SINGLE PARTICLE POTENTIAL OF A $\boldsymbol{\Sigma}$ HYPERON IN NUCLEAR MATTER

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The isospin and spin dependence of the real part of the single-particle potential of a Σ hyperon in nuclear matter, $\hat{V}_\Sigma$, is investigated. The isospin, spin, and spin-isospin dependent parts V_τ, V_σ, and $V_{\sigma\tau}$ of $\tilde{V}_\Sigma$ are expressed in terms of an effective ΣN interaction in nuclear matter. With suitable approximations numerical results for V_τ, V_σ, and $V_{\sigma\tau}$ are obtained for four models of the Nijmegen baryon-baryon interaction. A comparison with recent $(K^-,\pi^\pm)$ experiments favors model F as a realistic representation of the ΣN interaction.

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1. Introduction

We want to investigate the single particle (s.p.) potential felt by a Σ hyperon moving in nuclear matter composed of $Z_\uparrow$ protons with spin up, $Z_\downarrow$ with spin down, $N_\uparrow$ neutrons with spin up, and $N_\downarrow$ with spin down. Instead of $Z_\uparrow, Z_\downarrow, N_\uparrow, N_\downarrow$, we may use the following parameters to characterize the composition of nuclear matter: the total number of nucleons $A = Z_\uparrow + Z_\downarrow + N_\uparrow + N_\downarrow$, the proton or isospin excess parameter $\alpha_\tau = (Z_\uparrow + Z_\downarrow - N_\uparrow - N_\downarrow)/A$, the spin excess parameter $\alpha_\sigma = (Z_\uparrow + N_\uparrow - Z_\downarrow - N_\downarrow)/A$, and the spin-isospin excess parameter $\alpha_{\sigma\tau} = (Z_\uparrow + N_\downarrow - Z_\downarrow - N_\uparrow)/A$.

The s.p. potential of a Σ^+ hyperon with spin up/down moving with momentum $\boldsymbol{k}_\Sigma$ in nuclear matter, is in the linear approximation in the three excess parameters:

$$V_{\Sigma^+_{\uparrow/\downarrow}}(k_\Sigma) = V_0(k_\Sigma) + \tfrac{1}{2}\alpha_\tau V_\tau(k_\Sigma) \pm \tfrac{1}{4}\alpha_\sigma V_\sigma(k_\Sigma) \pm \tfrac{1}{2}\alpha_{\sigma\tau} V_{\sigma\tau}(k_\Sigma). \qquad (1)$$

Assuming charge-independence of the baryon-baryon interaction, we have for the s.p. potential of the Σ^- and Σ^0 hyperons:

$$V_{\Sigma^-_{\uparrow/\downarrow}}(k_\Sigma) = V_0(k_\Sigma) - \tfrac{1}{2}\alpha_\tau V_\tau(k_\Sigma) \pm \tfrac{1}{4}\alpha_\sigma V_\sigma(k_\Sigma) \mp \tfrac{1}{2}\alpha_{\sigma\tau} V_{\sigma\tau}(k_\Sigma), \qquad (2)$$

$$V_{\Sigma^0_{\uparrow/\downarrow}}(k_\Sigma) = V_0(k_\Sigma) \pm \tfrac{1}{4}\alpha_\sigma V_\sigma(k_\Sigma). \qquad (3)$$

In principle V_σ and $V_{\sigma\tau}$ depend on the angle between $\boldsymbol{k}_\Sigma$ and the spin quantization axis, as is discussed in the case of the nucleon s.p. potential in [1]. In the present paper this dependence on the direction on $\boldsymbol{k}_\Sigma$ is ignored, because we consider the case of a pure central ΣN effective interaction, in which this dependence does not appear.

Similarly as it was noticed first by Lane [2] in the case of the nucleon s.p. potential in nuclear matter with isospin excess, the s.p. potentials $V_{\Sigma^+_{\uparrow/\downarrow}}$, $V_{\Sigma^-_{\uparrow/\downarrow}}$, and $V_{\Sigma^0_{\uparrow/\downarrow}}$ are averaged versions of a more fundamental formula:

$$\hat{V}_\Sigma(k_\Sigma) = V_0(k_\Sigma) + V_\tau(k_\Sigma)\boldsymbol{t}_\Sigma \boldsymbol{T}_A/A + V_\sigma(k_\Sigma)\boldsymbol{s}_\Sigma \boldsymbol{S}_A/A + V_{\sigma\tau}(k_\Sigma)\hat{Y}/A, \quad (4)$$

where $\boldsymbol{t}_\Sigma$ is the Σ isospin ($t_{\Sigma 3} = \pm 1, 0$ for $\Sigma^\pm, \Sigma^0$) and $\boldsymbol{T}_A$ is the nuclear isospin, $\boldsymbol{T}_A = \sum_{i=1}^A \boldsymbol{t}_i$ [$\boldsymbol{t}_i$ is the isospin of the i-th nucleon ($t_{i3} = \frac{1}{2}$ for protons and $-\frac{1}{2}$ for neutrons), $T_{A3} = (Z-N)/2$], $\boldsymbol{s}_\Sigma$ is the Σ spin, $\boldsymbol{S}_A$ is the nuclear spin, $\boldsymbol{S}_A = \sum_{i=1}^A \boldsymbol{s}_i$ [$\boldsymbol{s}_i$ is the spin of the i-th nucleon], and $\hat{Y} = 4\sum_{i=1}^A (\boldsymbol{s}_i \boldsymbol{s}_\Sigma)(\boldsymbol{t}_i \boldsymbol{t}_\Sigma)$.

