KAON CONDENSATION IN NEUTRON STARS AND HIGH DENSITY BEHAVIOUR OF NUCLEAR SYMMETRY ENERGY *

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We study the influence of a high density behaviour of the nuclear symmetry energy on a kaon condensation in neutron stars. We find that the symmetry energy typical for several realistic nuclear potentials, which decreases at high densities, inhibits kaon condensation for weaker kaon–nucleon couplings at any density. There exists a threshold coupling above which the kaon condensate forms at densities exceeding some critical value. This is in contrast to the case of rising symmetry energy, as *e.g.* for relativistic mean field models, when the kaon condensate can form for any coupling at a sufficiently high density. Properties of the condensate are also different in both cases.

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1. Introduction

The possibility that a charged kaon condensate is present in the ground state of dense baryon matter has been suggested by Kaplan and Nelson [1]. The presence of a kaon condensate would strongly affect astrophysically important properties of dense matter in neutron stars [2]. For example, the proton abundance of neutron star matter could increase, exceeding the direct URCA threshold. This would strongly accelerate cooling of neutron stars. Also, the formation of metastable neutron stars could be allowed, as the equation of state of hot matter with trapped neutrinos would stiffen, supporting larger maximum mass than the cold one [2]. Metastable neutron stars with masses exceeding the maximum mass for the cold equation of

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state would collapse to black holes after the neutron star matter becomes transparent to neutrinos.

The formation of a kaon condensate is expected because of the presence of strongly attractive interactions between kaons and nucleons, mainly from the so-called sigma term, Σ^{KN}. These interactions lower the kaon effective mass at higher densities leading to the condensation of kaons above a critical density, $n_{\rm kaon}$, which is estimated in Ref. [1] to be $\sim 3n_0$, where $n_0 = 0.16$ fm^{-3} is the nuclear saturation density.

To study the possibility of a kaon condensation in neutron stars one should account for the β-equilibrium of the neutron star matter which requires that kaon and electron chemical potentials are equal, $\mu_{K^-} = \mu_e$, where the electron chemical potential is given by a difference of proton and neutron chemical potentials, $\mu_e = \mu_P - \mu_N$. This formula shows that also nucleon–nucleon interactions, which determine both μ_P and μ_N, are of crucial importance for the existence of a kaon condensation in neutron stars. Thorsson, Prakash and Lattimer [3] have studied the role of nuclear interactions in a kaon-condensed neutron star matter using simple parametrizations of nuclear forces. However, parametrizations used in Ref. [3] are rather similar as far as the corresponding nuclear symmetry energy, $E_{\rm sym}(n_B)$, is concerned. For all models in Ref. [3], $E_{\rm sym}(n_B)$ monotonically increases with increasing baryon density n_B. This behaviour at high densities differs considerably from that corresponding to several realistic phenomenological interactions, such as *e.g.* UV14 + TNI [4], or AV14 + UVII [4], for which the symmetry energy actually saturates and then decreases at high densities. In our paper we study consequences of this kind of behaviour of the nuclear symmetry energy for the formation of a kaon condensate in neutron stars.

The electron chemical potential, μ_e, and the proton abundance in the neutron star matter are very sensitive to the nuclear symmetry energy at high densities [5]. Thus, also the critical density for kaon condensation, $n_{\rm kaon}$, depends on $E_{\rm sym}(n_B)$. The latter quantity is subject to large uncertainties at higher densities [5]. As indicated above, different models which fit the saturation properties of nuclear matter give rather incompatible predictions of the symmetry energy at high densities. We find that this uncertainty affects strongly the critical density of the kaon condensation. In particular, for the symmetry energy which decreases at high densities, as *e.g.* for the UV14 + TNI interactions, the kaon condensation is inhibited, at least for weaker kaon–nucleon couplings. This is in contrast to the case of monotonically increasing symmetry energy when the kaon condensate can form for any value of the kaon–nucleon coupling at a sufficiently high density.

