ON THE EARLY STAGE OF NUCLEUS–NUCLEUS COLLISIONS

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A statistical model of the early stage of central nucleus–nucleus (A+A) collisions is developed. We suggest a description of the confined state with several free parameters fitted to a compilation of A+A data at the AGS. For the deconfined state a simple Bag model equation of state is assumed. The model leads to the conclusion that a Quark Gluon Plasma is created in central nucleus–nucleus collisions at the SPS. This result is in quantitative agreement with existing SPS data on pion and strangeness production and gives a natural explanation for their scaling behaviour. The localization and the properties of the transition region are discussed. It is shown that the deconfinement transition can be detected by observation of the characteristic energy dependence of pion and strangeness multiplicities, and by an increase of the event–by–event fluctuations. An attempt to understand the data on J/ψ production in Pb+Pb collisions at the SPS within the same approach is presented.

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1. Introduction

At the final state of high energy nuclear collisions many new particles appear. They are massive and extended objects: hadrons and hadronic resonances. What is the nature of particle creation in strong interactions? What is form of matter in a state of very high energy density which is created during the collision of two nuclei? These questions motivate a broad experimental programme in which nuclear collisions at high energy are investigated [1].

Due to lack of a calculable theory of strong interactions the interpretation of experimental results has to rely on phenomenological approaches. The first models attempting to describe high energy collisions were statistical models of the early stage [2,3], the stage in which excitation of the incoming matter takes place. In their original formulations they failed to reproduce experimental results. However, when a broad set of data became available [4, 5], it was realized [6, 7] that after necessary generalization a statistical approach to the early stage gives surprising agreement with the results. It could therefore be used as a tool to identify the properties of the state created at the early stage. The aim of this paper is to further develop the statistical model of the early stage and to apply it to study the properties of the high energy density state created in nucleus–nucleus collisions.

A special role in this study is played by entropy [8] (in collisions at high energy carried mainly by final state pions) and heavy flavours (strangeness, charm) production [9–11]. It can be argued that they are insensitive to the late stages of the collision and therefore carry information about the early stage.

It is experimentally well established that hadrons consist of more elementary subhadronic objects: quarks and gluons [12]. It is therefore natural to assume that at very high energy density, higher than a typical energy density inside a hadron, matter is in the form of a gas of subhadronic degrees of freedom called Quark Gluon Plasma (QGP) [13]. When a quasi-ideal gas of quarks and gluons is assumed in the statistical model of the early stage, the results on strangeness and pion production in nucleus–nucleus collisions at the SPS are reproduced in an essentially parameter free way [7].

This surprising agreement should be contrasted with the problems [14,15] of microscopic non-equilibrium models in describing the same set of data. Scaling properties of the data, natural in the thermodynamical approach, arise in non-equilibrium models as an accidental cancelation of many non-scaling dependences.

All that motivates further development of the experimental programme [16]. The main goal is to localize the collision energy region in which the deconfinement transition takes place and study the properties of the transi-

tion itself. In this region many anomalies are expected. Their experimental observation will lead us to definite conclusion concerning the early stage of nucleus–nucleus collisions. The corresponding experimental study at the SPS should start in 1999 when beams of lower than maximal SPS energy will become available. The experiment NA49 [16] is now being prepared for this investigation.

The main goal of this paper is to further develop the statistical model of the early stage in order to provide a description of the transition region, with special attention paid to the observables which can be measured in the experimental programme. This can be done only when the partition function of the confined state is given. We argue that this state should not be modeled as a gas of hadrons and hadronic resonances. Consequently we have to introduce an effective parametrization of the confined state and try to extract its properties from the comparison with the experimantal data.

Finally we attempt to include in the analysis the production of charm and understand experimental data on J/ψ production [17]. The standard approach is based on the assumption that $c\overline{c}$ states are created in hard QCD processes and later partially destroyed by interactions with the surrounding matter. This approach leads to the conclusion [18] of creation of a QGP only in central Pb+Pb collisions at SPS, but not in the collisions of lighter nuclei. Pion and strangeness data show however no essential difference between central S+S and Pb+Pb collisions at the SPS. They are consistent, within the statistical model analysis, with the creation of a QGP already in central S+S collisions at the SPS. Thus the crucial question is to what extent is this contradiction caused by the use of different approaches for data interpretation? Can a consistent description of the data including the production of charm be obtained using the statistical approach?

The paper is organized as follows. The model used further on for data interpretation and analysis of the transition region is defined in Section 2. The basic features of the model are presented in Section 3, where the approximate analytical formulae are given together with the numerical results obtained using the full version of the model. The model is confronted with the experimental data in Section 4. In Section 5 the discussion of the different approaches to the strangeness and J/ψ production is given. Summary and conclusions close the paper.

For simplicity reasons we consider only central collisions of two identical nuclei (central A+A collisions). The nuclear mass number is denoted by A. The whole discussion is done in the center of mass system. The c.m. energy of nucleon–nucleon pair and nucleon mass are denoted by $\sqrt{s}_{NN}$ and m_N, respectively.

2. A model of the early stage of A+A collisions

1. The basic assumption of our model is that the production of new degrees of freedom in the early stage of nucleus–nucleus collisions is a statistical process. Thus formation of all microscopic states allowed by conservation laws is equally probable. This means that the probability to produce a given macroscopic state is proportional to the total number of its microscopic realizations, *i.e.* a macroscopic state probability P is
$$P \sim e^S, \tag{1}$$
where S is the entropy of the macroscopic state.

2. As the particle creation process does not produce net baryonic, flavour and electric charges only states with the total baryon, flavour and electric numbers equal to zero should be considered. Thus the properties of the created state are entirely defined by the volume in which production takes place, the available energy and a partition function. In the case of collisions of large nuclei the thermodynamical approximation can be used and the dependence on the volume and the energy reduces to the dependence on the energy density. The state properties can be given in the form of an equation of state.

3. We assume that the creation of the early stage entropy in central A+A collisions takes place in the volume equal to the Lorentz contracted volume occupied by the colliding nucleons (participant nucleons) from a single nucleus:
$$V = \frac{V_0}{\gamma}, \tag{2}$$
where $V_0 = 4/3\pi r_0^3 A_p$ and $\gamma = \sqrt{s}_{NN}/(2m_N)$ and A_p is the number of participant nucleons from a single nucleus. The r_0 parameter is taken to be 1.30 fm in order to fit the mean baryon density in the nucleus, $\rho_0 = 0.11$ fm^{-3}.

4. Only a fraction of the total energy in A+A collision is transformed into the energy of new degrees of freedom created in the early stage. This is because a part of the energy is carried by the net baryon number which is conserved during the collision. The released energy can be expressed as:
$$E = \eta(\sqrt{s}_{NN} - m_N)\ A_p\,. \tag{3}$$
The parameter η defines the fraction of the available energy used in the production process. It is assumed to be independent of the collision energy and the system size for A+A collisions discussed in this paper.

This assumption is in agreemnet with the experimental data providing that a correction for pion absorption effects (see point 15 below) is taken into account. It is usually justified by quark–gluon structure of the nucleon [19]. The value of η used for the numerical calculations is 0.67 (see Section 4 for details).

5. In order to predict a probability of creation of a given macroscopic state all possible degrees of freedom and interaction between them should be given in the form of the partition function. In the case of large enough volume the grand canonical approximation can be used and the state properties can be given in the form of an equation of state. The question of how one can use this equation of state to calculate the space-time evolution (hydrodynamics) of the created system requires a separate study.

6. The most elementary particles of strong interaction are quarks and gluons. In the following we consider u, d and s quarks and the corresponding antiquarks with the internal number of degrees of freedom equal to 6 (3 colour states $\times$ 2 spin states). In the entropy evaluation the contribution of c, b and t quarks can be neglected due to their large masses. The charm production is discussed separately in Section 5. The internal number of degrees of freedom for gluons is 16 (8 colour states $\times$ 2 spin states). The masses of gluons and nonstrange (anti)quarks are taken to be 0, the strange (anti)quark mass is taken to be 175 MeV [20].

