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ISI/FSI IN THRESHOLD MESON PRODUCTION — ONSHELL APPROACH AND COULOMB PROBLEM*

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The onshell description of Initial and Final State Interactions (ISI and FSI) of threshold meson production reactions is reviewed. Existing onshell models and their offshell extension are discussed. Unitarity constraints on enhancement factors are formulated. A strategy for the treatment of essential singularities connected to Coulomb-like FSI is given.

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ISI and FSI strongly determine qualitatively and quantitatively the energy dependence of total cross sections of particle production processes close to threshold. Following the Watson–Migdal approach [1–3] it is assumed that the T-matrix T_{fi} can be "factorised" into a product of a short ranged production amplitude $T^{(0)}_{fi}$ and "enhancement factors" $T_{\rm ISI}(i)$ and $T_{\rm FSI}(f)$ for ISI and FSI respectively, *i.e.* $T_{fi} = \langle f\,|\,T\,|\,i\rangle \simeq T_{\rm FSI}(f)\; T^{(0)}_{fi}\; T_{\rm ISI}(i)$. Commonly used enhancement factors describe the elastic *onshell* scattering problem of the incoming or outgoing particles, so they do not have any reminder in the short ranged production process due to original time-reversal arguments of Watson. Yet from $\Delta E\, \Delta t \geq \hbar/2$ we know that the scattering system is going for a certain time of order Δt offshell, while the amount of offshellness ΔE is strongly determined by the short range interaction process. It will be shown that the reminder of the enhancement factors in the threshold value of the short range production amplitude $T^{(0)}_{fi,{\rm thr}}$ can be estimated by unitarity constraints on the T-matrix T_{fi}. To test the validity of

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enhancement factors presently in use one has to check three minimal constraints: (a) enhancement factors have to be dimensionless, (b) for *no* ISI or FSI the respective enhancement factors have to be 1, (c) the S-matrix $S = 1 + i\,(2\pi)^4\,\delta^4(P_f - P_i\,)\,T$ has to be unitary. Constraint (c) yields unitarity constraints on the T-matrix: assuming time-reversal–invariance $(\langle f\,|\,T\,|\,i\rangle \stackrel{!}{=} \langle i\,|\,T\,|\,f\rangle = \langle f\,|\,T^+\,|\,i\rangle^*)$ I obtain after insertion of a complete set of relativistically normalized states [4, p. 645 ff]:

$$\mathrm{Im}\,\langle f\,|\,T\,|\,i\rangle \;=\; \frac{1}{2}\,\sum_n\,(2\pi)^4\;\delta^4(P_f - P_i\,)\;\;\langle f\,|\,T\,|\,n\rangle\;(\langle n\,|\,T\,|\,i\rangle)^*\,.$$

In the approximation that the contribution via Fock-states being different from the initial and final states is very small (which is not valid for no ISI *and* no FSI) one can formulate an approximate unitarity relation replacing the sum $\sum_n$ by $\sum_{n\,\in\{i,f\}}$. Reexpressing the symbolic sum by integrals and applying Watson-Migdal factorisation with $T^{(0)}_{fi} \simeq T^{(0)}_{fi,\mathrm{thr}}$ I arrive at the following unitarity constraint on the enhancement factors for a particle production process $1 + 2 \to 1' + 2' + \ldots + n'$ close to threshold:

$$\begin{aligned}
&\mathrm{Im}(T_{\mathrm{FSI}}(f)\;T^{(0)}_{fi,\mathrm{thr}}\;T_{\mathrm{ISI}}(i)) \;\simeq \frac{1}{2}\,\Bigg[\;T^{*\,(0)}_{fi,\mathrm{thr}}\;T^{*}_{\mathrm{ISI}}(i)\\
&\times \int \frac{d^3p_{1'}}{(2\pi)^3\,2\,\omega_{1'}(|\vec{p}_{1'}|)}\cdot\ldots\cdot\frac{d^3p_{n'}}{(2\pi)^3\,2\,\omega_{n'}(|\vec{p}_{n'}|)}\langle f\,|\,T\,|\,1'\ldots n'\rangle T^{*}_{\mathrm{FSI}}(1'\ldots n')\\
&+T_{\mathrm{FSI}}(f)\;T^{(0)}_{fi,\mathrm{thr}}\;\int \frac{d^3p_1}{(2\pi)^3\,2\,\omega_1(|\vec{p}_1|)}\;\frac{d^3p_2}{(2\pi)^3\,2\,\omega_2(|\vec{p}_2|)}\\
&\times T_{\mathrm{ISI}}(12)\;(\langle 12\,|\,T\,|\,i\rangle)^*\;\Bigg]\;(2\pi)^4\;\delta^4(P_f - P_i)\,.
\end{aligned}$$

In the limit of no ISI ($T_{\mathrm{ISI}}(i) = 1$, $\langle 12\,|\,T\,|\,i\rangle \simeq 0$) *or* no FSI ($T_{\mathrm{FSI}}(f) = 1$, $\langle f\,|\,T\,|\,1'\ldots n'\rangle \simeq 0$) it is just an integral constraint on the enhancement factors $T_{\mathrm{FSI}}(f)$ or $T_{\mathrm{ISI}}(i)$, respectively.

As $T^{(0)}_{fi} \simeq$ const. close to threshold, the energy dependence of the total cross section of a process with an n-particle final state close to threshold is mainly determined by a FSI-modified phasespace integral [2, 3]($s = P^2$):

$$R^{\mathrm{FSI}}_n(s) = \int \frac{d^3p_{1'}}{2\omega_{1'}(|\vec{p}_{1'}|)}\cdots\frac{d^3p_{n'}}{2\omega_{n'}(|\vec{p}_{n'}|)}\delta^4(p_{1'} + \ldots + p_{n'} - P)|T_{\mathrm{FSI}}(f)|^2\,.$$

Without loss of generality I assume only FSI between particle $1'$ and $2'$. The enhancement factor now can be reexpressed either by the phaseshifts $\delta_\ell(\kappa)$

or Jost-functions $f_\ell(\kappa) \stackrel{!}{=} f_\ell^*(-\kappa^*)$ of the $1'2'$–onshell–scattering problem $(\kappa := \sqrt{\lambda(s_{12}, m_{1'}^2, m_{2'}^2)}/(2(m_{1'}+m_{2'})))(\kappa^{2\ell+1}\cot\delta_\ell(\kappa) = -a_\ell^{-1} + O(\kappa^2))$ [2]:

$$T_{\text{FSI}}(f) \simeq T_{\text{FSI}}(1'2') = \frac{\cot\delta_\ell(\kappa) - \dfrac{\mathcal{P}(\kappa)}{a_\ell\kappa^{2\ell+1}}}{\cot\delta_\ell(\kappa) - i} \stackrel{!}{=} \frac{\text{Re}f_\ell(\kappa)}{f_\ell(\kappa)} + \frac{\mathcal{P}(\kappa)}{a_\ell\kappa^{2\ell+1}}\frac{\text{Im}f_\ell(\kappa)}{f_\ell(\kappa)}\,.$$