In the next section we describe the model of the kaon condensate which includes realistic nucleon–nucleon interactions. In Sect. 3 we discuss the high density behaviour of the nuclear symmetry energy. Main results concerning the dependence of the condensate properties on the symmetry energy are presented in Sect. 4.

2. The kaon condensate in neutron stars

The Kaplan and Nelson model [1] of the kaon condensate employs an SU(3)×SU(3) chiral Lagrangian, obtained in the chiral perturbation theory, which involves octets of pseudoscalar mesons and baryons. Brown, Kubodera and Rho [6] have shown that interactions leading to the kaon condensation in the Kaplan and Nelson model are dominated by the s-wave kaon–nucleon coupling. Contributions due to remaining interactions are less important and we neglect them here.

When only the s-wave kaon–nucleon interactions are relevant, the kaon condensate is spatially uniform and its time dependence is $\langle K^- \rangle = v_K \exp(-i\mu_{K^-} t)$ [7], where v_K is the amplitude of the mean K^--field. For such a condensate the effective Kaplan–Nelson Lagrangian in the sector including only nucleons and kaons, that is relevant to the neutron star matter, reads [3]:

$$L_{KN} = \frac{f^2}{2}\mu^2 \sin^2\theta - 2m_K^2 f^2 \sin^2\frac{\theta}{2} + n^\dagger n\Big(\mu \sin^2\frac{\theta}{2} - (2a_2+4a_3)m_s \sin^2\frac{\theta}{2}\Big) + p^\dagger p\Big(2\mu \sin^2\frac{\theta}{2} - (2a_1+2a_2+4a_3)m_s \sin^2\frac{\theta}{2}\Big) + L_N\,. \qquad (1)$$

Here $\mu \equiv \mu_{K^-}$ is the kaon chemical potential, $\theta = \sqrt{2}v_K/f$ is the "chiral angle", where $f = 93$ MeV is the pion decay constant, and L_N is the free nucleon Lagrangian. The parameters, a_1, a_2, and a_3 are coefficients of the interaction terms in the original Kaplan–Nelson Lagrangian which provide splitting of the masses in the baryon octet [1], and m_s is the strange quark mass.

The effective Lagrangian (1) contains three effective kaon–nucleon coupling parameters, $a_1 m_s$, $a_2 m_s$, and $a_3 m_s$ (our notation follows that of Ref. [3]). The first two are determined by fitting the strange baryon masses [3]. In the following we adopt the values $a_1 m_s = -67$ MeV and $a_2 m_s = 134$ MeV from Ref. [3]. The third parameter, $a_3 m_s$, is related to the kaon–nucleon sigma term,

$$\Sigma^{KN} = -\frac{1}{2}(a_1 + 2a_2 + 4a_3)m_s\,, \qquad (2)$$

which is poorly known and thus the value of $a_3 m_s$ is subject to a considerable uncertainty. We use here values in the range -134 MeV $> a_3 m_s > -310$ MeV which correspond to the sigma term in the range 170 MeV $< \Sigma^{KN} <$ 520 MeV [3].

The Lagrangian (1) describes only the kaon–nucleon interactions in the condensate. To account for nucleon–nucleon interactions, which are crucial in a dense nucleon matter in neutron stars, we use realistic models of the nucleon matter [4]. The energy density of the neutron star matter with the kaon condensate is composed of three contributions,

$$\varepsilon_{ns} = \varepsilon_{KN} + \varepsilon_N + \varepsilon_{\rm lep} \,, \tag{3}$$

where ε_{KN} is the energy of the kaon condensate described by the Lagrangian (1), ε_N includes both the nucleon–nucleon interaction contribution and the nucleon Fermi energy, and ε_{lep} is the electron and muon contribution.