7. In the case of creation of colour quarks and gluons the equation of state is assumed to be the ideal gas equation of state modified by the bag constant B in order to account for the strong interaction between quarks and gluons and the surrounding vaccum (see *e.g.* [21]):
$$p = p^{\mathrm{id}} - B \ , \quad \varepsilon = \varepsilon^{\mathrm{id}} + B, \tag{4}$$
where p and ε denote pressure and energy density, respectively, and B is the so called bag constant. The equilibrium state defined above is called Quark Gluon Plasma or Q-state.

8. At the final freeze-out stage of the collision the degrees of freedom are hadrons — extended and massive objects composed of (anti)quarks and gluons. Due to their finite proper volume hadrons can exist in their well defined asymptotic states only at rather low energy densities. Estimates $\epsilon < 0.1 \div 0.4$ GeV/fm^3 for the hadron gas with van der Waals excluded volume have been found in Ref. [22]. In the early stage of A+A collisions such low energy density is created only at very low collision energies of a few GeV per nucleon. We note also that

asymptotic hadronic states can be questioned as possible degrees of freedom in the early stage on the basis of our current understanding of $e^+ + e^-$ anihilation processes, where the initial degrees of freedom are found to be colourless $q\overline{q}$ pairs [12]. The above statements lead us to the conclusion that there is no satisfactory model of the confined state at the early stage of A+A collisions at the AGS.

Guided by these considerations we introduce an effective parametrization of the confined state. We assume that at collision energies lower than the energy needed for a QGP creation the early stage effective degrees of freedom can be approximated by point-like colourless bosons. This state is denoted as W-state (White-state). The nonstrange degrees of freedom which dominate the entropy production are taken to be massless, as seems to be suggested by the original analysis of the entropy production in N+N and A+A collisions [6]. Their internal number of degrees of freedom was fitted to the same data [6,7] to be about 3 times lower than the internal number of effective degrees of freedom in A+A collisions at SPS, where in our model creation of QGP takes place. The internal number of degrees of freedom for a QGP is $16+(7/8)\cdot 36 \cong 48$ and therefore the internal number of nonstrange degrees of freedom for low energy collisions is taken to be $48/3 = 16$. The mass of strange degrees of freedom is assumed to be 500 MeV, equal to the kaon mass. The internal number of strange degrees of freedom is estimated to be 14 as suggested by the fit to the strangeness and pion data at the AGS (see Section 4). The phenomenological reduction factor 3 is used in our numerical calculations between the total number of degrees of freedom for Q-state and nonstrange degrees of freedom of W-state because of the different magnitude of strangeness suppression due to different masses of strangeness carriers in both cases. The ideal gas equation of state is selected. We would like to underline once more that the above description of the confined state should be treated only as an effective parametrization. Its parameters are fixed by fitting A+A data at the AGS. This parametrization is needed for the extrapolation to higher collision energies where the transition between the confined and deconfined state is expected. It is, however, intresting to speculate (see Appendix A) about the possible physical meaning of the obtained parameters of the degrees of freedom in the W-state.

9. For large enough volume the grand canonical approximation can be used and the calculation of the entropy is significantly simplified. In a large system only one macroscopic state is produced — the state with the maximum entropy density, s. This is because the relative

probability of the state with the entropy density $s' < s$ is given by:

$$\frac{P}{P_{\max}} = \exp\ [\ V\ (s' - s)\ ]\ . \tag{5}$$

Thus the relative probability decreases to zero when the volume increases to infinity for any value of $s' < s$.

10. In the case of finite (small) volume the conservation laws should be accounted for in a strict way (canonical or microcanonical treatment). The macroscopic states with an entropy density lower than the maximum one are created with final probabilities. As the physical properties of various states can be significantly different (see Section 3) sizeable nontrivial event–by–event fluctuations are expected.

11. The maximum entropy state is called equilibrium state. In the model with two different states (W and Q) the form of the maximum entropy state changes with the collision energy. The regions, in which the equilibrium state is in the form of a pure W or a pure Q state, are separated by the region in which both states coexist (mixed phase or W–Q-state).

12. It is important to note that the formation of a state in global equilibrium in the early stage of nuclear collisions is a consequence of our basic assumption that all possible microscopic states are created with equal probability. Thus it is due to the assumed statistical nature of the primary creation process and it is not due to equilibration by a long lasting sequence of secondary interactions.

13. The globally equilibrated state created in the early stage expands and finally freezes-out into hadrons and hadronic resonances. Recent analysis suggests that this hadronization process can be described by a statistical model [23,24]. Thus phase-space seems to govern not only the production of entropy and the flavour content of the state in the early stage, as discussed in this paper, but also its conversion to hadrons which happens at a significantly lower energy scale.

14. Note that due to the Lorentz contraction the shape of the early stage volume is non-spherical. This causes the isotropic angular distribution of particles in the early stage to be converted during an anisotropic expansion into a forward–backward peaked distribution as observed in the experimental data.

15. We assume that the only process which changes the entropy content of the produced matter during the expansion, hadronization and freeze-out is an interaction with the baryonic subsystem. It was argued that

it leads to an entropy transfer to baryons which corresponds to the effective absorption of about 0.35 π-mesons per baryon [6, 25]. This interaction causes also that the produced hadrons in the final state do not obey symmetries of the early stage production process, *i.e.* a final hadronic state has non-zero baryonic number and electric charge.

16. It is assumed that the total number of s and $\overline{s}$ quarks created in the early stage is conserved during the expansion, hadronization and freeze-out.

The above assumptions are selected to obtain the simplest model needed for the analysis of the experimental results in terms of the statistical mechanics of the confined and deconfined matter. The (approximate) validity of the assumptions is difficult to prove (or disprove) in a direct way due to nonperturbative nature of the discussed processes.

3. Calculations

In the first part of this section we analyze the simplified version of the model which allows us to perform calculations in an analytical way. The results of the numerical calculations done within the full version of the model are presented in the second part of the section.

The calculation are performed in the grand canonical formulation which is justified for large enough systems discussed in this paper. All chemical potentials have to be equal to zero, as we consider only systems with all conserved charges equal to zero. Thus the temperature T remains the only independent thermodynamical variable in the thermodynamical limit when the system volume goes to infinity. It is convenient to define the system equation of state in terms of the pressure function $p = p(T)$ as the entropy and energy densities can be caluclated from the thermodynamical relations:

$$s(T) \;=\; \frac{dp}{dT}\;, \qquad \varepsilon(T) \;=\; T\frac{dp}{dT} \;-\; p\;. \tag{6}$$

In the case of an ideal gas the pressure of the particle species 'j' is given by:

$$p^j(T) \;=\; \frac{g^j}{2\pi^2}\int\limits_0^\infty k^2 dk\;\frac{k^2}{3(k^2+m_j^2)^{1/2}}\;\frac{1}{\exp\left(\frac{\sqrt{k^2+m_j^2}}{T}\right)\;\pm\;1}\;, \tag{7}$$

where g^j is the internal number of degrees of freedom (degeneracy factor) for j-th species, m_j is a particle mass, '–1' appears in Eq. (7) for bosons and '+1' for fermions. The pressure $p(T)$ for an ideal gas of several particle species is additive: $p(T) = \sum_j p^j(T)$. The same is valid for the entropy and energy densities (6).

3.1. Analytical calculations

In order to perform analytical calculations of the system entropy and illustrate the model properties we simplify our consideration assuming that all degrees of freedom are massless. In this case the pressure function (Eq. (7)) is equal to:

$$p^j(T) \;=\; \frac{\sigma^j}{3}\, T^4 \;, \tag{8}$$

where σ^j is the so called Stephan–Boltzmann constant, equal to $\pi^2 g^j/30$ for bosons and $\frac{7}{8}\pi^2 g^j/30$ for fermions. The total pressure in the ideal gas of several massless species can be presented then as $p(T) = \pi^2 g T^4/90$ with the effective number of degrees of freedom g given by

$$g \;=\; g^b + \frac{7}{8}\, g^f, \tag{9}$$

where g^b and g^f are internal degrees of freedom of all bosons and fermions, respectively. The g parameter is taken to be $g_{\rm W}$ for W-state and $g_{\rm Q}$ for Q-state, with $g_{\rm Q} > g_{\rm W}$.