Particularly important for the structure of Σ hypernuclear states is the so called Lane potential V_τ. The existence of the only observed Σ hypernuclear bound state of ${}^4_\Sigma$He is strictly connected with a strong Lane potential V_τ [3, 4].

In this paper, in Sec. 2, I present the calculation of V_τ, V_σ, and $V_{\sigma\tau}$, which starts from the effective ΣN interaction in nuclear matter, $\mathcal{K}$. In Sec.3, I present and discuss the results for V_τ, V_σ, and $V_{\sigma\tau}$, obtained with $\mathcal{K}$ calculated in [5] in the low order Brueckner (LOB) theory for four models of the Nijmegen baryon-baryon interaction. The present discussion is restricted to the real Σ potential – the imaginary part of $\mathcal{K}$ in the isospin $\frac{1}{2}$ channel (due to the $\Sigma N \to \Lambda N$ conversion) is ignored.

The present paper is an extension of Ref. [6] in which the Lane potential V_τ has been discussed.

2. Expressions for V_τ, V_σ, and $V_{\sigma\tau}$

The derivation of the expressions for V_τ, V_σ, and $V_{\sigma\tau}$ is similar. I shall present the procedure more in detail only in the case of V_τ. In the remaining cases of V_σ and $V_{\sigma\tau}$, I shall only outline the procedure.

2.1. Expression for V_τ

While calculating V_τ we assume that

$$\alpha_\sigma = \alpha_{\sigma\tau} = 0. \tag{5}$$

In this case nuclear matter becomes a two-component system characterized by proton and neutron densities ρ_p and ρ_n, connected with the respective proton and neutron Fermi momenta κ_τ and λ_τ by:

$$\kappa_\tau = (3\pi^2\rho_p)^{1/3} = k_F(1+\alpha_\tau)^{1/3}, \quad \lambda_\tau = (3\pi^2\rho_n)^{1/3} = k_F(1-\alpha_\tau)^{1/3}. \tag{6}$$

where

$$k_F = [3\pi^2\rho/2]^{1/3}, \quad \rho = \rho_p + \rho_n. \tag{7}$$

Eqs (1), (2) and (3) take the simpler form:

$$V_{\Sigma^\pm}(k_\Sigma) = V_0(k_\Sigma) \pm \tfrac{1}{2}\alpha_\tau V_\tau(k_\Sigma), \quad V_{\Sigma^0}(k_\Sigma) = V_0(k_\Sigma). \tag{8}$$

Notice that the s.p. potential is now spin-independent, and we use the notation $V_{\Sigma^\pm}$ for $V_{\Sigma^\pm_\uparrow} = V_{\Sigma^\pm_\downarrow}$, and V_{Σ^0} for $V_{\Sigma^0_\uparrow} = V_{\Sigma^0_\downarrow}$.

To derive the expression for V_τ, we start from the definition of V_τ, which follows from Eq. (8):

$$V_\tau(k_\Sigma) = 2[\partial V_{\Sigma^+}(k_\Sigma)/\partial\alpha_\tau]_{\alpha_\tau=0}. \tag{9}$$

The s.p. potential V_{Σ^+} depends on the two densities ρ_p and ρ_n, or equivalently on the two Fermi momenta $\kappa_\tau = k_F(1+\alpha_\tau)^{1/3}$, and $\lambda_\tau = k_F(1-\alpha_\tau)^{1/3}$. This leads to the dependence of V_{Σ^+} on α_τ, which appears in Eq. (9). To determine this dependence, let us write the expression for V_{Σ^+} in terms of the ΣN ($N = p$ or n for protons or neutrons) effective interaction in our two-component nuclear matter, $\mathcal{K}_{\Sigma N}(\kappa_\tau\lambda_\tau)$:

$$\begin{aligned} V_{\Sigma^+}(k_\Sigma) \;=\; & \sum_{\boldsymbol{k}_N}^{k_N<\kappa_\alpha} (\boldsymbol{k}_\Sigma\boldsymbol{k}_N|\mathcal{K}_{\Sigma^+p}(\kappa_\tau\lambda_\tau)|\boldsymbol{k}_\Sigma\boldsymbol{k}_N) \\ & + \sum_{\boldsymbol{k}_N}^{k_N<\lambda_\alpha} (\boldsymbol{k}_\Sigma\boldsymbol{k}_N|\mathcal{K}_{\Sigma^+n}(\kappa_\tau\lambda_\tau)|\boldsymbol{k}_\Sigma\boldsymbol{k}_N), \end{aligned} \tag{10}$$

where spins are suppressed in our notation.

To obtain the expression for V_τ, we have to calculate — according to Eq. (9) — the derivative of V_{Σ^+} with respect to α_τ at $\alpha_\tau = 0$, taking into account the dependence of κ_τ and λ_τ on α_τ, as given by Eq. (6).

The derivative consists of two additive parts.