Here I used $(\cot\delta_\ell(\kappa) - i)^{-1} \stackrel{!}{=} (f_\ell(-\kappa) - f_\ell(\kappa))/(2if_\ell(\kappa))$ (see *e.g.* [4, p.286]). $\mathcal{P}(\kappa)$ is the offshell quantity defined in [5] carrying the reminder in the short ranged interaction. It either has to be calculated directly [6, 7] by a principle value integral or estimated from the unitarity constraint derived above. The limit $T_{\text{FSI}}(1'2') \to 1$ for $\delta_\ell(\kappa) \to 0$ yields $\mathcal{P}(0) \to 0$ *for no FSI.* As $\mathcal{P}(\kappa)$ depends on the nature of the short ranged interaction process, it has to be estimated for every single short ranged interaction diagram separately [7]. Most authors upto now are using the approximations $\text{Re}f_\ell(\kappa) = 0$ or $T_{\text{FSI}}(1'2') = f_\ell^{-1}(\kappa)$ [4, 8] conflicting strongly with the unitarity constraint. A review on related models can be found in [9]. The only existing approach driven by unitarity is called Fermi–Watson Theorem (see *e.g.* [4]). It states: $T_{fi} = \exp(i\,\text{Re}\,\delta_\ell(f))\,|T_{fi}|\,\exp(i\,\text{Re}\,\delta_\ell(i))$. For "well-behaved" FSI-potentials $|T_{\text{FSI}}(1'2')|^2$ can easily be Taylor-expanded in κ, *i.e.* $|T_{\text{FSI}}(1'2')|^2 = \sum_\alpha c_\alpha\,\kappa^\alpha$. If the outgoing $1'2'$-system is not bound, this Taylor-expansion is a threshold expansion of $R_n^{\text{FSI}}(s)$. *E.g.* for $n = 3$ I obtain an expansion in $\eta := \eta_{12} := \sqrt{\lambda(s\,, m_{3'}^2, (m_{1'} + m_{2'})^2)/(4\,s\,m_{3'}^2)}$ $(2\bar{\mu} := (m_{1'} + m_{2'})/m_{3'},\ \Delta := \sqrt{1 - (m_{1'} - m_{2'})^2/(m_{1'} + m_{2'})^2})$:

$$R_3^{\text{FSI}}(s) = \sum_\alpha c_\alpha \left(\frac{\pi^2 m_{3'}^{\alpha+2}}{2^{\alpha+1}(2\bar{\mu})}\eta^{\alpha+4} \int\limits_0^1 du \sqrt{u((1+2\bar{\mu})\Delta^2(1-u))^{\alpha+1}} + O(\eta^{\alpha+6}) \right)\,.$$

A similar expansion in $\eta_{23} := \sqrt{\lambda(s\,, m_{1'}^2, (m_{2'} + m_{3'})^2)/(4\,s\,m_{1'}^2)}$ (or η_{31}) can be performed for FSI between outgoing particles $2'3'$ (or $1'3'$). Close to threshold these expansions can be reformulated in terms of η by $(m_{1'} \geq m_{2'})$:

$$\left.\begin{matrix} \eta_{23} \\ \eta_{31} \end{matrix}\right\} = \left\{\begin{matrix} (m_{3'}/m_{1'}) \\ (m_{3'}/m_{2'}) \end{matrix}\right\} \cdot \frac{\eta}{2}\,\sqrt{2 \pm 2\sqrt{1-\Delta^2} + (2\,\bar{\mu})\,\Delta^2}\; + \; O(\eta^3)\,.$$

Of course, the treatment of FSI as a sum of FSI between the various two-particle subsystems is not sufficent [2]. A complete onshell-treatment would yield the the knowledge of the full outgoing elastic T-matrix. The offshell extension of this framework is non-trivial. Next to leading order terms have been attacked by [10]. For corrections due to ISI and the flux factor see [2].

Because of essential singularities due to the penetration factor $C_0^2(\gamma) = 2\pi\gamma\,(\exp(2\pi\gamma) - 1)^{-1}$ in a Coulomb–Modified Effective Range Expansion [11–13] with $\gamma := \alpha\,\sqrt{(\kappa^2 + (m_{1'} m_{2'}/(m_{1'} + m_{2'}))^2)/\kappa^2}$ ($\alpha \simeq 1/137$) the Taylor–expansion described above can't be performed for a Coulomb potential $V(r) = 2\gamma\,\kappa/r$. Yet by determining $f_\ell(\kappa)$ for a regularised Coulomb potential $V(r) = 2\gamma\,\kappa\exp(-\mu\,r)/(r + \varepsilon)$ using [14] a regularised Modified Effective Range Function can be derived [13], which does not suffer from essential singularities mentioned. Now the Taylor-expansion of $|T_{\text{FSI}}(1'2')|^2$ is well defined and the limits $\mu \to +0$ and $\varepsilon \to +0$ are straight forward.

