PHOTO- AND ELECTROPRODUCTION OF MESONS IN A GAUGE-INVARIANT FRAMEWORK*

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A comprehensive formalism is presented to describe the photo- or electroproduction of mesons off the nucleon. Taking the hadrons as extended objects described by form factors, we discuss the issues of gauge invariance and of off-shell form factors (both hadronic and electromagnetic). Specifically, we show how the gauge invariance of the production current is preserved by introducing auxiliary currents which take the place of the exchange currents usually neglected entirely in practical applications. The relation of the present formulation to tree-level-type prescriptions is discussed. Applications to kaon photoproduction are shown.

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1. Introduction

The simplest case of a meson production current concerns pion photo- or electroproduction off the nucleon. Although it is fairly straightforward to obtain a full field-theoretical treatment of this problem [1–4], it still is a formidable task numerically that up to now has not been solved in its entirety.

We will discuss here some issues — both practical and theoretical — related to this problem. First and foremost, we will give a method for preserving the gauge-invariance of the formalism when one needs to introduce approximations mandated by practical considerations. Second, we will

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discuss the implications of allowing for both electromagnetic and hadronic off-shell form factors. We will also present some numerical results for kaon photoproduction which exemplify the usefulness of the present approach.

2. The reaction $\gamma N \to \pi N$ for real or virtual photons

Assuming a complete solution of the reaction $\pi N \to \pi N$ (see Fig. 1), it is quite straightforward to write down an equally complete solution of the pion photoproduction problem $\gamma N \to \pi N$ (see Fig. 2) [1–4].

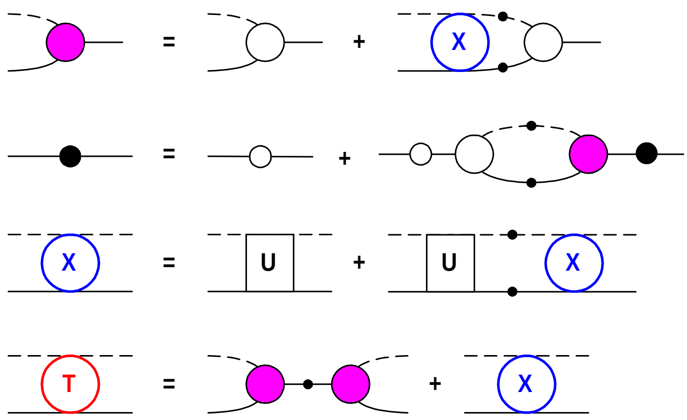

Fig. 1. Pion-nucleon scattering with fully dressed hadrons. Time proceeds from right to left. The full πN-amplitude is denoted by T, with X subsuming all of its nonpolar (*i.e.*, non-s-channel) contributions. The latter satisfies the Bethe–Salpeter-type integral equation depicted in the third line here, where the driving term U sums up all nonpolar irreducible contributions to πN-scattering, *i.e.*, all irreducible contributions which do not contain an s-channel pole (see Ref. [3] for full details). Diagram elements with open, unlabeled circles describe bare quantities.

2.1. Gauge invariance

Gauge invariance is one of the central issues when attempting to describe how photons interact with hadronic systems. The gauge invariance of the complete production current, as given in Fig. 2, follows as a matter of course from the full field-theoretical treatment [1–4]. In practice, however, approximations are inevitable which usually destroy the gauge invariance. To restore it, the neglected reaction mechanisms must be approximated by auxiliary currents constructed such that the gauge-invariance-violating contributions to the four-divergence of the total production amplitude are cancelled. Such a procedure cannot be unique, of course, since one may always add arbitrary transverse currents without affecting the four-divergence.

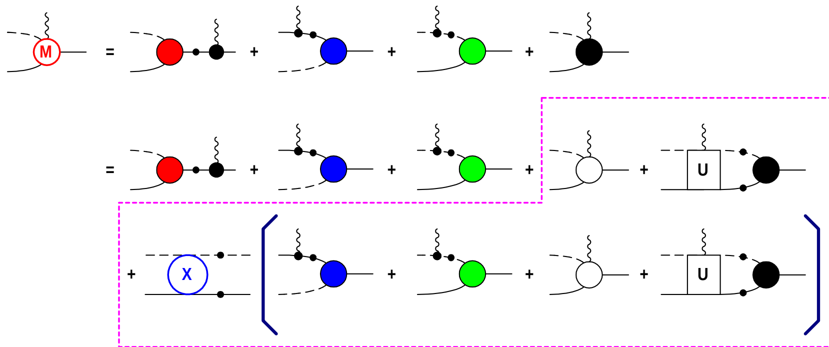

Fig. 2. Meson production current M^μ. The first line, which sums up the s-, u-, and t-channel diagrams and the interaction current $M^\mu_{\rm int}$ (right-most diagram), is referred to as the tree-level. The dynamical content of $M^\mu_{\rm int}$, including the final-state interaction mediated by the nonpolar πN amplitude X (see Fig. 1), is explicitly shown by the diagrams enclosed in the dashed box. The diagram element labelled U subsumes all exchange currents contributing to the process. The diagram with open circle depicts the bare current.