The pressure, energy and entropy densities are given then as:

$$p_{\rm W}(T) = \frac{\pi^2 g_{\rm W}}{90}\, T^4 \;, \quad \varepsilon_{\rm W}(T) = \frac{\pi^2 g_{\rm W}}{30}\, T^4 \;, \quad s_{\rm W}(T) = \frac{2\pi^2 g_{\rm W}}{45}\, T^3 \;, \tag{10}$$

$$p_{\rm Q}(T) = \frac{\pi^2 g_{\rm Q}}{90}\, T^4 - B \;, \quad \varepsilon_{\rm Q}(T) = \frac{\pi^2 g_{\rm Q}}{30}\, T^4 + B \quad , \; s_{\rm Q}(T) = \frac{2\pi^2 g_{\rm Q}}{45}\, T^3 \;, \tag{11}$$

for the pure W- and Q-state, respectively. Note the presence of the non-perturbative bag terms in addition to the ideal quark–gluon gas expressions for the pressure and energy density of the Q-state.

The 1-st order phase transition between W- and Q-state is defined by the Gibbs criterion

$$p_{\rm W}(T_c) \;=\; p_{\rm Q}(T_c) \;, \tag{12}$$

from which the phase transition temperature can be calculated as:

$$T_c \;=\; \left[\frac{90B}{\pi^2(g_{\rm Q} - g_{\rm W})}\right]^{1/4} \;. \tag{13}$$

At $T = T_c$ the system is in the *mixed* phase with the energy and entropy densities given by

$$\varepsilon_{\rm mix} = (1-\xi)\varepsilon^c_{\rm W} \;+\; \xi\varepsilon^c_{\rm Q} \;, \quad s_{\rm mix} = (1-\xi)s^c_{\rm W} \;+\; \xi s^c_{\rm Q} \;, \tag{14}$$

where $(1-\xi)$ and ξ are the relative volumes occupied by the W- and Q-state, respectively. From Eqs. (10), (11) one finds the energy density discontinuity ('latent heat')

$$\Delta\varepsilon \equiv \varepsilon_{\rm Q}(T_c) - \varepsilon_{\rm W}(T_c) \equiv \varepsilon^c_{\rm Q} - \varepsilon^c_{\rm W} = 4B \,. \tag{15}$$

In our model the early stage energy density is an increasing function of the collision energy and it is given by (see Eqs. (2), (3)):

$$\varepsilon \equiv \frac{E}{V} = \frac{\eta\,\rho_0\,(\sqrt{s}_{NN} - 2m_N)\,\sqrt{s}_{NN}}{2m_N} \,. \tag{16}$$

According to our basic assumption (point 1 in Section 2) the created macroscopic state should be defined by the entropy density maximum condition:

$$s(\varepsilon) = \max\,\{\, s_{\rm W}(\varepsilon),\; s_{\rm Q}(\varepsilon),\; s_{\rm mix}(\varepsilon)\,\} \,. \tag{17}$$

In Appendix B we prove a remarkable equivalence of the Gibbs criterion (largest pressure function p_i in the pure i-phase and equal pressures (12) in the mixed phase) and the maximum entropy criteria (17) for an arbitrary equation of state $p = p(T)$ with a 1-st order phase transition. From this fundamental equivalence it follows that for $\varepsilon < \varepsilon^c_{\rm W}$ or $\varepsilon > \varepsilon^c_{\rm Q}$ the system consists of pure W- or Q-state, respectively, with entropy density given by the following equations:

$$s_{\rm W}(\varepsilon) = \frac{4}{3}\left(\frac{\pi^2 g_{\rm W}}{30}\right)^{1/4} \varepsilon^{3/4} \,, \tag{18}$$

$$s_{\rm Q}(\varepsilon) = \frac{4}{3}\left(\frac{\pi^2 g_{\rm Q}}{30}\right)^{1/4} (\varepsilon - B)^{3/4} \,. \tag{19}$$

For $\varepsilon^c_{\rm W} < \varepsilon < \varepsilon^c_{\rm Q}$ the system is in the mixed phase (14) and its entropy density can be expressed as:

$$s_{\rm mix}(\varepsilon) = \frac{\varepsilon^c_{\rm Q} s^c_{\rm W} - \varepsilon^c_{\rm W} s^c_{\rm Q}}{4B} + \frac{s^c_{\rm Q} - s^c_{\rm W}}{4B}\,\varepsilon \equiv a + b\,\varepsilon \,. \tag{20}$$

The ratio of the total entropy of the created state to the number of nucleons paticipating in A+A collisions is given as

$$\frac{S}{2A_p} = \frac{V\,s}{2A_p} = \frac{m_N\,s}{\rho_0\sqrt{s}_{NN}} \tag{21}$$

and it is independent on the number of participant nucleons. The entropy density s in Eq. (21) is given by our general expressions (17) with ε defined

by Eq. (16). For small $\sqrt{s}_{NN}$ the energy density (16) corresponds to the pure W-state and one finds

$$\left(\frac{S}{2A_p}\right)_W = C\, g_{\rm W}^{1/4}\, F\ , \tag{22}$$

where

$$C = \frac{2}{3}\left(\frac{\pi^2 m_N}{15\rho_0}\right)^{1/4} \eta^{3/4}\ , \quad F = \frac{(\sqrt{s}_{NN}-2m_N)^{3/4}}{(\sqrt{s}_{NN})^{1/4}}\ . \tag{23}$$

Thus for low collision energies, where the W-state is created, the entropy per participant nucleon is proportional to F. For high $\sqrt{s}_{NN}$ the pure Q-state is formed and Eq. (21) leads to

$$\begin{aligned}\left(\frac{S}{2A_p}\right)_{\rm Q} &= C\, g_{\rm Q}^{1/4}\, F\, \left(1 - \frac{2m_N B}{\eta\rho_0(\sqrt{s}_{NN}-2m_N)\sqrt{s}_{NN}}\right)^{3/4} \\ &\cong C\, g_{\rm Q}^{1/4}\, F\, \left(1-\frac{3m_N B}{2\eta\rho_0 F^4}\right)\ .\end{aligned} \tag{24}$$

For large values of F the entropy per participant nucleon in Q-state is also proportional to F. The slope is, however, larger than the corresponding slope for the W-state by a factor $(g_{\rm Q}/g_{\rm W})^{1/4}$. In the interval of F in which the mixed phase is formed the energy dependence of the entropy per participant nucleon is given by:

$$\left(\frac{S}{2A_p}\right)_{\rm mix} = \frac{C_1}{\sqrt{s}_{NN}} + C_2\,(\sqrt{s}_{NN}-m_N)\ , \tag{25}$$

where

$$C_1 = \frac{m_N}{\rho_0}\, a\ , \quad C_2 = \eta\, b\ . \tag{26}$$

Eq. (25) gives approximately a F^2 increase of the entropy per participant nucleon in the mixed phase region.

Let us now turn to strangeness and assume that $g_{\rm W}^{\rm s}$ and $g_{\rm Q}^{\rm s}$ are the numbers of internal degrees of freedom of (anti)strangness carriers in W- and Q-state, respectively. The total entropy of the considered state is given by a sum of entropies of strange and nonstrange degrees of freedom. Provided that all particles are massless the fraction of entropy carried by strange (and antistrange) particles is proportional to the number of strangeness degrees of freedom:

$$S_s = \frac{g^{\rm s}}{g}\, S\ . \tag{27}$$

Eq. (27) is valid for both W- and Q-state. Note that all degeneracy factors are calculated according to the general relation (9). For massless particles of the j-th species the entropy is proportional to the particle number

$$S_j \;=\; 4N_j \;. \tag{28}$$

Thus the number of strange and antistrange particles can be expressed as

$$N_s + N_{\overline{s}} \;=\; \frac{S}{4}\,\frac{g^{\rm s}}{g}\;, \tag{29}$$

and the strangeness to entropy ratio is equal to

$$\frac{N_s + N_{\overline{s}}}{S} \;=\; \frac{1}{4}\,\frac{g^{\rm s}}{g}\;. \tag{30}$$

We conclude therefore that the strangeness to entropy ratio for the ideal gas of massless particles is dependent only on the ratio of strange to all degrees of freedom, $g^{\rm s}/g$. This ratio is expected to be equal to $g^{\rm s}_{\rm Q}/g_{\rm Q} \cong 0.22$ in Q-state and $g^{\rm s}_{\rm W}/g_{\rm W} \cong 0.5$ in W-state (see the next subsection). Therefore a phase transition from W- to Q-state should lead to a decrease of the strangeness to entropy ratio by a factor of about 2. This simple picture will be modified essentially because of the large value of the mass of strange degrees of freedom in W-state ($m^{\rm s}_{\rm W} \cong 500$ MeV) in comparison to T. In this case the left hand side of Eq. (30) is a strongly increasing function of T. The right hand side of Eq. (30) gives then only its asymptotic value approached for $T >> m^{\rm s}_{\rm W}$. The numerical calculations for the selected parameters of W- and Q-state are given below.