An interesting test for FSI is the η-dependence of $\sigma(pp \to pp\pi^0)$ close to threshold. Experiments yield $\sigma(s) \propto \eta^{\,\alpha+4}$ with $\alpha \geq 0$ for $\eta < 0.15$ and $\alpha \simeq -2$ for $0.2 \leq \eta \leq 0.5$. As the $pp\pi^0$-system is not bound and theoretical models seem to provide no mechanism for $|T_{fi}|^2 \propto \kappa^{\,\alpha}$ with $\alpha < 0$, it seems that present models [15] generate by hand inverse powers of κ as it was done by [16], who replaced $(1 + (a\,\kappa)^2)^{-1}$ by $(a\,\kappa)^{-2}$ in the Effective Range Factor. The real origin for $\alpha < 0$ still has to be explored. The ansatz $\sigma(pp \to pp\pi^0) \simeq D\,\eta\,(\zeta\eta)^3(1 + \sqrt{1 + \zeta^2\eta^2}\,)^{-2}$ of [17] has desired properties, if ζ is chosen such that $\zeta\eta \ll 1$ for $\eta < 0.15$ and $\zeta\eta \gg 1$ for $\eta \geq 0.2$.

REFERENCES

[1] K.M. Watson, *Phys. Rev.* **88**, 1163 (1952); A.B. Migdal, *Sov. Phys. JETP* **1**, 2 (1955); E. Fermi, *Nuovo Cimento Suppl.* **2**, 17 (1955).

[2] F. Kleefeld, πN-NEWSLETTER **15**, 250 (1999).

[3] F. Kleefeld, Doctoral Thesis, Univ. Erlangen-Nürnberg, Germany, 1999.

[4] C.J. Joachain, *Quantum Collision Theory* (reprinted 1987), Elesevier, 1975.

[5] C. Hanhart, K. Nakayama, *Phys. Lett.* **B454**, 176 (1999).

[6] H. Garcilazo, M.T. Peña, *Phys. Rev.* **C61**, 064010 (2000).

[7] V.V. Baru *et al.*, *Acta Phys. Pol.* **B31**, 2127 (2000).

[8] A. Sibirtsev, W. Cassing, *Eur. Phys. J.* **A2**, 333 (1998); R. Shyam, *Phys. Rev* **C60**, 055213 (1999); A.I. Titov *et al.*, `nucl-th/0001027`.

[9] J.A. Niskanen, *Phys. Lett.* **B456**, 107 (1999).

[10] A. Moalem *et al.*, `hep-ph/9505264`.

[11] G.J.M. Austen, Doctoral Thesis, Catholic Univ. Nijmegen, Netherlands, 1982.

[12] H. van Haeringen, *Charged-particle interactions*, Coulomb Press, Leyden 1985.

[13] H. van Haeringen, L.P. Kok, *Phys. Rev.* **A26**, 1218 (1982).

[14] L. Trlifaj, *J. Math. Phys.* **A21**, 1724 (1980).

[15] J.A. Niskanen, *Phys. Lett.* **B289**, 227 (1992); C.J. Horowitz *et al.*, *Phys. Rev.* **C49**, 1337 (1994); Ch. Hanhart, Doctoral Thesis, Univ. Bonn, Germany, 1998; V. Bernard *et al.*, *Eur. Phys. J.* **A4**, 259 (1999).

[16] D.S. Koltun, A. Reitan, *Phys. Rev.* **141**, 1413 (1966).

[17] G. Fäldt *et al.*, *Phys. Lett.* **B382**, 209 (1996); *Nucl. Phys.* **A604**, 441 (1996).