# RECONSTRUCTION OF THE $\pi^0$ KINEMATICS FROM $\gamma\gamma$ DECAY*

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The paper raises the problem of the reconstruction quality of $\pi^0$ kinematics from the detection of both decay photons in a electromagnetic segmented calorimeter. The standard method of photon energy-scaling is compared with the kinematically-constrained fit procedure. The $\chi^2$ value from a fit to a given event was a subject of quantitative confidence level analysis. This analysis rejects only a negligible fraction of events in the case of the kinematic fit procedure, while a large fraction of events are discarded using the energy-scaling method. The width of the reconstructed pion energy distribution from the kinematic fit is reduced by 14% compared to the results of the energy scaling method. Applying the kinematical fit method to the experimental data, the reconstructed pion energy spectrum does not violate the kinematical limit imposed by the energy conservation, in contrast to the energy-scaling method.

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## 1. Introduction

The identification of neutral pions (and other neutral mesons) is usually based on the invariant mass analysis of two-photon events detected in an electromagnetic calorimeter. In this paper we address the question of the quality of the reconstruction of the kinematics of $\pi^0$ , question important when the goal of the measurement is not only the $\pi^0$ yield but their kinematical properties. We discuss the properties of the constrained fitting procedure (kinematical fit), which is necessary to account for the effect of finite angular resolution of the detector. The method is applied to the results of Monte-Carlo simulation of monoenergetic neutral pions and to the experimental data obtained with the TAPS spectrometer in 190 MeV proton-induced $\pi^0$ production on nuclei.

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## 2. $\pi^0$ identification and simulations

The observables needed for the identification of $\pi^0$ through the invariant mass $m_{\gamma\gamma} = \sqrt{2E_{\gamma_1}E_{\gamma_2}(1-\cos\theta_{\gamma\gamma})}$ analysis are two photon energies ($E_{\gamma_1}$, $E_{\gamma_2}$) and the opening angle $\theta_{\gamma\gamma}$ between them. In heavy-ion collisions at intermediate energies, the detection of photons following neutral pion decay is now done with $BaF_2$ scintillation detectors. The response of the $BaF_2$ based detectors are very well reproduced in Monte-Carlo simulations of the electromagnetic showers with the GEANT code. Therefore, the simulations can be treated very reliably for the detailed study of response function to particles, like *e.g.*, monoenergetic neutral pions. The geometry used in the simulations was the geometry of the TAPS spectrometer [1] during the experiments performed at the AGOR cyclotron of KVI Groningen, the Netherlands. 384 $BaF_2$ hexagonal crystals of 5.9 cm inner diameter placed at a distance of 66 cm from the target covered about 20% of the full solid angle. The angular distribution of monoenergetic $\pi^0$ was isotropic in the laboratory. The calculations were done in 5 MeV energy steps up to 55 MeV pion kinetic energy (roughly the maximum $\pi^0$ energy available in this experiment) and also for pions of 100 MeV kinetic energy.

## 3. Reconstruction of $\pi^0$ kinematics

The angular resolution depends on the granularity of the detectors and the distance from the target. In a segmented electromagnetic calorimeter the electromagnetic shower induced by an energetic photon usually spreads over several modules, allowing then to determine the first interaction point with a precision better than the size of a single module [2]. When the detectors are placed far away from the target, the angular resolution is much better compared to the energy resolution of a scintillation detector. Let us consider the case a two-photon event, that was accepted as a neutral pion decay on the basis of invariant mass analysis. Since its invariant mass is not exactly equal to the known pion mass, the two experimentally-measured photon energies have to be appropriately scaled in order to reproduce particle mass. Basing on the corrected photon energies and fixed angular information, the kinematical properties of the decaying particle can be calculated. This method, described in Ref. [3], will be called energy-scaling method.