3.2. Numerical calculations

The results of the calculations performed within the full version of the model as defined in Section 2 are presented below. As all nonstrange degrees of freedom are assumed to be massless, their thermodynamical functions obtained in the previous subsections can be used. For the number of nonstrange degrees of freedom we get:

$$g^{\rm ns}_{\rm Q} \;=\; 2\cdot 8 \;+\; \frac{7}{8}\cdot 2\cdot 2\cdot 3\cdot 2 \;=\; 37\;; \qquad g^{\rm ns}_{\rm W} \;=\; 16\;. \tag{31}$$

The strange degrees of freedom are considered as massive ones. The Eq. (7) is used with

$$g^{\rm s}_{\rm Q} \;=\; 2\cdot 2\cdot 3 \;=\; 12\;,\;\; m^{\rm s}_{\rm Q} \;\cong\; 175\ {\rm MeV}\;; \qquad g^{\rm s}_{\rm W} \;=\; 14\;,\;\; m^{\rm s}_{\rm W} \;\cong\; 500\ {\rm MeV}\;. \tag{32}$$

Note that there is no factor '7/8' in the expression for $g_{\rm Q}^{\rm s}$ (32) as the Eq. (7) with Fermi momentum distribution is taken. The contributions of strange degrees of freedom to the entropy and energy densities are calculated using thermodynamical relations (6).

In order to demonstrate properties of the equation of state the ratios of ε/T^4 and s/T^4 are plotted in Fig. 1 as a function of the temperature. The bag constant $B = 600$ MeV/fm^3 was adjusted such that the critical temperature, T_c is equal to 200 MeV. This choice of T_c was suggested by the results of the analysis of hadron multiplicities in A+A collisions at SPS energies. They indicate that the hadron chemical freeze-out (or hadronization) occurs at a temperature 160–190 MeV [23,24,26–28].

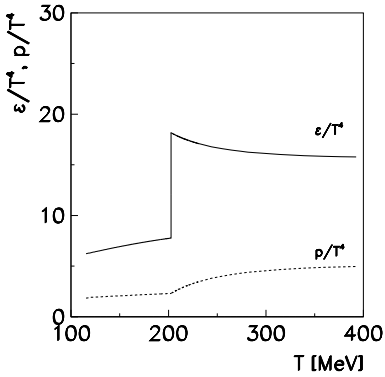

Fig. 1. Energy density and pressure divided by T^4 as a function of temperature T.

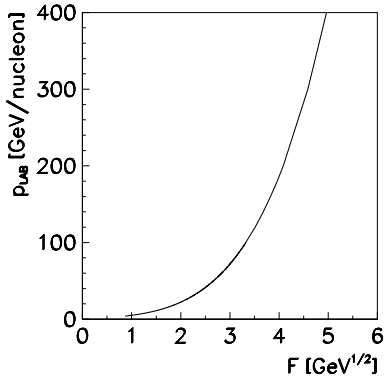

Fig. 2. Relation between laboratory momentum per nucleon and the Fermi–Landau energy variable F. The values of F for $p_{\rm lab} = 5$, 10, 15, 40, 80, 160 and 200 A·GeV are 0.99, 1.43, 1.71, 2.47, 3.10, 3.82 and 4.08 GeV$^{1/2}$, respectively.

As pointed out in the previuos section a conveinient variable to study collision energy dependence is the Fermi–Landau variable F. This variable is used for the further analysis. The relation of the F variable to the laboratory momentum p_{lab} is shown in Fig. 2. The values of F for the top SPS and AGS energies are about 4 $\mathrm{GeV}^{1/2}$ and 1.7 $\mathrm{GeV}^{1/2}$, respectively.

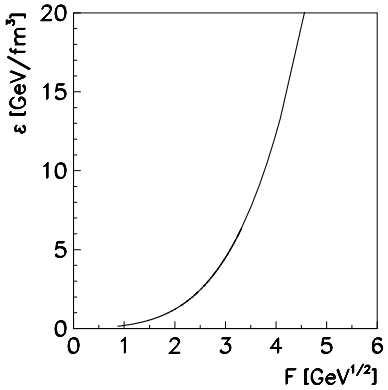

Fig. 3. The early stage energy density as a function of F. The values of ϵ for F = 0.99, 1.43, 1.71, 2.47, 3.10, 3.82 and 4.08 $\mathrm{GeV}^{1/2}$ are 0.20, 0.47, 0.77, 1.71, 2.36, 5.03, 10.53, and 13.32 $\mathrm{GeV/fm}^3$, respectively.

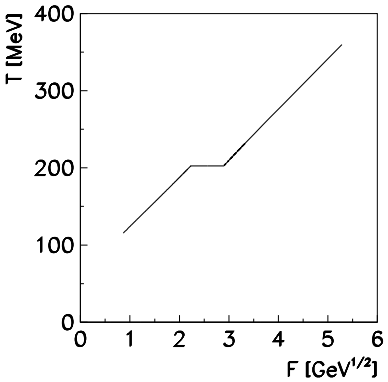

Fig. 4. The early stage temperature as a function of F. The values of T for F = 0.99, 1.43, 1.71, 2.47, 3.10, 3.82 and 4.08 $\mathrm{GeV}^{1/2}$ are 123, 151, 169, 203, 217, 264 and 281 MeV, respectively.

The energy density can be calculated in a unique way on the base of assumptions '3' and '4' from Section 2. The energy density (16) obtained in this way is plotted in Fig. 3 as a function of F. The energy densites for the SPS and AGS energies are about 12 $\mathrm{GeV/fm}^3$ and 0.7 $\mathrm{GeV/fm}^3$, respectively.

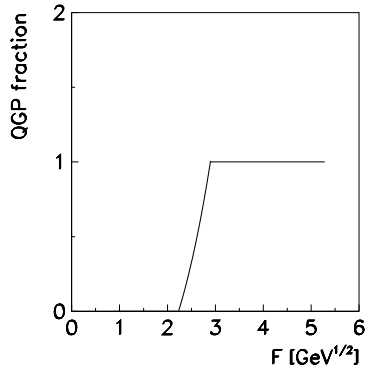

Fig. 5. The fraction of volume occupied by a QGP as a function of F.

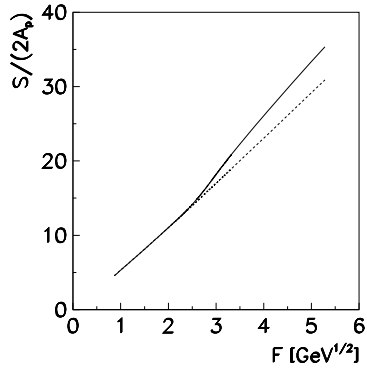

Fig. 6. The entropy per participant nucleon as a function of F (solid line). Dashed line indicates the dependence obtained assuming that there is no transition to the QGP.

The dependence of the early stage temperature T on F is shown in Fig. 4. Outside the transition region T increases in an approximately linear way. Inside the transition region T is constant ($T = T_c = 200$) MeV. The transition region begins at $F = 2.23$ GeV$^{1/2}$ ($p_{\rm lab} = 30$ A·GeV) and ends at $F = 2.90$ GeV$^{1/2}$ ($p_{\rm lab} = 64$ A·GeV).

The fraction of the volume occupied by the Q-state, ξ, increases rapidly in the transition region, as shown in Fig. 5.

The dependence of the entropy per participant nucleon on F is shown in Fig. 6. Outside the transition region the entropy increases approximately proportionaly to F, but the slope in the Q-state region is larger than the slope in the W-state region. The ratio between the value of entropy obtained in our model and the entropy calculated assuming that only W-state exists is shown in Fig. 7.