In describing in the following how we propose to restore gauge invariance, we assume that the gauge-invariance violation is due to the neglect of exchange currents [4]. However, the construction of auxiliary gauge-invariance-preserving (GIP) currents given here may easily be adapted to other cases.

The pion photoproduction current of the nucleon is shown in Fig. 2 [3]. According to the first line of this figure, the total current M^μ may be broken up into four main contributions: The three Born terms due to the s-, u-, and t-channel currents stemming from the photon coupling to the three external legs of the πNN vertex, and the interaction current $M^\mu_{\rm int}$ where the photon attaches itself to an internal leg of the πNN vertex, *i.e.*,

$$M^\mu = M^\mu_s + M^\mu_u + M^\mu_t + M^\mu_{\rm int}\,. \tag{1}$$

The interaction current $M^\mu_{\rm int}$ explicitly involves the full complexity of the internal reaction dynamics of the underlying πN scattering problem summarized in Fig. 1.

For gauge invariance of the total current M^μ to hold true, its four-divergence must satisfy a generalized Ward–Takahashi identity [1–4]

$$k_\mu M^\mu \;=\; \Delta_q^{-1} e_\pi \Delta_{q-k}[\gamma_5 G_t] + S_{p'}^{-1} e_f S_{p'-k}[\gamma_5 G_u] - [\gamma_5 G_s] S_{p+k} e_i S_p^{-1}\,, \tag{2}$$

where p and k are the four-momenta of the incoming nucleon and photon, respectively, and p' and q are the four-momenta of the outgoing nucleon and pion, respectively, related by momentum conservation $p' + q = p + k$. S and Δ are the propagators of the nucleons and pions, respectively, with their subscripts denoting the available four-momentum for the corresponding hadron; e_i, e_f, and e_π are the initial and final nucleon and the pion charges, respectively. $[\gamma_5 G_x]$ symbolically denotes the πNN vertex, with the subscript x labeling the kinematic situation appropriate for the s-, u-, or t-channel diagrams appearing in Fig. 2.

Combining this with the condition to be satisfied by the interaction current, *i.e.*,

$$k_\mu M^\mu_{\mathrm{int}} = e_\pi[\gamma_5 G_t] + e_f[\gamma_5 G_u] - [\gamma_5 G_s]e_i \,, \tag{3}$$

it was shown in Ref. [4] that, by introducing auxiliary currents designed to provide the GIP effects of the exchange currents contained in M^μ_{int}, one may approximate the interaction current by

$$M^\mu_{\mathrm{int}} \to j^\mu_{\mathrm{GIP}} + XG_0\Big(M^\mu_{\mathrm{T},u} + M^\mu_{\mathrm{T},t}\Big) \,, \tag{4}$$

where

$$\begin{aligned} j^\mu_{\mathrm{GIP}} \;=\; & -g_{\mathrm{pv}}\frac{\gamma_5\gamma^\mu}{2m}\Big\{(1-\alpha)G_{\mathrm{pv},t}e_\pi + \alpha G_{\mathrm{pv},s}e_i + \alpha G_{\mathrm{pv},u}e_f\Big\} \\ & - \frac{(2p+k)^\mu}{s-m^2}\gamma_5\Big(G_s[\alpha] - \widehat{G}\Big)e_i - \frac{(2p'-k)^\mu}{u-m^2}\gamma_5\Big(G_u[\alpha] - \widehat{G}\Big)e_f \\ & - \frac{(2q-k)^\mu}{t-\mu^2}\gamma_5\Big(G_t[\alpha] - \widehat{G}\Big)e_\pi \,, \end{aligned} \tag{5}$$

and

$$XG_0\Big(M^\mu_{\mathrm{T},u} + M^\mu_{\mathrm{T},t}\Big) \;=\; XG_0\Big\{T^\mu_N S_{p_N-k}[\gamma_5 G_u] + T^\mu_\pi \Delta_{q_\pi-k}[\gamma_5 G_t]\Big\} \,. \tag{6}$$

