THEORETICAL CONSTRAINTS ON INTERACTION AMPLITUDES OF LIGHT MESONS*

R. Kamiński, L. Leśniak, and K. Rybicki

H. Niewodniczański Institute of Nuclear Physics
31–342 Kraków, Poland

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We impose unitarity constraints on the S-wave isoscalar $\pi\pi$ amplitudes extracted from the analysis of the $\pi^- p \to \pi^+\pi^- n$ data which have been measured by the CERN–Cracow–Munich collaboration on a transversely polarized target at 17.2 GeV$/c$ π^- momentum. Two "steep" solutions contain a narrow S-wave $f_0(750)$ resonance under the $\rho(770)$ and exhibit a considerable inelasticity η which is in disagreement with the four pion production data below the $K\overline{K}$ threshold. We impose $\eta \equiv 1$ for all data points and examine four sets of solutions for the S-wave isoscalar phase-shifts. The "down-flat" and "up-flat" solutions easily pass the $\eta \equiv 1$ constraint but the remaining "down-steep" and "up-steep" are eliminated. We conclude that the 17.2 GeV data cannot be described by a relatively narrow $f_0(750)$.

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Scalar meson spectroscopy is a subject of many phenomenological analyses in which a construction of interaction amplitudes between light pseudoscalar mesons (like $\pi^+\pi^-$, K^+K^- and other pairs of mesons) is very important. The spectrum of scalars is poorly known [1] but an agreement on the existence of its lowest member $f_0(400–1200)$, also called σ meson, is now rather common. At higher energies there exist isoscalars $f_0(980)$, $f_0(1370)$ and $f_0(1500)$ found in various production processes. Nature of scalar mesons is naturally related to a spectrum of scalar glueballs since a mixing of the $q\bar{q}$ states with gluonia can enrich a number of the observed scalar resonances [2].

A final success of phenomenological analyses in systematization of the existent experimental data depends quite substantially on application of the appropriate theoretical constraints on multichannel amplitudes. For example, using relations coming from parity or isospin symmetry of strong

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interactions can lead to an important reduction of a number of independent scattering amplitudes. In some channels like $\pi^+\pi^-$ one can apply chiral symmetry constraints and the relations following from the crossing symmetry. Analyticity of the coupled channel amplitudes is also a very important property. The masses and widths of the resonances can be essentially obtained in a model-independent way if they are extracted from positions of the T-matrix poles present in all the relevant decay and production channels. The dispersion relations serve as a tool to construct mesonic amplitudes like those appearing in Roy's equations of the $\pi\pi$ S and P waves. One should also mention a particular role played by constraints following from unitarity of the S-matrix. Limitations on the phenomenological amplitudes coming from unitarity requirement will be discussed in the analysis presented below.

Let us briefly recall the results of our phenomenological analysis [3] of the CERN–Cracow–Munich data [4] on the reaction $\pi^-p \to \pi^+\pi^-n$ obtained at 17.2 GeV/c. In this reaction several $\pi^+\pi^-$ partial waves (S, P, D and F) are important. There are significant contributions of three scalar resonances in addition to leading resonances $\rho(770)$, $f_2(1270)$ and $\rho_3(1690)$. Using the same data Svec claimed that a narrow scalar resonance $f_0(750)$ exists below the $K\overline{K}$ threshold [5]. In [3] an energy independent separation of the S-wave pseudoscalar and pseudovector amplitudes has been performed and we have extracted four solutions of the $\pi\pi$ scalar–isoscalar phase shifts called "down-flat", "down-steep", "up-flat" and "up-steep". The labels "down" and "up" refer to a behaviour of the S-wave intensity which in the effective $\pi^+\pi^-$ mass range between 800 MeV and 980 MeV is smaller for the case "up" than for the case "down". The other two-fold ambiguity is related to the fact that a sign of the S–P phase difference can be chosen in two ways, so the "flat" phase shifts are smaller and the "steep" phase shifts are larger than the P-phases near the $\rho(770)$ resonance. Thus the "steep" solutions could be related to the $f_0(750)$ while the "flat" solutions to a broad $f_0(500)$ postulated in [6].

The S-wave isospin 0 $\pi\pi$ amplitude can be written as

$$a_0 = \frac{\eta e^{2i\delta_0} - 1}{2i}\,, \tag{1}$$

where η is inelasticity and δ_0 is the isoscalar phase shift. The unitarity constraint leads to inequality $\eta \leq 1$ which in phenomenological analysis can sometimes be violated due to experimental errors. In [3] we have, however, eliminated the solution "down-steep" since the values η reached 2 at the $\pi^+\pi^-$ effective mass $m_{\pi\pi}$ near 900 MeV. However, the "up-steep" solution cannot be eliminated in the same way since the values of η are smaller in that range and their errors are substantial. Nevertheless, a general behaviour of η for the "up-steep" solution is similar to the "down-steep" solution showing a two-bump character. In contrast to two previous solutions the remaining "down-flat" and "up-flat" solutions exhibit a very smooth behaviour of η very close to 1.