The first part comes from the dependence on α_τ of the limits of the sums in Eq. (10). Its contribution to V_τ, which we denote by $V_\tau^{(0)}$, can be easily calculated, and the result is:

$$V_\tau^{(0)}(k_\Sigma) = \tfrac{1}{2}A\left[\int \frac{d\hat{k}_N}{4\pi}(\boldsymbol{k}_\Sigma \boldsymbol{k}_N|\mathcal{K}_{\Sigma^+p}(k_F) - \mathcal{K}_{\Sigma^+n}(k_F)|\boldsymbol{k}_\Sigma \boldsymbol{k}_N)\right]_{k_N=k_F}, \tag{11}$$

where $\mathcal{K}_{\Sigma^+p(n)}(k_F)=\mathcal{K}_{\Sigma^+p(n)}(\kappa_\tau=k_F, \lambda_\tau=k_F)$ is defined for $Z=N=A/2$.

The second part comes from the intrinsic dependence of $\mathcal{K}$ on κ_τ and λ_τ. Its contribution to V_τ, which we denote by $V_\tau^{(I)}$, is:

$$\begin{aligned} V_\tau^{(I)}(k_\Sigma) = \tfrac{2}{3}\textstyle\sum_{\boldsymbol{k}_N}^{k_N<k_F}(\boldsymbol{k}_\Sigma \boldsymbol{k}_N|k_F\Bigg[\left(\frac{\partial}{\partial\kappa_\tau} - \frac{\partial}{\partial\lambda_\tau}\right) \\ \left\{\mathcal{K}_{\Sigma^+p}(\kappa_\tau\lambda_\tau) + \mathcal{K}_{\Sigma^+n}(\kappa_\tau\lambda_\tau)\right\}\Bigg]_{\kappa_\tau=\lambda_\tau=k_F}|\boldsymbol{k}_\Sigma \boldsymbol{k}_N). \end{aligned} \tag{12}$$

The most difficult part of $V_\tau = V_\tau^{(0)} + V_\tau^{(I)}$ is $V_\tau^{(I)}$. Whereas $V_\tau^{(0)}$ is expressed by $\mathcal{K}_{\Sigma N}(k_F)$, the effective ΣN interaction in nuclear matter with $Z = N$, to determine $V_\tau^{(I)}$ we should know $\mathcal{K}_{\Sigma N}(\kappa_\tau\lambda_\tau)$, the effective ΣN interaction in a two component nuclear matter with $Z \neq N$. A calculation of $\mathcal{K}_{\Sigma N}(\kappa_\tau\lambda_\tau)$ starting from realistic ΣN interaction would be very tedious and so far has not been performed. On the other hand, results for the simpler effective interaction $\mathcal{K}_{\Sigma N}(k_F)$, obtained with the Brueckner theory, are available in the literature. To be able to use the results obtained for $\mathcal{K}_{\Sigma N}(k_F)$ in our theory of the Lane potential V_τ, we introduce the single density (or single Fermi momentum) approximation, applied originally in the problem of spin symmetry energy of liquid ^{3}He [7], and later in the problem of the nuclear isospin [8] and spin [9, 1] symmetry energy. Namely, we introduce the following simplifying assumptions:

$$\mathcal{K}_{\Sigma^+p}(\kappa_\tau\lambda_\tau) \approx \mathcal{K}_{\Sigma^+p}(\kappa_\tau), \tag{13}$$

$$\mathcal{K}_{\Sigma^+n}(\kappa_\tau\lambda_\tau) \approx \mathcal{K}_{\Sigma^+n}(\lambda_\tau). \tag{14}$$

Approximation (13) says that the effect of the proton excess on the Σ^+p effective interaction in nuclear matter is determined primarily by the shift in Fermi momentum of protons. This assumption seems to be physically plausible and it corresponds exactly to the way in which the action of the exclusion principle is altered by the proton excess. The motivation of approximation (14), which applies to the Σ^+n interaction, is analogous.

By applying approximations (13) and (14) in Eq. (12), we get the following approximate expression for $V_\tau^{(I)}$:

$$V_\tau^{(I)}(k_\Sigma) \approx \frac{2}{3} \sum_{\boldsymbol{k}_N}^{k_N<k_F} (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | k_F \frac{d}{dk_F} \Big\{ \mathcal{K}_{\Sigma^+ p}(k_F) - \mathcal{K}_{\Sigma^+ n}(k_F) \Big\} | \boldsymbol{k}_\Sigma \boldsymbol{k}_N). \quad (15)$$

Now, we introduce explicitly spins suppressed so far in our notation. We denote by sm_s the total ΣN spin and its z-projection. Furthermore, let us introduce the total ΣN isospin T $(= \frac{1}{2}, \frac{3}{2})$. Eqs (11) and (15) take the form (with $x = \tau$):

$$V_x^{(0)}(k_\Sigma) = \tfrac{1}{6} A \Big[\int \frac{d\hat{k}_N}{4\pi} (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | \mathcal{A}_x(k_F) | \boldsymbol{k}_\Sigma \boldsymbol{k}_N) \Big]_{k_N = k_F}, \quad (16)$$

$$V_x^{(I)}(k_\Sigma) \approx \frac{2}{9} \sum_{\boldsymbol{k}_N}^{k_N<k_F} (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | k_F \frac{d}{dk_F} \mathcal{A}_x(k_F) | \boldsymbol{k}_\Sigma \boldsymbol{k}_N), \quad (17)$$

where

$$\mathcal{A}_\tau(k_F) = \sum_{sm_s} \Big[\mathcal{K}(sm_s T = \tfrac{3}{2}; k_F) - \mathcal{K}(sm_s T = \tfrac{1}{2}; k_F) \Big], \quad (18)$$

where $\mathcal{K}(sm_s T; k_F)$ is the diagonal matrix element of the effective ΣN interaction in the $sm_s T$ representation in the case of $Z_\uparrow = Z_\downarrow = N_\uparrow = N_\downarrow = A/4$.