The energy per particle, ε_N/n_B, obtained in variational many-body calculations with realistic nucleon–nucleon interactions, can be written as a function of the baryon number density, n_B, and the proton fraction, $x = n_P/n_B$, in the form [8]

$$E(n_B,x) = T_F(n_B,x) + V_0(n_B) + (1-2x)^2 V_2(n_B)\,, \tag{4}$$

where $T_F(n_B,x)$ is the Fermi-gas energy, and $V_0(n_B)$ and $V_2(n_B)$ are the interaction energy contributions.

The energy density of the neutron star matter with the kaon condensate including nucleon–nucleon interactions reads

$$\begin{aligned}\varepsilon_{ns}(n_B,x,\mu,\theta) &= T_F(n_B,x)n_B + n_B V_0(n_B) \\ &\quad + n_B(1-2x)^2 V_2(n_B) + 2m_K^2 f^2 \sin^2\frac{\theta}{2} + f\frac{\mu^2}{2}\sin^2\theta \\ &\quad + (2a_1 m_s x + 2a_2 m_s + 4a_3 m_s) n_B \sin^2\frac{\theta}{2} + \varepsilon_e + \varepsilon_\mu \,, \end{aligned} \tag{5}$$

where the electron Fermi sea contribution is

$$\varepsilon_e = \frac{\mu^4}{4\pi^2}\,. \tag{6}$$

Muons are present only when the electron chemical potential, μ, exceeds the muon rest mass, $\mu > m_\mu$. The energy density of the muon Fermi sea is

$$\varepsilon_\mu = m_\mu^4 f\left(\frac{p_{F_\mu}}{m_\mu}\right)\,, \tag{7}$$

with

$$f(y) = \frac{1}{8\pi^2}\left((y+2y^3)\sqrt{1+y^2} - \mathrm{arsinh}\, y\right). \tag{8}$$

The above formulae allow us to determine the critical density of kaon condensation and to study properties of the neutron star matter with a developed kaon condensate. We obtain the ground state parameters, the proton fraction, x, the kaon chemical potential, μ, the kaon condensate amplitude, θ, and the energy density, ε_{ns}, as functions of the baryon number density, n_B, by optimization of the thermodynamical potential $\tilde{\varepsilon} = \varepsilon_{ns} - \mu(n_K^- + n_e + n_\mu - n_P)$ which reads

$$\begin{aligned}
\tilde{\varepsilon}(n_B, x, \mu, \theta) &= T_F(n_B, x)n_B + n_B V_0(n_B) + n_B(1-2x)^2 V_2(n_B) \\
&\quad - f^2 \frac{\mu^2}{2} \sin^2\theta + 2m_K^2 f^2 \sin^2\frac{\theta}{2} + \mu n_B x - \mu n_B(1+x)\sin^2\frac{\theta}{2} \\
&\quad + (2a_1 m_s x + 2a_2 m_s + 4a_3 m_s) n_B \sin^2\frac{\theta}{2} + \tilde{\varepsilon}_e + \tilde{\varepsilon}_\mu \,. \qquad (9)
\end{aligned}$$

This thermodynamical potential is related to the (Landau and Lifshitz) potential, $\Omega/V = \varepsilon - \Sigma\mu_i n_i = \tilde{\varepsilon} - \mu_N n_B$, where μ_N is the neutron chemical potential.

The critical density, n_{kaon}, is defined as a density at which the condensate amplitude starts to deviate from zero. Our procedure provides the chiral angle of the condensate as a function of density, $\theta(n_B)$. The value of n_{kaon} is thus found from the condition $\theta(n_{\text{kaon}}) = 0$.