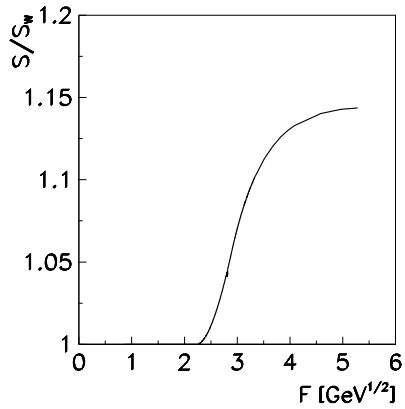

Fig. 7. The ratio between the entropy calculated within our model and the entropy obtined assuming absence of the phase transition to the QGP.

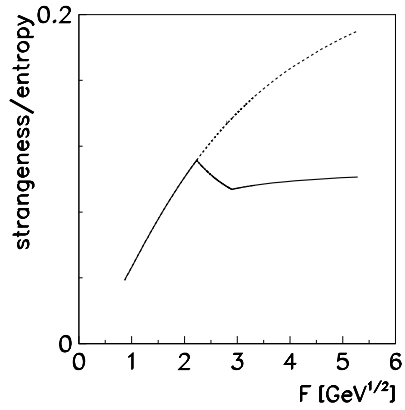

Fig. 8. The ratio of the total number of s and $\overline{s}$ quarks and antiquarks to the entropy (solid line) as a function of F. The dashed line indicates the corresponding ratio calculated assuming absence of the phase transition to the QGP.

We are interested in the collision energy region between the AGS and SPS. At 'low' collision energies (when a pure W-state is formed) the strangeness to entropy ratio increases with F. This is due to the fact that the mass of the strange degrees of freedom is significantly higher than the system temperature. At $T = T_c$ the ratio is higher in the W-state than in the Q-state, this causes the decrease of the ratio in the mixed phase to the level characteristic for Q-state. In the Q-state, due to the low mass of strange quark in comparison to the system temperature, only a weak dependence of the ratio on F is observed. The F dependence of strangeness/entropy ratio is shown in Fig. 8.

Within the model one can estimate the lower limit for T_c assuming that the transition starts just above top AGS energy (15 A·GeV). In this case one obtaines $T_c = 170$ ($B = 300$ MeV/fm^3) and the non-monotonic behaviour of the strangeness production is substituted by a rapid saturation. Remaining signatures of the phase transition are unchanged.

4. Comparison with data

The comparison of the model with the experimental data on pion and strangeness production is presented below. The results are taken from the compilations [4, 5, 7] where the references to the original experimental publications can be found.

During the evolution of the system the equilibration between newly created matter and baryons takes place. It is argued that this equilibration causes transfer of entropy from the produced matter to baryons. The analysis of the pion suppression effect at low collision energies indicates that this transfer corresponds to the effective absorption of about 0.35 pion per participant nucleon [25]. We assume that there are no other processes which change the entropy content of the state produced in the early stage.

For the comparison with the model it is convenient to define the quantity:

$$\langle S_\pi \rangle = \langle \pi \rangle + \kappa \langle K + \overline{K} \rangle + \alpha \langle N_P \rangle, \tag{33}$$

where $\langle \pi \rangle$ is the measured total multiplicity of final state pions and $\langle K + \overline{K} \rangle$ is the multiplicity of kaons and antikaons. The factor $\kappa = 1.6$ is the approximate ratio between mean entropy carried by a single kaon to the corresponding pion entropy at chemical freeze-out. The term $\alpha \langle N_P \rangle$ with $\alpha = 0.35$ is a correction for the above discussed transfer of the entropy to baryons. The quantity $\langle S_\pi \rangle$ can thus be interpreted as the early stage entropy measured in pion entropy units. The conversion factor between S and $\langle S_\pi \rangle$ is choosen to be 4 ($\approx$ pion entropy at chemical freeze-out).

The number of baryons which take part in the collision ($2A_p$ in the model calculations) is identified now with the experimentally measured number of participant nucleons, $\langle N_P \rangle$. The fraction of energy carried by the produced particles (η in Eq. (1)) is taken to be 0.67 as measured by the NA35 Collaboration [29] for central S+S collisions at 200 A·GeV. Production of pions and kaons scales with the number of participant nucleons when central Pb+Pb and S+S collisions at SPS are compared [30]. This suggests that η can be assumed to be independent of the size of the colliding nuclei. Similar values of η are obtained when central A+A collisions at the AGS are analyzed [31] and the correction for the pion absorption is taken into account.

The comparison between the data on $\langle S_\pi \rangle / \langle N_P \rangle$ and the model is shown in Fig. 9. The parametrization of the W-state has been chosen to fit the

AGS data and, therefore, an agreement with low energy A+A data is not surprising. On the other hand the description of the high energy (SPS) results obtained by the NA35 and NA49 Collaborations is essentially parameter free, as the properties of the early stage state, Quark Gluon Plasma, are rather well defined. The change of the slope in F depednence of the pion multiplicity was previously proposed as a signature of the transition region [6].

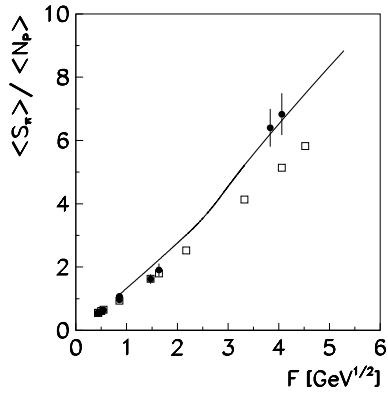

Fig. 9. The $\langle S_\pi \rangle / \langle N_P \rangle$ ratio as a function F. Experimental data on central collisions of two identical nuclei are indicated by closed circles. These data should be compared with the model predictions shown by the solid line. The open boxes show results obtained for nucleon–nucleon interactions.

The comparison between the model and the data on strangeness production is done under assumption that the strangeness content defined in the early stage is preserved till the hadronic freeze-out. It simplifies a picture as the gluon contribution to the strangeness production during the QGP hadronization is neglected. We do not expect a significant number of strange $(s, \bar{s})$-pairs or/and strange-antistrange hadron pairs produced by massless gluons with typical momenta of several hundred MeV at $T = T_c$.

The total strangeness production is usually studied using the experimental ratio:

$$E_s \;=\; \frac{\langle \Lambda \rangle + \langle K + \overline{K} \rangle}{\langle \pi \rangle}, \tag{34}$$

where $\langle \Lambda \rangle$ is the mean multiplicity of Λ hyperons. Within the model E_s (34) is calculated as:

$$E_s \;=\; \frac{(N_s + N_{\overline{s}})/\zeta}{(S - S_s)/4 - \alpha \langle N_P \rangle}, \tag{35}$$

where $\zeta = 1.36$ is the experimentally estimated ratio between total strangeness and strangeness carried by Λ hyperons and $K + \overline{K}$ mesons [32] and

S_s is the fraction of the entropy carried by the strangeness carriers. The comparison between the calculations and the data is shown in Fig. 10. The description of the AGS data is again a consequence of our parametrization of W-state: $g_{\rm W}^{\rm s} = 14$, $m_{\rm W}^{\rm s} = 500$ MeV. As in the case of the pion multiplicity, the description of the strangeness results at the SPS (NA35 and NA49 Collaborations) can be considered as being essentially parameter free[1]. The agreement with the SPS data is obtained assuming creation of globally equilibrated QGP in the early stage of nucleus–nucleus collisions. The characteristic non-monotonic energy dependence of the E_S ratio was proposed in Ref. [5] as a signature of the phase transition and it is confirmed here by calculations in our model. Measurements of strangeness and pion production in the transition region are obviously needed.