We conclude this Subsection with an approximate expression for $V_\tau^{(0)}$. First, let us write the expression for V_0,

$$\begin{aligned} V_0(k_\Sigma) &= \frac{1}{6} \sum_{\boldsymbol{k}_N}^{k_N<k_F} (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | \sum_T (2T+1) \sum_{sm_s} \mathcal{K}(sm_s T; k_F) | \boldsymbol{k}_\Sigma \boldsymbol{k}_N) \\ &= \sum_T V_0(T, k_\Sigma) \, , \end{aligned} \quad (19)$$

which follows from expression (10). Now, the approximate expression for $V_\tau^{(0)}$ is:

$$V_\tau^{(0)}(k_\Sigma) \approx \tilde{V}_\tau^{(0)}(k_\Sigma) = V_0(\tfrac{3}{2}, k_\Sigma) - 2V_0(\tfrac{1}{2}, k_\Sigma) \, . \quad (20)$$

To obtain this approximate relation between $V_\tau^{(0)}(k_\Sigma)$ and the parts $V_0(T, k_\Sigma)$ of $V_0(k_\Sigma)$ produced by the effective interaction in the ΣN states with isospin T, one approximates $V_\tau^{(0)}$, Eq. (16), by its value averaged over the nucleon momenta k_N in nuclear matter. That means, one introduces on the right hand side of expression (16) the sum $4 \sum_{\boldsymbol{k}_N} / A$, which leads immediately to expression (20).

2.2 Expression for V_σ

Here we assume that $\alpha_\tau = \alpha_{\sigma\tau} = 0$, and the resulting two-component system is characterized by the densities $\rho_\uparrow$ and $\rho_\downarrow$ of spin up and spin down nucleons, connected with the respective Fermi momenta κ_σ and λ_σ.

Eqs (1), (2) and (3) take the form:

$$V_{\Sigma_{\uparrow/\downarrow}}(k_\Sigma) = V_0(k_\Sigma) \pm \tfrac{1}{4}\alpha_\sigma V_\sigma(k_\Sigma). \tag{21}$$

The s.p. potential is now charge-independent, and we use the notation $V_{\Sigma_{\uparrow/\downarrow}}$ for $V_{\Sigma^+_{\uparrow/\downarrow}} = V_{\Sigma^-_{\uparrow/\downarrow}} = V_{\Sigma^0_{\uparrow/\downarrow}}$. Eq. (21) implies that

$$V_\sigma(k_\Sigma) = 4[\partial V_{\Sigma_\uparrow}(k_\Sigma)/\partial\alpha_\sigma]_{\alpha_\sigma=0}. \tag{22}$$

The s.p. potential $V_{\Sigma_\uparrow}$ is determined by:

$$\begin{aligned} V_{\Sigma_\uparrow}(k_\Sigma) \; = \; & \sum_{\boldsymbol{k}_N}^{k_N<\kappa_\sigma} (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | \mathcal{K}_{\Sigma_\uparrow N_\uparrow}(\kappa_\sigma\lambda_\sigma) | \boldsymbol{k}_\Sigma \boldsymbol{k}_N) \\ & + \sum_{\boldsymbol{k}_N}^{k_N<\lambda_\sigma} (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | \mathcal{K}_{\Sigma_\uparrow N_\downarrow}(\kappa_\sigma\lambda_\sigma) | \boldsymbol{k}_\Sigma \boldsymbol{k}_N)\,, \end{aligned} \tag{23}$$

where isospins (*i.e.*, charges) are suppressed in the notation.

By inserting expression (23) into Eq. (22) we get $V_\sigma = V_\sigma^{(0)} + V_\sigma^{(I)}$, with

$$V_\sigma^{(0)}(k_\Sigma) = A\left[\int \frac{d\hat{k}_N}{4\pi} (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | \mathcal{K}_{\Sigma_\uparrow N_\uparrow}(k_F) - \mathcal{K}_{\Sigma_\uparrow N_\downarrow}(k_F) | \boldsymbol{k}_\Sigma \boldsymbol{k}_N)\right]_{k_N=k_F}, \tag{24}$$

$$\begin{aligned} V_\sigma^{(I)}(k_\Sigma) = \tfrac{4}{3} \textstyle\sum_{\boldsymbol{k}_N}^{k_N<k_F} & (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | k_F \Bigg[\left(\tfrac{\partial}{\partial\kappa_\sigma} - \tfrac{\partial}{\partial\lambda_\sigma}\right) \\ & \left\{\mathcal{K}_{\Sigma_\uparrow N_\uparrow}(\kappa_\sigma\lambda_\sigma) + \mathcal{K}_{\Sigma_\uparrow N_\downarrow}(\kappa_\sigma\lambda_\sigma)\right\}\Bigg]_{\kappa_\sigma=\lambda_\sigma=k_F} | \boldsymbol{k}_\Sigma \boldsymbol{k}_N), \end{aligned} \tag{25}$$

where $\mathcal{K}_{\Sigma_\uparrow N_{\uparrow/\downarrow}}(k_F) = \mathcal{K}_{\Sigma_\uparrow N_{\uparrow/\downarrow}}(\kappa_\sigma = k_F, \lambda_\sigma = k_F)$. Now we introduce the single density approximation:

$$\mathcal{K}_{\Sigma_\uparrow N_\uparrow}(\kappa_\sigma\lambda_\sigma) \approx \mathcal{K}_{\Sigma_\uparrow N_\uparrow}(\kappa_\sigma), \tag{26}$$

$$\mathcal{K}_{\Sigma_\uparrow N_\downarrow}(\kappa_\sigma\lambda_\sigma) \approx \mathcal{K}_{\Sigma_\downarrow N_\uparrow}(\lambda_\sigma). \tag{27}$$