3. The nuclear symmetry energy at high densities

The nucleon–nucleon interaction energy is parametrized in Eq. (4) in terms of the isoscalar and isovector contributions, $V_0(n_B)$ and $V_2(n_B)$. As one can notice, at a given baryon density, n_B, the interaction energy density corresponding to the isoscalar part, $n_B V_0(n_B)$ in Eq. (9), is constant and thus it does not play any role in the optimization of the potential $\tilde{\varepsilon}$ which determines the kaon condensate parameters. It is the isovector contribution, $V_2(n_B)$, which is crucial for the onset of the condensation. This component is directly related to the nuclear symmetry energy which expressed in terms of $V_2(n_B)$ reads

$$E_{\text{sym}}(n_B) = \tfrac{5}{9} T_F\left(n_B, \tfrac{1}{2}\right) + V_2(n_B)\,. \qquad (10)$$

As we already mentioned, the high density behaviour of $E_{\text{sym}}(n_B)$ is not well known at present. Different model calculations give incompatible extrapolations away from the empirically determined value at n_0, $E_{\text{sym}}(n_0) = 31 \pm 4$ MeV [9]. In some approaches [5], $E_{\text{sym}}(n_B)$ is a monotonically increasing function of the baryon number density, whereas other models [5] predict $E_{\text{sym}}(n_B)$ to saturate and then decrease at high densities. Physically, in the former case the energy of pure neutron matter is always higher than the energy of symmetric nuclear matter, while in the latter one pure neutron matter becomes eventually the ground state of dense baryon matter.

This difference has profound consequences for neutron star matter. Here we study how it influences the onset of the kaon condensation.

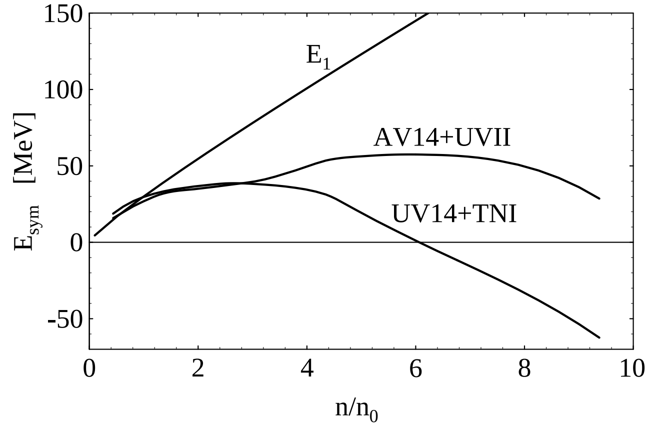

Fig. 1. The nuclear symmetry energy as a function of density for indicated models of nuclear interactions. The curve E_1 corresponds to a linear parametrization of Ref. [3].

In Fig. 1 we show the symmetry energy corresponding to three different models of nucleon matter which we use in our calculations. The curve labelled E_1 corresponds to a linear parametrization used in Ref. [3]. It fits the empirical value, $E_{\rm sym}(n_0)$, and then extrapolates linearly to higher densities. Such a linear dependence of the symmetry energy on density is typical for relativistic mean field models of nucleon matter [5]. Other curves in Fig. 1 show the symmetry energy from Ref. [4] obtained in variational many-body calculations with realistic two-body nucleon–nucleon potentials and three-body interactions. The curves labelled UV14 + TNI and AV14 + UVII correspond to the Urbana v_{14} two-body potential with the TNI three-body term [4] and the Argonne v_{14} potential with the UVII three-body interaction [4], respectively. As one can notice, realistic models predict that the symmetry energy saturates and then decreases with increasing baryon density. This behaviour is markedly different from that of the curve E_1.

4. The critical density and properties of the kaon-condensed neutron star matter

In this section we present results for three models of the nuclear symmetry energy shown in Fig. 1. Particularly interesting is the corresponding critical density, $n_{\rm kaon}$. To best illustrate the influence of the symmetry energy on $n_{\rm kaon}$ we use a different definition of the critical density which states that $n_{\rm kaon}$ is the lowest density for which the energy of the lowest state of K^- in the neutron star matter, ω_-, becomes equal to the electron chemical potential,

$$\omega_- = \mu \, . \tag{11}$$

At higher densities, $n > n_{\rm kaon}$, kaons form a Bose–Einstein condensate of finite amplitude. This definition of the critical density is equivalent to that given at the end of Sect. 2.