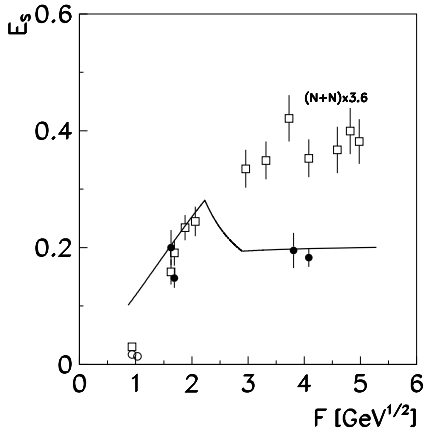

Fig. 10. The ratio E_S as a function F. Experimental data on central collisions of two identical nuclei are indicated by closed circles. These data should be compare with the model predictions shown by the solid line. The open boxes show results obatined for nucleon–nucleon interaction, scaled be a factor 3.6 to match A+A data at AGS energy.

The entropy and strangeness production in central A+A collisions considered here satisfies well the conditions needed for thermodynamical treatment. Therefore one expects that the measures of the entropy per participant nucleon, $\langle S_\pi \rangle / \langle N_P \rangle$, and strangeness per entropy, E_s, are independent

[1] The E_s value resulting from a QGP can be estimated in a simple way. Assuming that $m_s = 0$, and neglecting small ($< 5\%$) effect of pion absorption at SPS, one gets from (30) and (35) $E_s \approx (g_{\rm Q}^{\rm s}/1.36)/g_{\rm Q}^{\rm ns} \approx 0.21$, where $g_{\rm Q}^{\rm s} = (7/8) \cdot 12$ is the effective number of degrees of freedom of s and $\overline{s}$ quarks and $g_{\rm Q}^{\rm ns} = 16 + (7/8) \cdot 24$ is the corresponding number for $u, \overline{u}, d, \overline{d}$ quarks and gluons. Here we use also the approximation that the pion entropy at freeze-out is equal to the mean entropy of q, $\overline{q}$ and g in a QGP.

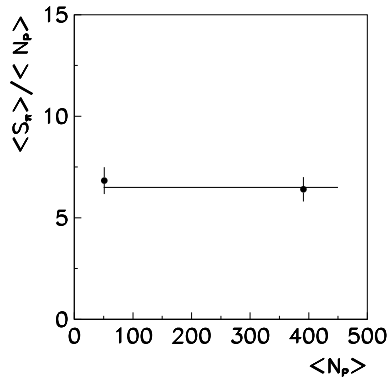

Fig. 11. The $\langle S_\pi \rangle / \langle N_P \rangle$ ratio as a function $\langle N_P \rangle$ for central S+S and Pb+Pb collisions at 200 A·GeV and 158 A·GeV. The results are not corrected for a small difference in the collision energy (see Fig. 9). The model prediction is shown by the solid line.

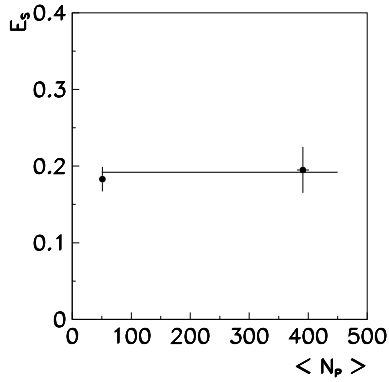

Fig. 12. The ratio E_S as a function $\langle N_P \rangle$ for central S+S and Pb+Pb collisions at 200 A·GeV and 158 A·GeV. The results are not corrected for a small difference in the collision energy (see Fig. 10). The model prediction is shown by the solid line.

of the number of participants for large enough values of $\langle N_P \rangle$. In order to check this in an explicit way we show $\langle S_\pi \rangle / \langle N_P \rangle$ (Fig. 11) and E_s (Fig. 12) as a function of $\langle N_P \rangle$ at SPS energy for central S+S and Pb+Pb collisions.

5. Discussion

Relations between our approach and two widely discussed aspects of nucleus–nucleus collisions strangeness and J/ψ production are presented. Finally we comment on the event–by–event fluctuations.

5.1. Strangeness production

The enhanced production of strangeness was considered by many authors as a potential signal of QGP formation [9–11]. The line of arguments is the following. One estimates that the strangeness equilibration time in QGP is comparable to the duration of the collision process (< 10 fm/c) and about 10 times shorter than the corresponding equilibration time in hadronic matter. It is further assumed that in the early stage strangeness density is much below the equilibrium density *e.g.* it is given by the strangeness obtained from the superposition of nucleon–nucleon interactions. Thus it follows that during the expansion of the matter the strangeness content increases rapidly and approaches its equilibrium value provided the matter is in the QGP state. In the case of hadronic matter the modification of the initial strangeness content is less significant due to the long equilibration time. This leads to the expectation that strangeness production should rapidly increase when the energy transition region is crossed from below.

In the model presented in this paper the role of strangeness is different. The reason for that is different assumption concerning the early stage properties. We assume that due to the statistical nature of the creation process the strangeness in the early stage is already in equilibrium and therefore possible secondary processes do not modify its value. As at $T = T_c$ the strangeness density is similar or even lower (depending on the T_c value) in the QGP than in the confined matter, saturation or suppression of strangeness production is expcted to occur when crossing the transition energy range from below.

In our model the low level of strangeness production in N+N interactions as compared to the strangeness yield in central A+A collisions, called strangeness enhancement, can be understood as due to the effect of strict strangeness conservation (canonical suppression factor) imposed on the degrees of freedom in the confined matter in the early stage.

5.2. J/ψ production

Suppression of J/ψ production was proposed as a signal of creation of the QGP in nuclear collisions [33]. The details of the models used to describe this process changed with time, but the main line of arguments, important here, is the same since the first proposal (for a recent review see [17]). The interpretation of J/ψ results is done within hard production QCD model. It is assumed that the creation of J/ψ follows the dependence given by the production of Drell–Yan pairs, *i.e.* the inclusive cross section in A+A collisions increases with A as A^2. Deviations from this dependence are interpreted as due to interaction of the J/ψ (or 'pre–J/ψ' state) with the surrounding matter. Suppression of J/ψ observed in p+A and O(S)+A collisions at SPS is considered to be caused by the interactions with participant nucleons and

produced particles. The rapid increase of the suppression observed only for central Pb+Pb collisions is attributed [18, 34, 35] to the formation of the QGP which leads to a strong additional desintegration of the J/ψ mesons.

This interpretation is in contradiction to the conclusions based on the analysis of pion and strangeness results within the statistical model. No anomalous change in this observables is seen between central S+S and cental Pb+Pb collisions (see Figs. 11 and 12). It is therefore essential to understand whether this contradiction can be removed when the same approach is used to interpret the whole set of data.

It is natural to extend our statistical model for charm production assuming that like entropy and strangeness, charm in the early stage is produced according to the phase space. We take the mass of the charm quark to be $m_{\rm Q}^c \cong 1.5$ GeV and calculate the mean number of c and $\overline{c}$ quarks for central Pb+Pb collisions at 158 A·GeV. The early stage volume (2) for the experimental number of participant nucleons $\langle N_P \rangle$ in central Pb+Pb collisions is approximately $V \cong 200$ fm^3 and the early stage temperature is 264 MeV (see Fig. 4). For the number density of charm quarks and antiquarks ($g_{\rm Q}^c = 2 \cdot 2 \cdot 3 = 12$) we get:

$$\rho_c = \frac{g_{\rm Q}^c}{2\pi^2} \int\limits_0^\infty k^2 dk \; \frac{1}{\exp\left(\frac{\sqrt{k^2+(m_{\rm Q}^c)^2}}{T}\right)+1} \cong \; g_{\rm Q}^c \left(\frac{m_{\rm Q}^c T}{2\pi}\right)^{3/2} \exp\left(-\frac{m_{\rm Q}^c}{T}\right) . \tag{36}$$

Finally the total average number of charm quarks and antiquarks can be estimated as:

$$N_c \; = \; \rho_c \, V \; \cong \; 0.085 \text{ fm}^{-3} \times 200 \text{ fm}^3 \; \cong \; 17 \, . \tag{37}$$

Note that contribution of c and $\overline{c}$ to thermodynamical functions of the QGP was neglected in the calculations presented in the previous Sections. This is indeed justified by a large value of charm quark mass. The contribution of charm quarks and antiquarks to the energy density can be estimated as $\varepsilon_c \cong \rho_c m_{\rm Q}^c \cong 0.13$ GeV/fm^3. This value is much smaller than total energy density of the QGP, $\varepsilon_{\rm Q} \cong 11$ GeV/fm^3, in the early stage of Pb+Pb at the SPS. The inclusion of charm into the QGP equation of state causes a decrease of less than 1 MeV of the early stage temperature.