The first term on the right-hand side of Eq. (5) describes a dressed Kroll–Ruderman current. It is assumed here that the πNN vertex is given by a mixture of pseudoscalar (ps) and pseudovector (pv) couplings, *i.e.*,

$$G_x[\alpha_x] = g_{\mathrm{ps}}G_{\mathrm{ps},x} + g_{\mathrm{pv}}\frac{\not{q} - \alpha_x\not{k}}{2m}G_{\mathrm{pv},x} \,, \tag{7}$$

where $\alpha_s = \alpha_u = 0$ for $x = s$, u and $\alpha_t = 1$ for $x = t$. The parameter α appearing in Eq. (5) provides a straightforward generalization of these cases by allowing α in j^μ_{GIP} to be chosen independent of the kinematic situation. (This slightly generalizes the procedure of Ref. [4], where $\alpha = 1$.) The function $\widehat{G}$ is undetermined; we choose it as [5]

$$\widehat{G} = a_s G_s[\alpha] + a_u G_u[\alpha] + a_t G_t[\alpha] \,, \quad \text{with} \quad a_s + a_u + a_t = 1 \,. \tag{8}$$

In practical calculations, this provides three fit parameters for the GIP current (5), namely α and two of the a_x's. Equation (6) provides the remaining contribution of the final-state interaction mediated by the non-polar πN amplitude X (see Fig. 1). G_0 is the product of the intermediate pion and nucleon propagators and T_N^μ and T_π^μ denote the transverse parts of the electromagnetic nucleon and pion currents, respectively. In the GIP scheme described here, the contributions due to the final-state interaction, therefore, are entirely transverse.

We emphasize once again that the GIP current given here is not unique. One may always add a transverse current without affecting the gauge invariance. In particular, choosing $\alpha = 1$ and $\widehat{G} = g_{\pi NN}$, the present procedure subsumes also the GIP recipe by Ohta [6].

2.2. Application to kaon production

In Fig. 3, typical results for the photoproduction $p(\gamma, K^+)\Lambda$ are shown. This result was obtained at the tree level (*i.e.*, $X=0$). The figure shows a comparison of the present method to Ohta's recipe. It is found that, in general, one obtains a better description of experimental data with our prescription. This is also corroborated by the results reported in Refs. [5,9,10].

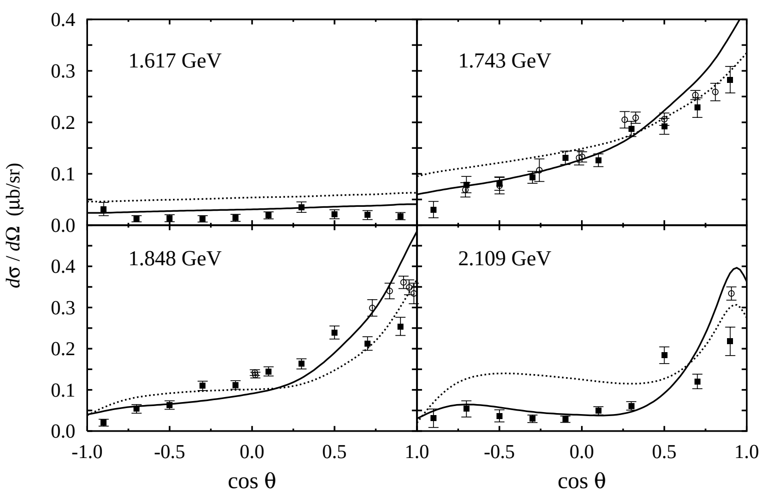

Fig. 3. New Bonn SAPHIR data [7] for $p(\gamma, K^+)\Lambda$ (new data: solid squares; old data: open squares) compared to theoretical predictions using the present formalism (solid line) and the one by Ohta [6] (dotted lines).

2.3. Gauge invariance vs current conservation

The procedure for obtaining a GIP current described above rests upon satisfying the basic generalized Ward–Takahashi identity (2). We would like to emphasize that the latter is an *off-shell* condition. To make a given approximate current J^μ conserved, one often relies on a subtraction of the longitudinal component, *i.e.*,

$$J^\mu \to J^\mu_{\rm cc} = J^\mu - k^\mu \frac{k\cdot J}{k^2}\,, \tag{9}$$

which obviously provides $k\cdot J_{\rm cc}=0$. Often this conserved current is confused with a gauge-invariant current. In general, however, this is not justified. We emphasize that $J^\mu_{\rm cc}$ will only be gauge-invariant if the difference between the true current and the conserved current $J^\mu_{\rm cc}$ can be shown to be transverse when all hadron legs are off-shell; transversality of the on-shell difference is *not* sufficient.

3. Off-shell form factors

In the following, we will briefly consider the implications of explicitly allowing off-shell degrees of freedom in both the hadronic and electromagnetic vertices.