The lowest kaon energy in the nucleon medium reads:

$$\begin{aligned}\omega_- &= -n_B(1+x)\frac{1}{4f^2} \\ &\quad + \left(\frac{1}{16f^4}n_B^2(1+x)^2 + m_K^2 + \frac{n_B}{2f^2}(2a_1x + 2a_2 + 4a_3)m_s\right)^{1/2} . \end{aligned} \tag{12}$$

The energy ω_- decreases with baryon number density, Figs 2 and 3. The

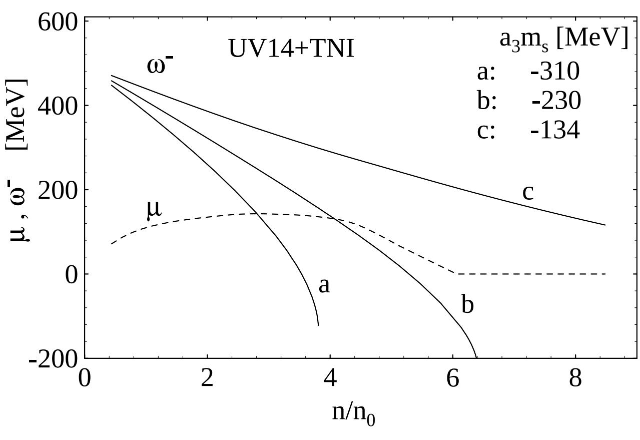

Fig. 2. The lowest kaon energy in the nucleon matter, ω_-, (solid curves) and the electron chemical potential, μ, (dashed curve) for the UV14+TNI interactions. Curves labelled a, b, and c correspond to the coupling parameter a_3m_s of -310 MeV, -230 MeV, and -134 MeV, respectively.

condition (11) can be satisfied only at a sufficiently high density provided the electron chemical potential does not decrease too fast. In Fig. 2 we show the lowest kaon energy, ω_-, for three values of the coupling parameter a_3m_s, and for the electron chemical potential, μ, corresponding to the UV14 + TNI interactions. The electron chemical potential is rather low, it has a maximum at $n = 3n_0$ and then decreases to zero at a density of about $6n_0$, where electron and proton densities vanish. The critical condition (11) can be satisfied only for the coupling values at least as strong as in case (b), $a_3 m_s \leq -230$ MeV. For weaker coupling the ω_- curve does not cross the chemical potential and the critical condition (11) can never be satisfied. Hence, in the case of the symmetry energy of the UV14 + TNI potential there exists a threshold value of the kaon–nucleon coupling parameter, $a_3 m_s$, below which there exists no kaon condensation in the neutron star matter.

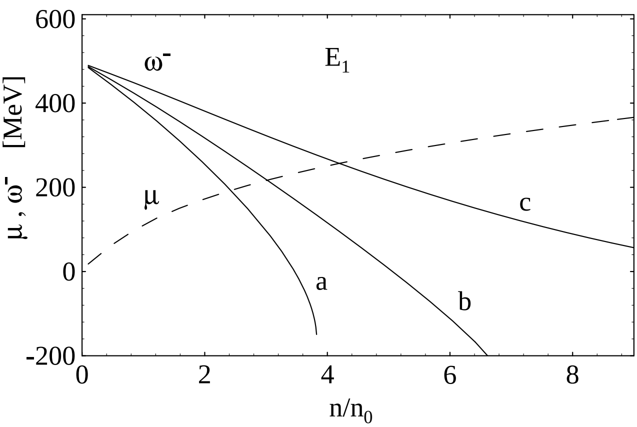

Fig. 3. The same as in Fig. 2, for the E_1 symmetry energy

Such a threshold does not exist in Fig. 3 where we show the kaon energy ω_- and the electron chemical potential for the symmetry energy E_1. Since the electron chemical potential monotonically increases with density, the critical condition (11) can be satisfied for any value of the coupling parameter $a_3 m_s$ at a sufficiently high density. Critical densities corresponding to indicated values of $a_3 m_s$ in Fig. 2 differ by a factor of about two.