By applying approximations (26) and (27) in Eq. (25), we get the following approximate expression for $V_\sigma^{(I)}$:

$$V_\sigma^{(I)}(k_\Sigma) \approx \frac{4}{3} \sum_{\boldsymbol{k}_N}^{k_N<k_F} \left(\boldsymbol{k}_\Sigma \boldsymbol{k}_N | k_F \frac{d}{dk_F} \Big\{ \mathcal{K}_{\Sigma_\uparrow N_\uparrow}(k_F) - \mathcal{K}_{\Sigma_\uparrow N_\downarrow}(k_F) \Big\} | \boldsymbol{k}_\Sigma \boldsymbol{k}_N \right). \tag{28}$$

After introducing explicitly isospins suppressed so far in our notation, we get for $V_\sigma^{(0)}$ and for the single density approximation of $V_\sigma^{(I)}$ expressions (16) and (17) with

$$\mathcal{A}_\sigma = \sum_T (2T+1) \Big[2\mathcal{K}(11T; k_F) - \mathcal{K}(10T; k_F) - \mathcal{K}(00T; k_F) \Big]. \tag{29}$$

The approximate expression for $V_\sigma^{(0)}$, analogous to approximation (20) is:

$$\begin{aligned} V_\sigma^{(0)}(k_\Sigma) \approx \tilde{V}_\sigma^{(0)}(k_\Sigma) = 4[&2V_0(s{=}1 m_s{=}1, k_\Sigma) \\ &-V_0(s{=}1 m_s{=}0, k_\Sigma) - V_0(s{=}0, k_\Sigma)], \end{aligned} \tag{30}$$

where $V_0(sm_s, k_\Sigma)$ is the part of $V_0(k_\Sigma)$ produced by the effective interaction in the ΣN spin state sm_s.

2.2. Expression for $V_{\sigma\tau}$

Here we assume that $\alpha_\tau = \alpha_\sigma = 0$, and the resulting two-component system is characterized by $Z_\uparrow = N_\downarrow$ and $Z_\downarrow = N_\uparrow$, or by the corresponding densities $2Z_\uparrow/\Omega$ and $2Z_\downarrow/\Omega$ (where Ω is the volume of the system), connected with the respective Fermi momenta $\kappa_{\sigma\tau}$ and $\lambda_{\sigma\tau}$.

Eqs (1), (2) and (3) take the form:

$$\begin{aligned} V_{\Sigma^+_{\uparrow/\downarrow}}(k_\Sigma) &= V_0(k_\Sigma) \pm \tfrac{1}{2}\alpha_{\sigma\tau} V_{\sigma\tau}(k_\Sigma), \\ V_{\Sigma^-_{\uparrow/\downarrow}}(k_\Sigma) &= V_0(k_\Sigma) \mp \tfrac{1}{2}\alpha_{\sigma\tau} V_{\sigma\tau}(k_\Sigma), \\ V_{\Sigma^0_{\uparrow/\downarrow}}(k_\Sigma) &= V_0(k_\Sigma), \end{aligned} \tag{31}$$

and we have

$$V_{\sigma\tau}(k_\Sigma) = 2[\partial V_{\Sigma^+_\uparrow}(k_\Sigma)/\partial \alpha_{\sigma\tau}]_{\alpha_{\sigma\tau}=0}. \tag{32}$$

The s.p. potential $V_{\Sigma^+_\uparrow}$ is given by:

$$V_{\Sigma^+_\uparrow}(k_\Sigma) = \sum\nolimits_{\boldsymbol{k}_N}^{k_N<\kappa_{\sigma\tau}} (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | \mathcal{K}_{\Sigma^+_\uparrow p_\uparrow}(\kappa_{\sigma\tau}\lambda_{\sigma\tau}) + \mathcal{K}_{\Sigma^+_\uparrow n_\downarrow}(\kappa_{\sigma\tau}\lambda_{\sigma\tau}) | \boldsymbol{k}_\Sigma \boldsymbol{k}_N) \\ + \sum\nolimits_{\boldsymbol{k}_N}^{k_N<\lambda_{\sigma\tau}} (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | \mathcal{K}_{\Sigma^+_\uparrow p_\downarrow}(\kappa_{\sigma\tau}\lambda_{\sigma\tau}) + \mathcal{K}_{\Sigma^+_\uparrow n_\uparrow}(\kappa_{\sigma\tau}\lambda_{\sigma\tau}) | \boldsymbol{k}_\Sigma \boldsymbol{k}_N). \tag{33}$$