When the detectors are placed close to the target, the reduced precision of angular mesurements might influence the procedure of reconstruction of pion kinematics. In that case, not only the photon energies have to be corrected, but also their emission angles, to make an event compatible with the known meson mass. This is realised through a constrained fit, called the kinematical fit. While satisfying the kinematical criteria of reproducing the meson mass, the corrected photon momenta are determined by minimizingthe $\chi^2$-value

$$\chi^2 = \sum_{i_1,i_2=1}^{3} \sum_{j_1,j_2=1}^{2} (q_{i_1j_1} - p_{i_1j_1}) G_{i_1i_2j_1j_2} (q_{i_2j_2} - p_{i_2j_2}) \,, \tag{1}$$

where $p_{ij}$ is the measured $i$-th momentum component of $j$-th photon and $q_{ij}$ its corrected value. Since the momenta are not measured directly, but calculated from the photon energy and angle, the $\chi^2$ calculations takes into account the error propagation through the matrix $G_{i_1i_2j_1j_2}$ (more details in Ref. [4]). The invariant mass calculated from the reconstructed momenta has to be equal to $\pi^0$ mass, so only 5 of the 6 components of Eq. (1) are free parameters of minimization. The last parameter, for example $p_z$ of the second photon, can be expressed analytically by the 5 other parameters. This expression, which mimics the constraint of the proper meson mass, allowed to develop a fast fitting procedure with the minimization library MINUIT.

The aim of this analysis is the comparison of the quality of the reconstruction procedures and their resolutions. In order to answer this question, the quantity to be compared is the $\chi^2$ value. The quantitative interpretation of $\chi^2$ values requires an insight in the properties of measurement errors. The error distributions of three components of photon momenta is non-gaussian when all $\gamma\gamma$ pairs from $\pi^0$ decay are taken, regardless of their invariant mass value. As expected, reduction of the sample of events to those of higher value of $m_{\gamma\gamma}$ improves the shape of error distributions. A minimum of the $\chi^2$/NDF value of the gaussian fit to this distribution is at 112 MeV. This value is significantly higher than the values of the low-mass cut commonly used for neutral pion identification (for example 90 MeV in Ref. [5]). We claim that such a strong condition (which depends of course on the experimental set-up) is necessary when the data are to be used for the reconstruction of pion kinematics. The gaussian shape of the error distributions of quantities adjusted in the kinematical fit procedure as well as in the energy-scaling method allows now to make a quantitative comparison of the reconstruction quality.

Assuming a $\chi^2$ confidence level of 10%, we obtain a condition allowing to reject those reconstructed events, for which the $\chi^2$ value is above the one determined by the confidence level. Application of this condition shows (Fig. 1) dramatically different influence on the efficiency of the compared methods. The reconstructed pion total energy for both analyses are well centered around the known energy that was used as input to the simulations. The shape of the total reconstructed energy distribution (Fig. 1), when no conditions are applied, are characterized by significant long tails on both sides of the peak. The introduction of the $m_{\gamma\gamma} > 112$ MeV condition,

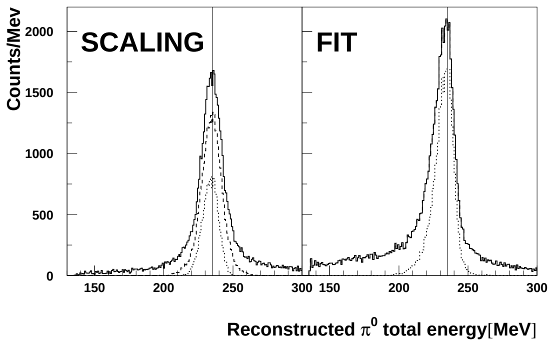


Fig. 1. Spectra of the reconstructed pion total energy with the energy scaling method and the kinematical fit (left and right panels, respectively), obtained from simulated $\gamma\gamma$ decay of neutral pions of 100 MeV kinetic energy (235 MeV total energy). The solid lines represent all $\gamma\gamma$ coincidencies' The events having $m_{\gamma\gamma} >$ 112 MeV are represented by the dashed line. From these, events with $\chi^2$ value within 10% confidence level are shown as dotted line (not distinguishable from the dashed one for the kinematical fit method).

which corresponds to the request of statistical nature of measurement errors, removes this tail in both cases. Now, the condition of 10% confidence level, while significantly reducing the number of accepted events in the energy-scaling method, has almost no effect for the kinematical fit procedure. This result points that in spite of the strong rejection of events characterized by low invariant-mass value, the corrections done by the energy-scaling methods to the measured photon energies are still large compared to the known resolution of the detector. The kinematical fit procedure offers a better energy resolution compared to the energy-scaling method. The resolution of the kinematical fit procedure is reduced by 14% (averaged over the energy range of the present analysis) compared to the energy-scaling method.