Values of the critical density can also be read off from Fig. 4 and Fig. 5, where we show the amplitude of the condensate, θ, as a function of density, for the UV14 + TNI and AV14 + UVII interactions, respectively. In both figures, results corresponding to the symmetry energy E_1 are also shown. The values of $n_{\rm kaon}$ found from Fig. 2 and Fig. 3 coincide with those from Fig. 4. It is also interesting to compare how the angle θ varies with density. In Fig. 4, for the UV14 + TNI model the amplitude grows very fast with density for densities above the critical one. It reaches a maximum when the proton fraction becomes $x = 1$, and decreases at higher densities. A similar behaviour is displayed in Fig. 5 for the AV14 + UVII interactions for the strongest coupling (solid curve a). This is in contrast to the E_1 case, when the condensate amplitude monotonically grows with density for all indicated values of the coupling parameters. In Fig. 6 the proton fraction of the neutron star matter with the kaon condensate is shown. One can notice that for the UV14 + TNI interactions the kaon-condensed phase

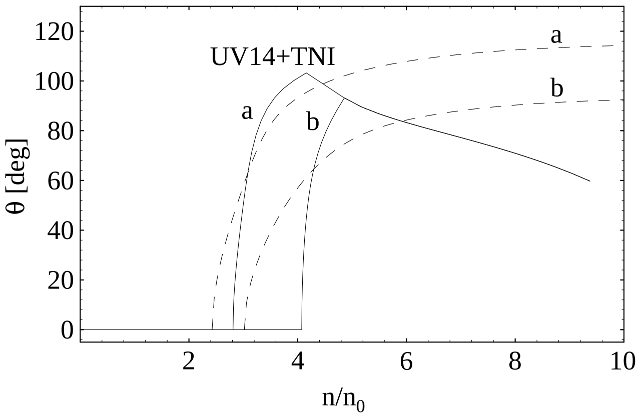

Fig. 4. The condensate amplitude as a function of density for the $UV14 + TNI$ interactions, solid curves, and for the symmetry energy E_1, dashed curves. The coupling parameters corresponding to labels a and b are the same as in Fig. 2

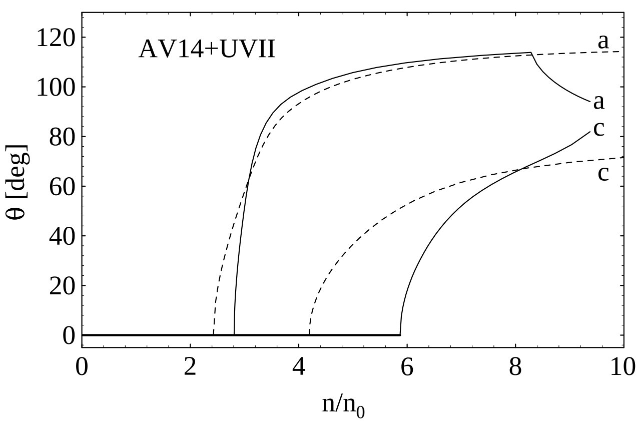

Fig. 5. The same as in Fig. 4 for the AV14 + UVII interactions.

contains only protons. Neutrons are fully converted into protons at densities only slightly exceeding the critical value. This behaviour is similar for all coupling parameters stronger than the threshold value $a_3 m_s = -230$ MeV. In Fig. 7, where the same is shown for the AV14 + UVII interaction, one can notice that for the strongest coupling the proton fraction also reaches unity at high densities. For interactions with the symmetry energy E_1 the proton fraction at high densities tends to an asymptotic value $x = 0.6$, for all kaon–nucleon couplings.

