\gamma) = 8.36 \times 10^{-6}$. The model we have used [3,4] does not use the formalism of heavy-quark effective field theory (HQEFT), where results are obtained in terms of expansions in inverse powers of heavy c quark mass and in addition some bound state parameters [12].

Comparing our present results with our previous estimation for the two gamma decay amplitude of heavy D meson:

(i) $A(D_L \to 2\gamma) = 2.04 \times 10^{-11}$ MeV^{-1} (our present prediction)

(ii) $A(D_L \to 2\gamma) = 1.62 \times 10^{-11}$ MeV^{-1} (pole dominance model prediction)

(iii) $A(D_L \to 2\gamma) = 0.88 \times 10^{-11}$ MeV^{-1} (quark model prediction)

we can conclude that quark model results for the two-gamma decay amplitudes of heavy D meson is enhanced when we take into account the long distance effect by meson pole model. It is further enhanced if we estimate the decay amplitude by bound state model.

The parameters of the bound state model are determined by certain requirements for the D meson self-energy. Unfortunately, these requirements lead to imaginary parts for this amplitude. Due to confinement effects, such imaginary parts should not be there and we ignore these imaginary parts. This is of course a drawback of the model. Another drawback of this model is that the obtained value for f_D is of the order of $\frac{1}{3}$ of the measured value. Moreover, there is also an irreducible contribution to $c \to u\gamma\gamma$ of fourth order in the photon momenta which has to be taken into account, in order to obtain a precise value of the amplitude. This contribution is directly proportional to the matrix element of the axial vector current and thus proportional to f_D. Still the contributions we have considered give the comparable order of magnitude for the decay amplitude.

Since uptill now no experimental data is available for the two gamma decay width of heavy D meson so we have no scope to compare our results with the experiment. But we hope that our results should encourage experimentalist to estimate the two gamma decay width of heavy D meson by using modern upcoming machines.

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