## 4. Example of application

The two discussed methods for the reconstruction of pion kinematics were applied to the experimental data of $\pi^0$ production in 190 MeV proton-induced reactions on Ni target. The measurements were done with the TAPS spectrometer at the AGOR cyclotron, KVI Groningen.

The total energy spectrum of reconstructed pions spans a large dynamic range from the $\pi^0$ rest mass (Fig. 2). In the case of the energy-scaling method applied to $\gamma\gamma$ events with $m_{\gamma\gamma} > 90$ MeV the high-energy tail of the spec-

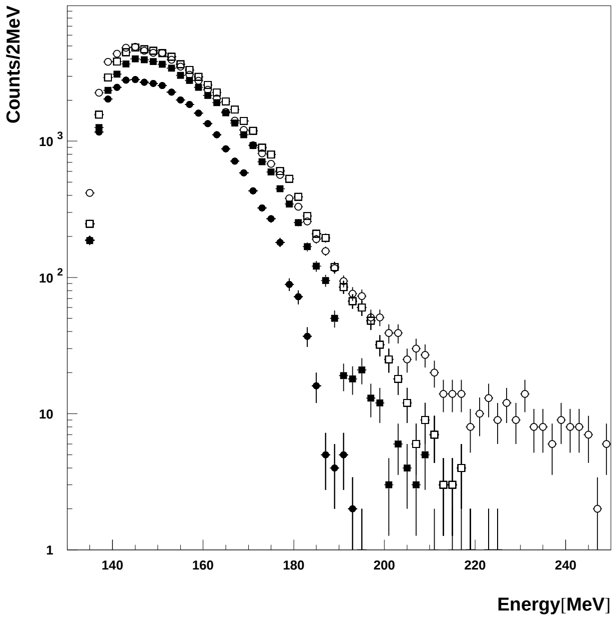


Fig. 2. The total energy spectrum of $\pi^0$ from the 190 MeV proton-induced reactions on Ni. The $\pi^0$ energy was reconstructed from the $\gamma\gamma$ decay applying the energy-scaling method (open squares) and kinematical fit (open circles) to $\gamma\gamma$ events with $m_{\gamma\gamma} > 90$ MeV. Pion energy reconstructed for events with $m_{\gamma\gamma} > 112$ MeV by the energy-scaling method are denoted as full squares. The above and the confidence level of 10% conditions applied within the kinematical fit procedure result in a spectrum denoted by full circles.

trum overcomes the kinematical limit imposed by the energy-conservation. It is even worse for the kinematical fit procedure. When the $m_{\gamma\gamma} > 112$ MeV condition is applied, the spectrum from the energy-scaling method continues to exceed the kinematical limit (almost 100 events above 190 MeV, 60 above 195 MeV). In the case of the above condition and the confidence-level analysis, the kinematical fit procedure provides us with a spectrum which does not effectively exceed the energy limit (10 events above 190 MeV, single one at 195 MeV). This property is obtained at a cost of reduced efficiency (only 54% of events accepted), due to the strong influence of the $m_{\gamma\gamma} > 112$ MeV condition.

## 5. Conclusions

The paper raises the problem of the quality of the reconstruction of $\pi^0$ kinematics from the detection of the two decay photons in an electromagnetic segmented calorimeter. Based on GEANT simulations of the $\pi^0 \to \gamma\gamma$ decay ($m_{\pi^0} = 135$ MeV), we compare the standard method of photon energy scaling with the kinematically-constrained fit procedure which allows for angular variations of the photon directions as well. Analyzing the properties of the measurement errors, we find that these errors have a statistical character only for events for which the invariant mass $m_{\gamma\gamma}$ is above 112 MeV. This value is much higher than the low-mass cuts commonly used. With the statistical nature of the measurement errors, the $\chi^2$ value from a fit to a given event can be a subject of quantitative confidence level analysis. We demonstrate that the confidence level analysis (10% confidence level) rejects only a negligible fraction of events in the case of the kinematic fit procedure, while a large fraction of events are discarded using the energy-scaling method. The width of the reconstructed pion energy distribution from the kinematic fit is reduced by 14% compared to the results of the energy scaling method. Also, applying the kinematical fit method to the experimental data, the reconstructed pion energy spectrum does not violate the kinematical limit imposed by the energy conservation, while the tail of the spectrum obtained with the energy-scaling method is over that limit. With appropriate scaling, these results can be applicable to the problem of the reconstruction of kinematics of other mesons decaying via two-photon channel.

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