# ELASTIC SCATTERING OF $\pi^{\pm}$ -NUCLEUS USING A LOCAL POTENTIAL\*

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Analysis of  $\pi^{\pm}$ -nucleus scattering has been done using an equivalent local potential to calculate the differential cross sections for pions elastically scattered from <sup>12</sup>C, <sup>40</sup>Ca and <sup>208</sup>Pb above the  $\Delta$ -resonance in the momentum range of 610 to 895 MeV/c. Our calculations of the local potential, without free parameters, give a reasonable description for the  $\pi^{\pm}$ -nucleus elastic scattering in the high momentum range. We observe that the EELL parameter effect on the calculations in the high pion momentum is smaller than in the  $\Delta$ -resonance.

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#### 1. Introduction

Recently, the angular distributions of elastically scattered  $\pi^{\pm}$  from <sup>12</sup>C, <sup>16</sup>O, <sup>28</sup>Si and <sup>40,44,48</sup>Ca were analyzed in the pion kinetic energy range from 30 to 292 MeV [1]. For the first time the local potential of Johnson and Satchler [2], where the local potentials are easier to visualize than the non-local versions, together with the DWUCK4 code [3] were employed in such calculations. Good fits to the experimental data were obtained. This success encouraged us to extend such calculations to higher energies than (3,3) resonance where the experimental data are available.

Here, the elastic scattering differential cross section angular distributions for pions scattered from  $^{12}$ C,  $^{40}$ Ca and  $^{208}$ Pb in the momentum range from 610 MeV/c to 895 MeV/c are calculated and compared with the corresponding experimental data. The local potential of Johnson and Satchler together with DWUCK4 code are used, since the DWUCK4 program is widely available. The derivation of the equivalent local pion–nucleus potential is

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to be found elsewhere [2, 4]. This local potential depends on S-wave and P-wave parts of the pion-nucleon interaction. Both of these two parts can be expressed in terms of the nuclear density of the target nucleus. They are complex and energy dependent.

For the nuclear matter density of  ${}^{12}C$ ,  ${}^{40}Ca$ , and  ${}^{208}Pb$  we use the two parameter Fermi. So that:

$$\rho(r) = \rho_n(r) + \rho_p(r). \tag{1}$$

The two-parameter Fermi (2PF) form is:

$$\rho_i(r) = \frac{\rho_{0i}}{1 + \exp(r - c_i)/a_i},$$
(2)

 $\rho_{\scriptscriptstyle 0i}~(i=n~{\rm for~neutrons},\,p~{\rm for~protons})$  can be evaluated from the normalization condition:

$$\int \rho_i(r) \, d\bar{r} \,=\, (A - Z) \text{ or } Z \,, \tag{3}$$

which yields

$$\rho_{0i}(r) = \frac{(A-Z) \text{ or } Z}{4 \pi I_1}, \qquad (4)$$

where

$$I_1 \simeq \frac{1}{3} \left[ c_i^3 + (a_i \pi)^2 c_i \right] \,,$$

and Z is the atomic number of the nucleus. The parameters  $c_i$  and  $a_i$  (i = n, p) were taken from Refs. [1,5].

The phase shifts are parametrized as a function of energy in the form of Ebrahim and Peterson [6], but without resonance terms. The form is:

$$\frac{\tan \delta_\ell}{q^{2\ell+1}} = b + cq^2 + dq^4 \,. \tag{5}$$

Parameters b, c, and d in powers of MeV/c were taken from Ref. [6], q is the center of mass momentum of the pion-nucleon  $(\pi-N)$  system, and  $\ell$ is the angular momentum quantum number. The first-order parameters  $b_i$ and  $c_i$  (i = 0,1) are calculated through the phase shift equation (5), as they are computed in the code of Ref. [6]. These parameters  $b_i$  and  $c_i$  are then used to generate the complex local potential using the expressions from Ref. [2]. The second-order parameters  $B_i$  and  $C_i$  (i = 0,1) are required only at lower energy  $T_{\pi} \leq 80$  MeV, but these parameters make no differences in the calculations at higher energies, so they were set to zero.

#### 2. Results and discussion

Examples are taken for a wide range of momenta from 610 MeV/c to 895 MeV/c. We used four values of Ericson-Ericson Lorentz-Lorentz (EELL) parameter  $\zeta$ , namely  $\zeta=0.0, 1.0, 1.2$  or 1.8, and the two parameter Fermi (2PF). The EELL parameter  $\zeta$  was found to play a significant role on the calculations in the  $\Delta$ -resonance, where the positions of the minima seen in the data were reproduced by our calculations [1] with  $\zeta=1.0$ , while these minima were found to move toward forward angles with the value  $\zeta=1.8$ .

In the present work, we use solid curves for  $\zeta = 1.0$ , dot-dashed curves for  $\zeta = 0.0$ , dashed curves for  $\zeta = 1.2$ , two dots-dashed for  $\zeta = 1.8$  of the present work.