Eqs (32) and (33) lead to $V_{\sigma\tau} = V_{\sigma\tau}^{(0)} + V_{\sigma\tau}^{(I)}$, with

$$V_{\sigma\tau}^{(0)}(k_\Sigma) = \frac{1}{2} A \Big[\int \frac{d\hat{k}_N}{4\pi} (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | \mathcal{K}_{\Sigma^+_\uparrow p_\uparrow}(k_F) + \mathcal{K}_{\Sigma^+_\uparrow n_\downarrow}(k_F) \\ - \mathcal{K}_{\Sigma^+_\uparrow p_\downarrow}(k_F) - \mathcal{K}_{\Sigma^+_\uparrow n_\uparrow}(k_F) | \boldsymbol{k}_\Sigma \boldsymbol{k}_N) \Big]_{k_N = k_F}, \tag{34}$$

$$V_{\sigma\tau}^{(I)}(k_\Sigma) = \frac{2}{3} \sum_{\boldsymbol{k}_N}^{k_N<k_F} (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | k_F \Big[\Big(\frac{\partial}{\partial \kappa_\sigma} - \frac{\partial}{\partial \lambda_\sigma} \Big) \\ \Big\{ \mathcal{K}_{\Sigma^+_\uparrow p_\uparrow}(\kappa_{\sigma\tau}\lambda_{\sigma\tau}) + \mathcal{K}_{\Sigma^+_\uparrow n_\downarrow}(\kappa_{\sigma\tau}\lambda_{\sigma\tau}) \\ + \mathcal{K}_{\Sigma^+_\uparrow p_\downarrow}(\kappa_{\sigma\tau}\lambda_{\sigma\tau}) + \mathcal{K}_{\Sigma^+_\uparrow n_\uparrow}(\kappa_{\sigma\tau}\lambda_{\sigma\tau}) \Big\} \Big]_{\kappa_{\sigma\tau} = \lambda_{\sigma\tau} = k_F} | \boldsymbol{k}_\Sigma \boldsymbol{k}_N). \tag{35}$$

The single density approximation is assumed in the form:

$$\mathcal{K}_{\Sigma^+_\uparrow p_\uparrow}(\kappa_{\sigma\tau}\lambda_{\sigma\tau}) \approx \mathcal{K}_{\Sigma^+_\uparrow p_\uparrow}(\kappa_{\sigma\tau}), \quad \mathcal{K}_{\Sigma^+_\uparrow n_\downarrow}(\kappa_{\sigma\tau}\lambda_{\sigma\tau}) \approx \mathcal{K}_{\Sigma^+_\uparrow n_\downarrow}(\kappa_{\sigma\tau}), \tag{36}$$

$$\mathcal{K}_{\Sigma^+_\uparrow p_\downarrow}(\kappa_{\sigma\tau}\lambda_{\sigma\tau}) \approx \mathcal{K}_{\Sigma^+_\uparrow p_\downarrow}(\lambda_{\sigma\tau}), \quad \mathcal{K}_{\Sigma^+_\uparrow n_\uparrow}(\kappa_{\sigma\tau}\lambda_{\sigma\tau}) \approx \mathcal{K}_{\Sigma^+_\uparrow n_\uparrow}(\lambda_{\sigma\tau}). \tag{37}$$

With approximations (36) and (37), Eq. (35) becomes:

$$V_{\sigma\tau}^{(I)}(k_\Sigma) \approx \frac{2}{3} \sum_{\boldsymbol{k}_N}^{k_N<k_F} (\boldsymbol{k}_\Sigma \boldsymbol{k}_N | k_F \frac{d}{dk_F} \Big\{ \mathcal{K}_{\Sigma^+_\uparrow p_\uparrow}(k_F) + \mathcal{K}_{\Sigma^+_\uparrow n_\downarrow}(k_F) \\ - \mathcal{K}_{\Sigma^+_\uparrow p_\downarrow}(k_F) - \mathcal{K}_{\Sigma^+_\uparrow n_\uparrow}(k_F) \Big\} | \boldsymbol{k}_\Sigma \boldsymbol{k}_N). \tag{38}$$

When we introduce the Tsm_s representation, we get for $V_{\sigma\tau}^{(0)}$ and for the single density approximation of $V_{\sigma\tau}^{(I)}$ expressions (16) and (17) with

$$\mathcal{A}_{\sigma\tau} = 2\mathcal{K}(11\tfrac{3}{2}; k_F) + \mathcal{K}(10\tfrac{1}{2}; k_F) + \mathcal{K}(00\tfrac{1}{2}; k_F) \\ - 2\mathcal{K}(11\tfrac{1}{2}; k_F) - \mathcal{K}(10\tfrac{3}{2}; k_F) - \mathcal{K}(00\tfrac{3}{2}; k_F)\,. \tag{39}$$

The approximate expression for $V^{(0)}_{\sigma\tau}$, analogous to approximations (20) and (30) is

$$\begin{aligned} & V^{(0)}_{\sigma\tau}(k_\Sigma) \approx \tilde{V}^{(0)}_{\sigma\tau}(k_\Sigma) = 2V_0(s{=}1 m_s{=}1 T{=}\tfrac{3}{2}, k_\Sigma) \\ & +2V_0(s{=}1 m_s{=}0 T{=}\tfrac{1}{2}, k_\Sigma) + 2V_0(s{=}0 T{=}\tfrac{1}{2}, k_\Sigma) - V_0(s{=}0 T{=}\tfrac{3}{2}, k_\Sigma) \\ & -4V_0(s{=}1 m_s{=}1 T{=}\tfrac{1}{2}, k_\Sigma) - V_0(s{=}1 m_s{=}0 T{=}\tfrac{3}{2}, k_\Sigma), \end{aligned} \tag{40}$$

where $V_0(sm_sT, k_\Sigma)$ is the part of $V_0(k_\Sigma)$ produced by the effective interaction in the ΣN spin-isospin state sm_sT.