3.1. Hadronic vertex: πNN

The linear combination of pseudoscalar and pseudovector coupling assumed in Eq. (7) is not the most general form for the πNN vertex. In general, this vertex has four form factors $G_{ij}=G_{ij}(q^2,p'^2,p^2)$, *i.e.*,

$$F=\gamma_5 G_{00}+\frac{\not p'-m'}{2m'}\gamma_5 G_{10}+\gamma_5\frac{\not p-m}{2m}G_{01}+\frac{\not p'-m'}{2m'}\gamma_5\frac{\not p-m}{2m}G_{11}\,. \tag{10}$$

In view of the inverse Dirac propagators appearing here, the last three terms give rise to contact-type contributions if employed inside a diagram. For practical purposes, and to allow one to make contact with the usual parameterizations, it seems reasonable to simplify the vertex by using

$$G_{00}-\frac{m+m'}{2m}G_{01}=G_{00}-\frac{m+m'}{2m'}G_{10}\;\longrightarrow\; g_{\rm ps}G_{\rm ps}\,, \tag{11}$$

$$\frac{m+m'}{2m}G_{01}=\frac{m+m'}{2m'}G_{10}=\frac{(m+m')^2}{4mm'}G_{11}\;\longrightarrow\; g_{\rm pv}G_{\rm pv}\,, \tag{12}$$

which provides

$$F=\gamma_5 g_{\rm ps}G_{\rm ps}+\frac{\not p'+m}{m'+m}\gamma_5\frac{\not p+m'}{m+m'}g_{\rm pv}G_{\rm pv}\,. \tag{13}$$

Half-on-shell this is exactly identical to Eq. (7). At the tree-level, therefore, there is no difference to the previous treatment of the πNN vertex.

3.2. Electromagnetic vertex: γNN

The most general current for the nucleon that satisfies gauge invariance contains eight form factors $F_n^{ij} = F_n^{ij}(p'^2, p^2; k^2)$, *i.e.*,

$$\begin{aligned}
\Gamma^\mu = \gamma^\mu \; &+ \; \left[\left(\gamma^\mu k^2 - k^\mu \not{k}\right)\frac{F_1^{00}}{k^2} + i\frac{\sigma^{\mu\nu}k_\nu}{2m}F_2^{00}\right] \\
&+ \; \left[\left(\gamma^\mu k^2 - k^\mu \not{k}\right)\frac{F_1^{01}}{k^2} + i\frac{\sigma^{\mu\nu}k_\nu}{2m}F_2^{01}\right]\frac{\not{p}-m}{2m} \\
&+ \; \frac{\not{p}'-m}{2m}\left[\left(\gamma^\mu k^2 - k^\mu \not{k}\right)\frac{F_1^{10}}{k^2} + i\frac{\sigma^{\mu\nu}k_\nu}{2m}F_2^{10}\right] \\
&+ \; \frac{\not{p}'-m}{2m}\left[\left(\gamma^\mu k^2 - k^\mu \not{k}\right)\frac{F_1^{11}}{k^2} + i\frac{\sigma^{\mu\nu}k_\nu}{2m}F_2^{11}\right]\frac{\not{p}-m}{2m}\,. \qquad (14)
\end{aligned}$$

If both nucleons are on-shell, the first line here corresponds to the usual parameterization in terms of the Dirac and Pauli form factors, *i.e.*, $F_1^{00} \to F_1 - 1$ and $F_2^{00} \to F_2$.

The remaining six form factors describe off-shell degrees of freedom which are not directly accessible by experiment in a model-independent way. As with the hadronic vertex above, these terms can be associated with contact-type interactions. The simplest ansatz which employs only two form factors, yet retains some of the off-shell dependence results from the identification

$$F_1^{11} = F_1^{01} = F_1^{10} = F_1^{00} \;\longrightarrow\; F_1 - 1\,, \qquad (15)$$
$$F_2^{11} = F_2^{01} = F_2^{10} = F_2^{00} \;\longrightarrow\; F_2 \qquad (16)$$

which reduces (14) to

$$\Gamma^\mu \;=\; \gamma^\mu + \frac{\not{p}'+m}{2m}\left[\left(\gamma^\mu - \frac{k^\mu \not{k}}{k^2}\right)(F_1-1) + i\frac{\sigma^{\mu\nu}k_\nu}{2m}F_2\right]\frac{\not{p}+m}{2m}\,. \qquad (17)$$

4. Summary and discussion

In summary, we have treated here the electromagnetic production current for mesons off the nucleon for both real and virtual photons. We used the constraints following from requiring the validity of the generalized Ward–Takahashi identities to construct auxiliary current pieces that ensure that gauge invariance is preserved even if — as it is invariably the case in practical applications — one does not treat the problem completely and consistently.

The present results remain equally valid whether the form factors F and the final-state amplitude X are obtained via some sophisticated Bethe–Salpeter-type formalism or are based on a simple phenomenological model ansatz. How these elements are obtained does not enter any of the present considerations and therefore has no bearing on the question of gauge invariance.

We have also discussed briefly the possibility of employing off-shell form factors for both hadronic and electromagnetic vertices. The simplifications we propose here may help determine how important the full off-shell dependence of these vertices may be in practical calculations.

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