The equilibrium number (37) exceeds substantially the estimate given in Ref. [36] and based on the assumption of perturbative production of open charm. Recently the NA50 Collaboration attempts to estimate open charm production in central Pb+Pb collisions at SPS using the measured invariant mass spectrum of dimuon pairs [37]. This estimate relies on the assumption (based on the PHYTIA model) that c and $\overline{c}$ quarks are produced correlated.

In our statistical model c and $\overline{c}$ quarks are independent and therefore their contribution to the dimuon spectrum can not be distinguished from the background contribution[2]. Thus results of NA50 on 'charm-like' enhancement can not be compared with our predictions.

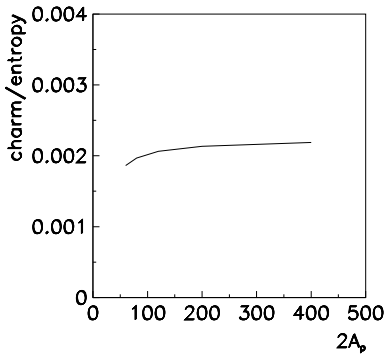

Fig. 13. The ratio of charm to entropy as a function of the the number of participant nucleons ($2A_p$) for A+A collisions at 158 A·GeV. The canonical suppression factor is included in the calculation.

Due to large mass of the charm quark one should consider a possible correction to the grand canonical approximation due to a strict charm conservation. In Fig. 13 we show the predicted dependence of the charm to entropy ratio on the number of participant nucleons ($2A_p$) including a correction for strict charm conservation calculated as in Ref. [38]. It is observed that even down to low values of $2A_p = 40$ the correction is small and therefore the charm/entropy ratio is approximately independent of the volume of the system, similar to the strangeness/entropy ratio.

Analysis of the hadron yields within a statistical hadronization model [23,24] shows that the hadronization is a local statistical process. Thus one expects that also the ratio of the mean J/ψ multiplicity to entropy (pion multiplicity) should be volume independent. This prediction of our model can be checked against experimental data. However as the results on J/ψ multiplicity are not published, we have to perform ourselves a conversion of the available data. We start from the E_T dependence of the ratio measured by the NA50 Collaboration [39]:

$$R(J/\psi) = \frac{B_{\mu\mu}\sigma(J/\psi)}{\sigma(DY)} \ , \tag{38}$$

where $\sigma(J/\psi)$ and $\sigma(DY)$ are inclusive cross sections for production of J/ψ and Drell–Yan pairs and $B_{\mu\mu}$ is the branching ratio for J/ψ decay into a

[2] We thank E. Scomparin for pointing to us this property of the NA50 procedure.

$\mu^+\mu^-$ pair. As at SPS energies pion production dominates particle production, the measured transverse energy is basically determined by the pion transverse energy. The mean transverse momentum of pions is indepedent of the centrality [40] and therefore E_T can be considered to be proportinal to the pion multiplicity or the number of participant nucleons. The multiplicity of Drell–Yan pairs increases with the centrality as $\langle N_P \rangle^{4/3}$ or equivalently as $E_T^{4/3}$. Thus the mean J/ψ multiplicity is expected to increase as:

$$\langle J/\psi \rangle \sim R(J/\psi) E_T^{4/3} \tag{39}$$

and consequently the J/ψ multiplicity per pion should be proportional to

$$\frac{\langle J/\psi \rangle}{E_T} \sim R(J/\psi) E_T^{1/3}. \tag{40}$$

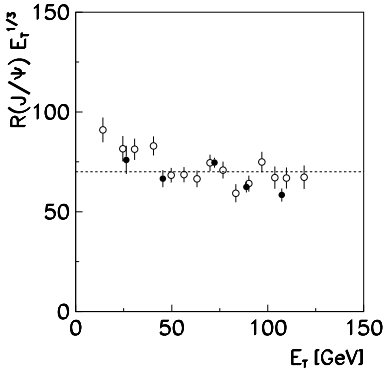

Fig. 14. The estimate of the transverse energy dependence of the J/ψ multiplicity per pion (arbitrary units) in Pb+Pb collisions at 158 A·GeV. The closed points show final 1995 data and the open points preliminary 1996 data of the NA50 Collaboration. The dashed line is drawn for the reference.

The values of $R(J/\psi)E_T^{1/3}$ are plotted as a function of E_T for Pb+Pb collisions at 158 A·GeV in Fig. 14. The ratio $R(J/\psi)E_T^{1/3}$ ($\sim \langle J/\psi \rangle / \langle \pi \rangle$) seems to be independent of E_T in the whole range of E_T. Thus we conclude that the experimental dependence of J/ψ production on E_T in Pb+Pb collisions is in the agreement with the expectation of our model. It just reflects the statistical character of charm production and the following hadronization process.

As pointed out in Ref. [24] the particle abundances resulting from the hadronization can be modified by the inelastic interactions in the freezeing-out hadronic matter. It is however argued [17] that the J/ψ hadronic cross

sections are small, thus no significant reduction of the J/ψ yield is expected due to hadronic interactions. This argumentation is however not valid for ψ' production for which hadronic cross sections are estimated to be 10 times larger than for J/ψ and therefore a significant suppression of ψ' mesons can be observed [17]. This is usually considered as the reason why the ratio $\sigma(\psi')/\sigma(J/\psi)$ decreases with E_T [17].

We summarize that the effect of the anomalous J/ψ suppression in central Pb+Pb collisions is a result of the interpretation of the data within the model of hard QCD production of J/ψ with following suppression. The same data analyzed within the statistical approach show no anomalous behaviour and lead to a consistent interpretation of the results on pion, strangeness and charm production.

5.3. Event–by–event fluctuations

It was recently measured by the NA49 Collaboration [41] that the event–by–event transverse momentum fluctuations in central Pb+Pb collisions at 158 A·GeV are much smaller than the fluctuations measured in the p+p interactions and expected in non-equilibrium models of nuclear collisions [42, 43]. A decrease of the global fluctuations with the increasing volume of the system and/or increasing number of internal degrees of freedom is a generic feature of the statistical models [44]. Therefore in our approach one expects a decrese of global fluctuations when going from p+p interactions to central Pb+Pb collisions. The same arguments lead to the conclusion that the flavour fluctuations should be also reduced in central A+A collisions in comparison to p+p interactions. The method to analyze these fluctuations was recently formulated [45].

Finally we expect an increase of the fluctuations in the transition region. This is because of the additional possibility of changing the relative content of W- and Q-state in the early stage. An important observable should be the strangeness to entropy ratio as it is significantly different in W- and Q-state at $T = T_c$.

6. Summary and conclusions

In this paper we continue a development of the statistical model of the early stage in high energy nucleus–nucleus collisions. We attempt to understand the possible meaning of the equilibration of the *created state*. We attribute the success of the statistical description of the early stage to the statistical nature of the strong interaction in A+A collisions rather than to the results of multiple secondary processes following the primary creation.

We show that the assumption that the state created in the early stage is in the form of the Quark Gluon Plasma gives an essentially parameter free description of the data on pion and strangeness production in central A+A collisions at SPS.

It is argued that the early stage degrees of freedom in the confined matter can not be modeled by hadrons and hadronic resonances. An effective statistical description of the confined state (W-state) is introduced and the parameters characterizing degrees of freedom are extracted from the comparison with the data.

The transition between W-state and QGP when increasing collision energy is discussed. It is proven that the condition of maximum entropy is equivalent to the Gibbs construction of the first order phase transition between W-state and QGP.

The transition region is localized to be between 30 A · GeV and 65 A · GeV for the set of parameters used in the paper. It is shown that the transition should be associated with a characteristic increase of pion multiplicity and a non-monotonic energy dependence of the strangeness to pion ratio. It is also argued that an increase of the event–by–event fluctuations can be expected in the transition region. Note that anomalies in the space-time pattern of the matter expansion are also expected due to softening of the equation of state in the mixed phase [46]. This can be detected by the analysis of single particle spectra and two particle correlations [47].