TABLE I

Different components (in MeV) of $V_\Sigma(k_\Sigma = 0)$ calculated at $k_F = 1.35$ fm^{-1} with the YNG interaction obtained from the indicated models of the ΣN interaction.

x	Model	V_0	$V_x^{(0)}$	$(\tilde{V}_x^{(0)})$	$V_x^{(I)}$	V_x
τ	D	-13.1	51.0	(51.3)	4.1	55.1
	F	23.5	67.4	(70.2)	13.1	80.4
	SC	-9.6	13.3	(14.4)	17.7	31.0
	NSC	-16.6	-26.3	(-25.8)	-10.4	-36.7
σ	D	-13.1	66.3	(56.2)	0.5	66.8
	F	23.5	68.3	(60.5)	4.0	72.3
	SC	-9.6	19.1	(17.1)	-15.8	3.3
	NSC	-16.6	-21.9	(-26.5)	-18.6	-40.4
$\sigma\tau$	D	-13.1	61.9	(61.1)	2.1	63.9
	F	23.5	88.3	(87.2)	7.3	95.6
	SC	-9.6	50.6	(52.1)	5.8	56.3
	NSC	-16.6	63.8	(65.0)	6.9	70.7

3. Results for the YNG interaction and discussion

In calculating $V_x^{(0)}$, Eq. (16), and $V_x^{(I)}$, Eq. (17), I have used the YNG effective ΣN interaction of Yamamoto *et al.* [5]. It is a configuration space representation of the G-matrix calculated in the LOB approximation from the model D [10], model F [11], and the soft-core (SC) model [12] of the Nijmegen baryon-baryon interaction. I use also the YNG interaction obtained from the new soft-core (NSC) model of Rijken, Stoks, and Yamamoto [13]. In the case of V_τ all these YNG interactions have been applied and discussed in [6].

The results for V_0, $V_x^{(0)}$, $\tilde{V}_x^{(0)}$, and $V_x = V_x^{(0)} + \tilde{V}_x^{(I)}$ (for $x = \tau, \sigma$, and $\sigma\tau$), calculated at $k_\Sigma = 0$ with the YNG interaction are shown in Table I.

The YNG interaction is constructed so that it simulates the G-matrix in the ground states of the system of a Σ hyperon in nuclear matter, and it is best suited for the case of $k_\Sigma = 0$, presented in Table I. Calculation for $k_\Sigma > 0$ show that the dependence of $V_x(k_\Sigma)$ on k_Σ is weak [*e.g.*, $V_\tau(k_\Sigma = 1$ fm^{-1}) differs from $V_\tau(k_\Sigma = 0)$ by less than 2 %].

Because of the LOB approximation applied in [5] and [13], the accuracy of our results may appear uncertain. In the LOB approximation pure kinetic energies are used in the intermediate states of the G-matrix equation, and the spectrum of these states has a gap at the Fermi momentum. As is well known, this approximation seriously affects V_0 (it shifts it towards more positive values — see, *e.g.*, [14]). However, V_x ($x = \tau, \sigma, \sigma\tau$) are determined by differences between the $\mathcal{K}$ matrices, and one may hope that the LOB approximation affects them to a much lesser degree, *i.e.*, that V_x is much less sensitive than V_0 to the choice of the spectrum of the intermediate states in the G-matrix equation. This may be demonstrated for V_τ in the case of the D model. For this model, the G-matrix calculation at $k_F = 1.35$ fm^{-1} performed with a continuous spectrum led to $V_0 = -36.2$ MeV [15] which differs essentially from the LOB value of -13.1 MeV in Table I. On the other hand, the results of Ref. [15] lead to $\tilde{V}_1^{(0)} = 60.5$ MeV which is reasonably close to the value of 51.3 MeV in Table I.

In calculating $V_x^{(I)}$, we applied the single density approximation, Eqs (13), (14), (26), (27), (36), (37). The single density approximation relies on the plausible assumption, that the effect of nuclear matter on the interaction between Σ hyperon and nucleon with given spin and isospin is dominated by the density of these nucleons. Unfortunately, we are not able to present a quantitative estimate of the accuracy of this approximation. It should be stressed however that in the important case of model F $V_x^{(I)}$ is only a correction to $V_x^{(0)}$ equal 20 % for $x = \tau$, 6 % for $x = \sigma$, and 8 % for $x = \sigma\tau$. (In other cases the situation is different. *E.g.*, in the case of the SC model $V_\tau^{(I)}$ is bigger than $V_\tau^{(0)}$.)

The experimental information on on V_Σ comes mainly from the strangeness exchange reactions (K^-, π). Namely, the observed pion spectrum is sensitive to the final state interaction of the produced Σ hyperons with the nuclear core (see, *e.g.*, [16]). Recently the (K^-, π) spectra have been measured at BNL [17–20] (at $p_K = 600$ MeV$/c$) with an order of magnitude better statistics than reported in the early CERN experiments. In our discussion, we shall restrict ourselves to the BNL results.