In Fig. 1 we show elastic scattering data with  $\pi^-$  at different momenta from 610 to 895 MeV/*c* from <sup>12</sup>C, compared to curves computed with our modified first order local optical potential calculations. This computed cross sections are in qualitative agreement with the experimental data [7]. We show also in this figure different choices of  $\zeta$ , they give the same shape as data [7].

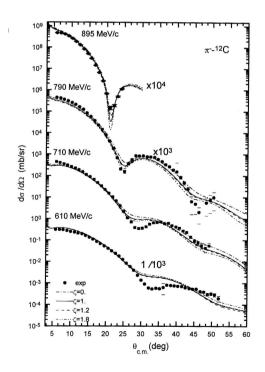


Fig. 1. Negative pion elastic scattering from <sup>12</sup>C data [7] are compared to calculations using the first-order local optical potential from the present work.

Fig. 2 shows the differential elastic cross sections of  $\pi^{\pm}$  from <sup>12</sup>C at 800 MeV/c. Data [8] are compared to the present local potential calculations. Our local optical potential calculations using the 2PF distribution and  $\zeta = 1.0$  give the same shape over the whole range of angles as the data [8].

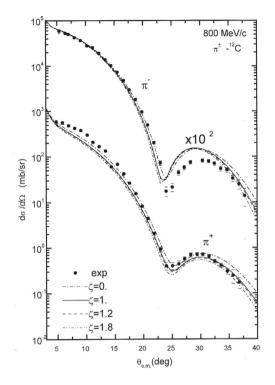


Fig. 2. Above negative pion elastic scattering from  $^{12}$ C data [8] at 800 MeV/c are compared to calculations using a first-order local optical potential from the present work. Below are shown the corresponding data and calculations for positive pions.

In Fig. 3 we show the differential elastic cross sections data [8] with both pion signs at 800 MeV/c from <sup>40</sup>Ca, compared to the local optical potential predictions computed by our modified DWUCK4 code. We have good agreement between the theoretical calculations and data [8].

In Fig. 4 we show the differential elastic cross sections data [9] with both pion signs at 790 MeV/c from <sup>208</sup>Pb, computed by our modified first-order local optical potential. Our calculations for  $\pi^-$  are in better agreement with the experimental data than for  $\pi^+$ . Our first-order local optical potential can also be used to predict total and reaction cross sections of pions on <sup>12</sup>C, <sup>40</sup>Ca, and <sup>208</sup>Pb in the momentum range 610–895 MeV/c. The comparisons between our computations and calculations estimated by others [7] are listed in Table I.

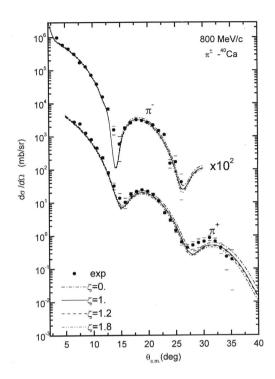


Fig. 3. As in Fig. 2 but for  $\pi^{\pm}-^{40}$ Ca differential elastic scattering cross sections at 800 MeV/c. The experimental data are taken from Ref. [8].

### TABLE I

		This calculation		From Ref. [7]	
Nucleus	$\mathrm{P}_{\mathrm{Lab}}$	$\sigma_{ m tot}~({ m mb})$	$\sigma_{\rm R}~({\rm mb})$	$\sigma_{\rm tot}~({\rm mb})$	$\sigma_{\rm R} \ ({\rm mb})$
$^{12}\mathrm{C}$	610	278.7	170.3	$270.8 {\pm} 8.6$	$178.6 {\pm} 5.9$
	710	280.3	201	$272.50{\pm}8.9$	$194.4 {\pm} 6.0$
	790	293.8	204.2	$294.2 {\pm} 8.9$	$217.9 {\pm} 5.0$
	800	305.0	222.0	—	
	895	316.6	230.8	$316.4 \pm 11.4$	$239.8 {\pm} 6.4$
$^{40}$ Ca	800	687.0	478.0	—	
$^{208}\mathrm{Pb}$	790	2955.0	1705.0	—	

Total and reaction cross sections are calculated for  $\pi^-$  by the present work, and compared to calculations from [7].

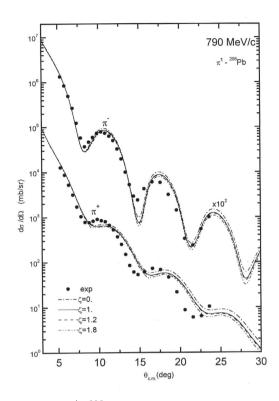


Fig. 4. As in Fig. 2 but for  $\pi^{\pm}-^{208}$ Pb differential elastic scattering cross sections at 790 MeV/c. The experimental data are taken from Ref. [9].

#### 3. Conclusion

We conclude that the EELL parameter has a small effect above the  $\Delta$ -resonance in the calculations compared to its effect in the resonance energy. In all cases of carbon, calcium and lead the minima in data are not well reproduced by calculations in spite of the data and calculations shown in Figs. 1–4 having the same shape. Increasing the energy of pions yields better agreement with the experimental data for the differential elastic cross sections. Recalculation of the parameters b, c, and d of equation (5) to be adequate for higher pion energies may seriously affect the results of calculations.

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