In [7] we have examined in more detail a range of $m_{\pi\pi}$ between 720 MeV and 820 MeV, where five points of η corresponding to the "up-steep" solution systematically lie below 1. The average value of inelasticity in that range is 0.67 ± 0.17, well below 1. The probability to find accidentally all five points below 1 is small, equal to 0.002. Therefore we have looked for inelastic reactions in which four pions can be produced below the $K\overline{K}$ threshold with the same quantum numbers as those of the $\pi\pi$ system. The reactions such as the central 4π production in the high energy proton–proton collisions, peripheral $4\pi^0$ or $2\pi^+2\pi^-$ production by high energy pion beams and multipion production in the antiproton annihilation have been considered. We have noticed that there were generally only a few 4π events below 1 GeV and that no peak was seen in the 4π effective mass distribution near $\rho(770)$. Thus a natural assumption in the analysis of the $\pi\pi$ production data below 990 MeV is that the inelasticity $\eta \equiv 1$. With this theoretical constraint we have made a new analysis of the $\pi^+\pi^-$ isoscalar–scalar phase shifts obtained from the $\pi^-p \to \pi^+\pi^-n$ data at 17.2 GeV/c. In [3] we have extracted the S-wave $\pi^+\pi^-$ elastic amplitude a_S which was related to the isoscalar a_0 and isotensor amplitude a_2 in the following way:

$$a_0 = 3a_S - \frac{1}{2}a_2\,. \tag{2}$$

We have also assumed that the a_2 amplitude is fully elastic and the isotensor phase shifts are known from the analysis of the $\pi^+p \to \pi^+\pi^+n$ data of [8]. Now in view of experimental errors we have to modify the values of a_S obtained in [3] to fulfill the postulated equality $\eta \equiv 1$. The minimum modification is to multiply a_S by a real factor n such that

$$\eta^2 = |1 + 2ia_0|^2 = \left|1 + 2i(3na_S - \frac{1}{2}a_2)\right|^2 \equiv 1\,. \tag{3}$$

This is a quadratic equation for n which has to be solved for each value of $m_{\pi\pi}$. Existence of roots is equivalent to the elastic unitarity condition satisfied by a_0, namely Im $a_0 = |a_0|^2$.

We have obtained the following numerical results for the solutions discussed in [7]: values of n exist for all twenty $m_{\pi\pi}$ points of the solution "up-flat" and for 19 points (except of the extreme point at 990 MeV) corresponding to the solution "down-flat". However for 7 points of the solution "up-steep" and for 12 points of the solution "down-steep" the elastic unitarity condition cannot be satisfied. The remaining points of n vary with $m_{\pi\pi}$

forming a parabolic shape with a maximum at about 800 MeV. In contrast to the "steep" solutions both "flat" solutions are well fitted by constants very close to unity in the whole $m_{\pi\pi}$ range between 600 and 1000 MeV. These facts represent a strong argument against our accepting the "steep" solutions as good physical solutions.

Let us now discuss common features of the "flat" solutions and the major differences between them. There is an initial slow growing of phase shifts with $m_{\pi\pi}$ above 600 MeV, then at the $K\overline{K}$ threshold there is a sudden jump up by more than $100°$ and then a further rise particularly steep near 1400 MeV. This behaviour of phases has been interpreted in terms of three scalar resonances $f_0(500)$, $f_0(980)$ and $f_0(1400)$. The parameters of these resonances have been recently determined from the positions of the T-matrix poles in the complex energy plane using a three coupled channel model [9]. The major differences between the "up-flat" and "down-flat" solutions exist for $m_{\pi\pi}$ between 800 MeV and 1000 MeV where the "up-flat" phase shifts are larger than the "down-flat" ones by a few tens of degrees, the difference reaching about $45°$. This fact has some consequences since the values of the $f_0(500)$ mass and width differ by about 50 MeV between two "flat" solutions (see [9]). Around the $K\overline{K}$ threshold both solutions approach each other but the "up-flat" phases are systematically larger than the "down-flat" phases up to about 1300 MeV.

In conclusion, we have demonstrated that theoretical constraints imposed on the meson–meson interaction amplitudes are very useful in elimination of some ambiguities found in phenomenological analyses of experimental data. Both "steep" solutions of the scalar–isoscalar $\pi\pi$ phase shifts show unphysical behaviour in contrast to the two "flat" solutions which satisfy well the unitarity tests. As a consequence a narrow resonance $f_0(750)$ can be eliminated and the broad $f_0(500)$ is confirmed.

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