Let us start our discussion with V_0. Except for YNG(Model F), all the remaining YNG interactions, when used in Eq. (19), give resulting negative values for V_0, *i.e.*, V_0 is attractive. The YNG(Model F) interaction on the other hand leads to an repulsive V_0 with $V_0(k_\Sigma = 0) = 23.5$ MeV. Now the (K^-, π^+) spectrum observed in the BNL experiments on the ^{9}Be target [17, 19] clearly indicates that the interaction of the produced Σ^- hyperon with the nuclear core is repulsive [16, 18]. The best agreement with the measured π^+ spectrum was obtained in [16] for $V_0 \approx 20$ MeV. This obviously favors model F as a realistic representation of the ΣN interaction. Let us notice that this conclusion is consistent with the analysis of the energy levels of Σ^- atoms, which also indicates that the Σ-nucleus interaction is repulsive [21].

When comparing our result for V_x with experimental data, we restrict ourselves to the Lane potential V_τ, because this is the part of V_Σ which is particularly important for Σ hypernuclear states, and which may be simply related to the existing BNL data. In the following discussion I follow the considerations presented in [6].

First of all, we shall discuss the BNL experiments on the ^{9}Be target [17–19].

Let us start with the simpler case of the (K^-, π^+) reaction in which only one direct elementary strangeness exchange process $K^- p \to \pi^+ \Sigma^-$ occurs. As mentioned before, the observed π^+ spectrum implies that the interaction of Σ^- with the nuclear core is repulsive, $V_{\Sigma^-} \approx 20$ MeV [16].

The π^- spectrum observed in the BNL (K^-, π^-) experiments indicates that the final state interaction of the Σ hyperon is less repulsive than in the (K^-, π^+) reaction or possibly even attractive [18]. Here, two elementary processes may occur: (A) $K^- p \to \pi^- \Sigma^+$, and (B) $K^- n \to \pi^- \Sigma^0$. The difference between the final state interaction in case (A) and that in the (K^-, π^+) reaction is [see Eq. (8)]: $\Delta V^{(A)} \equiv V_{\Sigma^+} - V_{\Sigma^-} = \alpha_\tau V_\tau = -\frac{1}{4} V_\tau$ $(Z = 3, N = 5$ in the nuclear core in the final state). For the same difference in case (B), we have $\Delta V^{(B)} \equiv V_{\Sigma^0} - V_{\Sigma^-} = \frac{1}{2}\alpha_\tau V_\tau = \frac{1}{2}\Delta V^{(A)}$ [1]. If we use for V_τ in the nuclear core our model F nuclear matter result from Table I, we get $\Delta V^{(A)} \simeq -20$ MeV and $\Delta V^{(B)} \simeq -10$ MeV. This are sizable decreases in the repulsion, required by the comparison of the π^+ and π^- BNL spectra. This is an additional argument in favor of model F of the Nijmegen baryon-baryon interaction, because the remaining Nijmegen models lead to a much weaker Lane potential (in this respect the worst is model NSC which gives a negative V_τ).

[1] Whereas the pion spectra in case (A) and in the (K^-, π^+) reaction depend on the s.p. states of the target protons, the pion spectrum in case (B) depends on the s.p. states of the target neutrons. The effect of this difference requires a more detailed investigation.

Finally, let us mention the $(K^-, \pi^\pm)$ experiments at BNL on the ^{4}He target [19, 20]. They confirmed the existence of of a bound state of $^4_\Sigma$He, originally reported in the ^{4}He(Stopped K^-, π^-) reaction [22]. The existence of this bound state was predicted by Harada *et al.* [3]. In their theoretical description of this state, Harada and Akaishi [4] apply phenomenological ΣN interactions, in particular the interaction SAP-F simulating at low energies the Nijmegen model F interaction. With this phenomenological interaction, they calculate the Σ s.p. potential in the $A = 3$ nuclear core, which turns out to have a strong Lane component V_τ which at the center of the nuclear core is equal 78.7 MeV[2]. This is close to the corresponding value of 80.4 MeV in Table I, although the $A = 3$ system can hardly be considered as a piece of nuclear matter.

Our general conclusion concerning V_0 and V_τ is that among the Nijmegen models of the baryon-baryon interaction only model F is compatible with the BNL data[3].

I conclude this paper with some remarks on the history of Σ hypernuclei. Let me remind you that it is model F that has led to the binding energy of the Λ hyperon in nuclear matter agreeing well with experiment thus solving the so called overbinding problem [23]. Naturally, when we started working on the problem of Σ hyperons in nuclear matter, we originally applied model F, and obtained a repulsive Σ s.p. potential. At that time, however, the CERN data on the strangeness exchange reactions seemed to reveal the existence of narrow Σ hypernuclear states which suggested an attractive Σ s.p. potential. Consequently after discussions at the 1979 Jabłonna Hypernuclear Conference, we switched to the earlier Nijmegen model D which leads to an attractive Σ-nucleus potential [15]. Actually, till the recent BNL experiments in which the narrow Σ states disappeared, everybody in the field used interactions compatible with an attractive Σ-nucleus potential. This was so because the CERN experimental results were accepted without reservations in spite of their insufficient statistics.

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[2] The analogical value of V_σ suggested in [4] is 23.1 MeV, *i.e.*, much less than our value of 72.3 MeV in Table I. As far as the $\sigma\tau$ term is concerned, it appears that the correct operator is $\hat{Y}$, Eq. (4), and not the operator $(\boldsymbol{S}_A \boldsymbol{s}_\Sigma)(\boldsymbol{T}_A \boldsymbol{t}_\Sigma)$ suggested in [4].

[3] Our conclusion is compatible with recent studies of $\Sigma^+ p$ scattering at KEK [24].

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