Finally we remind that the anomalous J/ψ suppression in central Pb+Pb collisions at SPS is a result of the data interpretation within a model assuming that the charm production is a hard QCD process. We show that the same experimental results are also consistent with the hypothesis that the J/ψ multiplicity per pion is independent of the centrality of Pb+Pb collisions, similar to the behaviour of the strangeness/pion ratio. This behaviour can be reproduced in our approach when the charm production is treated in the same statistical way as the production of strangeness. It allows for a consistent interpretation of the results on pion, strangeness and J/ψ production in A+A collisions at SPS. Data on total charm production are obviously needed to check our assumption of a statistical charm production.

We conclude that a broad set of experimental data is in agreement with the hypothesis that QGP is created in central A+A (S+S and Pb+Pb) collisions at the SPS. A study of the energy dependence of several basic observables (pion and strangeness multiplicities, expansion pattern and event–by–event fluctuations) should be able to uniquely prove the existence of the phase transition to the Quark Gluon Plasma.

The results obtained by the NA35 and NA49 Collaborations play a major role in the interpretation of the whole set of data. We would like specialy thank P. Seyboth and R. Stock spokesmen of these experiments. We thank K.A. Bugaev, D. Ferenc, L. Frankfurt, U. Heinz, C. Lourenco, St. Mrówczyński, H. Satz, H. Ströbele and B. Svetitsky for critical and vivid discussions and comments to the manuscript. M.I.G. is also grateful to the BMFT, DFG and GSI for the financial support.

Appendix A

The properties of W-state were obtained by an 'educated guess' procedure. It is however still intresting to note that the degrees of freedom in W-state can be identified with colourless $q\overline{q}$ pairs. Assuming that the light quarks (u, d) are almost massless we obtaine 4 nonstrange flavour–antiflavour combinations: $u\overline{u}$, $d\overline{d}$, $d\overline{u}$ and $u\overline{d}$. Each pair can be in 4 spin states, which gives 16 massless nonstrange internal degrees of freedom. Similar counting gives 16 different pairs with s or $\overline{s}$ quark. There are in addition 4 $s\overline{s}$ pairs which however can be considered strongly suppressed due to the large mass of strange quarks. Thus the numbers of non-strange and strange degrees of freedom obtained for the colourless $q\overline{q}$ pairs approximately coincide with the corresponding numbers extracted from the data for the W-state ($g_{\rm W}^{\rm ns} \approx 16$ and $g_{\rm W}^{s} \approx 14$).

The reduction of the effective number of degrees of freedom from colored q, $\overline{q}$ and g to colour neutral $q\overline{q}$ pairs may be understood as a result of the requirement of local colour neutrality imposed on the creation process at low energy density and/or for small systems.

We note also that the colourless $q\overline{q}$ pairs are identified as initial degrees of freedom in the $e^+ + e^-$ anihilation process by the well known analysis of the ratio $\sigma(e^+ + e^- \to hadrons)/\sigma(e^+ + e^- \to \mu^+ + \mu^-)$ [12].

Appendix B

We present here a general proof of the equivalence between Gibbs construction of the 1-st order phase transition and the basic condition that the equilibrium state is equal to the maximum entropy state. The proof is valid when all conserved charges are equal to zero as considered in the paper. In this case the pressure function, $p = p(T)$, defines completely the system thermodynamics providing that the system volume V goes to infinity (thermodynamical limit). The temperature T remains the only independent thermodynamical variable. The energy density, $\varepsilon(T)$, and entropy density, $s(T)$, are calculated as

$$\varepsilon(T) \;=\; T\frac{dp}{dT} - p\,, \qquad s(T) \;=\; \frac{dp}{dT}\,. \tag{41}$$

A discontinuity of the first derivative, dp/dT, at $T = T_c$ corresponds, by definition, to a 1-st order phase transition at temperature $T = T_c$. In physical terms one describes the system at $T < T_c$ by function $p = p_1(T)$ (low-temperature phase) and by $p = p_2(T)$ at $T > T_c$ (high-temperature phase). At $T = T_c$ pressures of the two phases are equal

$$p_1(T_c) \;=\; p_2(T_c) \;\equiv p_c \;, \tag{42}$$

and their first derivatives satisfy inequalities

$$\left(\frac{dp_2}{dT}\right)_{T=T_c} \;>\; \left(\frac{dp_1}{dT}\right)_{T=T_c} \;. \tag{43}$$

The energy density discontinuity (latent heat) as well as the entropy density discontinuity take place at $T = T_c$:

$$\Delta\varepsilon \;=\; \varepsilon_2(T_c) - \varepsilon_1(T_c) \;=\; T_c\,[s_2(T_c) - s_1(T_c)] \;>\; 0 \;. \tag{44}$$

At $T = T_c$ the system is in the mixed phase with

$$\varepsilon_{\rm mix} \;=\; (1-\xi)\varepsilon_1(T_c) \;+\; \xi\varepsilon_2(T_c) \;, \quad s_{\rm mix} \;=\; (1-\xi)s_1(T_c) \;+\; \xi s_2(T_c) \;, \tag{45}$$

where $1 - \xi$ and ξ are relative volumes occupied by phases '1' and '2', respectively. The above construction is known as the Gibbs creteria for a 1-st order phase transition: at a given temperature T the system occupies a pure phase which pressure is larger and the mixed phase is formed if both pressures are equal. One considers phases '1' at $T > T_c$ and '2' at $T < T_c$ as metastable states (superheated and supercooled, respectively). Such a consideration is physically important in the kinetic picture of a phase transition and for the studies of statistical fluctuations. We prove now the equivalence of the Gibbs creteria to the maximum entropy condition of the mixed phase. It claims that at any energy density ε from the interval $[\varepsilon_1(T_c), \varepsilon_2(T_c)]$ the entropy density of the mixed phase is maximal one:

$$s_{\rm mix}(\varepsilon) \;>\; s_i(\varepsilon) \;, \quad i = 1, 2 \;, \quad \varepsilon \in [\varepsilon_1(T_c), \varepsilon_2(T_c)] \;. \tag{46}$$

The following equations for the entropy densities of the pure and mixed phase can be easily obtained from Eq. (41):

$$s_i \;=\; \frac{\varepsilon + p_i(T)}{T} \;, \quad i = 1, 2 \;; \qquad s_{\rm mix} \;=\; \frac{\varepsilon + p_c}{T_c} \;. \tag{47}$$

Now the values of s_1 and s_2 should be compared to the value of $s_{\rm mix}$ at the same ε from the interval $[\varepsilon_1(T_c), \varepsilon_2(T_c)]$. This means that the comparison

is done at the temperature of a pure phase $T > T_c$ for i=1 and $T < T_c$ for i=2 in Eq. (47). The inequalities (46) can be transformed into

$$\frac{dp_1}{dT} > \frac{p_1(T) - p_c}{T - T_c}, \quad T > T_c, \tag{48}$$

$$\frac{dp_2}{dT} < \frac{p_c - p_2(T)}{T_c - T}, \quad T < T_c \tag{49}$$

by substituton of ε in Eq. (47) by $Tdp_i/dT - p_i(T)$ according to Eq. (41). Simple geometrical meaning of these inequalities is quite clear: they are satisfied for any convex (from below) function $p_i(T)$. Any physical pressure function $p(T)$ should have positive second derivative, $d^2p/dT^2 > 0$, and, therefore, is indeed a convex function. To prove this last statement we use the relation

$$\frac{d^2p}{dT^2} = \frac{1}{T}\frac{d\varepsilon}{dT}, \tag{50}$$

which follows from Eq. (41). Positive sign of $d\varepsilon/dT$ is a consequence of the definition of energy in statistical mechanics:

$$\varepsilon = \frac{\langle E\rangle}{V} = \frac{1}{V}\,\frac{\sum_n E_n \exp(-E_n/T)}{\sum_n \exp(-E_n/T)}. \tag{51}$$

From Eq. (51) one finds

$$\frac{d\varepsilon}{dT} = \frac{1}{V}\,\frac{d\langle E\rangle}{dT} = \frac{\langle E^2\rangle - \langle E\rangle^2}{VT^2} = \frac{\langle (E - \langle E\rangle)^2\rangle}{VT^2} > 0. \tag{52}$$

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