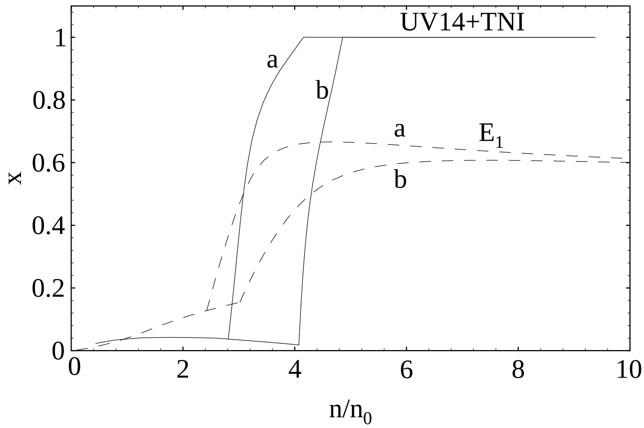

Fig. 6. The proton fraction of the neutron star matter with a kaon condensate for the UV14 + TNI interactions and for the symmetry energy E_1. The curves a and b correspond to values of the coupling parameter given in Fig. 2

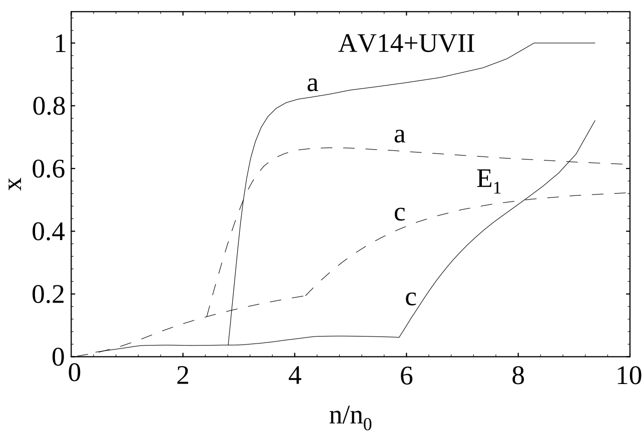

Fig. 7. The same as in Fig. 4 for the AV14 + UVII interactions. The curves a and c correspond to values of the coupling parameter given in Fig. 2

5. Discussion

The above results show that the critical density of kaon condensation in neutron stars is rather sensitive to the high density behaviour of the nuclear symmetry energy. For some realistic equations of state, such as the UV14 + TNI one, the existence of the condensate is even forbidden if the kaon–nucleon interaction is not strong enough. Also, properties of the neutron star matter with developed condensate are sensitive to the symmetry energy. One can notice in Fig. 4 and Fig. 5 that the condensate amplitude, measured by the angle θ, behaves in a quite different way for considered models of nucleon–nucleon interactions. In particular, the kaon condensate weakens at high densities for the UV14 + TNI interactions, Fig. 4.

For astrophysical applications, the proton fraction of the neutron star matter plays an important role. The influence of the symmetry energy on the proton abundance in the kaon–condensed phase is rather strong, as shown in Fig. 6 and Fig. 7. In fact, the proton abundance at higher densities is determined entirely by the nuclear symmetry energy. The kaon–nucleon coupling parameter, $a_3 m_s$, affects the proton fraction only at densities close to the critical value. The role of the symmetry energy is most spectacular for the UV14 + TNI interactions for which neutrons are fully converted into protons, Fig. 6.

Let us remark finally that recent claims [10] that all modern phase equivalent potentials which fit accurately the n–n and n–p scattering data yield a symmetry energy which increases with density, are unjustified. For the Argonne potential $AV18$ authors of Ref. [10] find increasing $E_{\rm sym}(n_B)$. Calculations reported in Ref. [11] show that the symmetry energy corresponding to this potential saturates and then decreases at high densities in a similar way as found by Wiringa, Fiks and Fabrocini [4] for Urbana UV14 and Argonne AV14 potentials. Premature conclusions of Ref. [10] stem from the use of the lowest order Brueckner approach which is inadequate at high densities. Hence the uncertainty as to the high density behaviour of the nuclear symmetry energy is still present. For astrophysical applications, both decreasing and increasing symmetry energy should be used in order to assess the role of this uncertainty.

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