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HIGHER ORDER COULOMB CORRECTIONS TO THE PRIMAKOFF MEASUREMENT OF THE π^0 LIFETIME *

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Corrections to single photon exchange in the π^0 Primakoff production on heavy nuclei are considered with an emphasis on multiple Coulomb corrections. Those can be shown to be negligible for small momentum transfers, the region most crucial for Primakoff measurements of Γ_{π^0}. For the η, which can be produced by the same mechanism, corrections are seen to be of the order of 1 %.

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1. Introduction

It has been known since 1969, that the coupling to the electromagnetic field causes an anomalous term of the form $(\alpha_{\rm em} N_c)/(12\pi) F_{\mu\nu}\tilde{F}^{\mu\nu}$ to appear in the third component of the divergence of the axial isospin current $\vec{A}_\mu = \bar{\Psi}\gamma_\mu\gamma_5\vec{\tau}\Psi$ [1, 2]. This anomalous term is stable against higher order corrections [3] and assuming pion pole dominance, a firm theoretical prediction for the two photon decay width can be made

$$\Gamma_{\pi^0} = \frac{\alpha_{\rm em}^2 N_c m_\pi^2}{576\pi^3 F_\pi^2}\,. \tag{1}$$

The pion decay constant F_π, defined via $< 0|A_\mu^k|\pi^j(p) >= iF_\pi p_\mu \delta^{kj}$, is known from charged pion decay. Modifications due to higher excited poles (π',..) can be estimated to be ≤ 1.3 % [4]. This results in a theoretical value $\Gamma_{\rm theo} = 7.72$ eV ($\pm 2\%$). A measurement of Γ_{π^0} can thus be regarded as a fundamental test of QCD.

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The present experimental situation is not satisfactory. The PDB gives a mean value Γ_π=7.8 ± 0.6 eV. Plans for a new precision experiment at Jefferson Lab aim at reaching an accuracy of ≈ 1.4 % with the Primakoff technique.

2. The Primakoff effect

The π^0 photoproduction in the nuclear Coulomb field via single photon exchange is known as the Primakoff effect [5] (left part of Fig. 1).

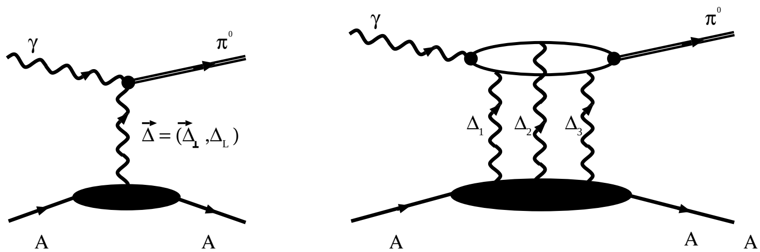

Fig. 1. The Primakoff effect and multiple Coulomb corrections.

Since the differential cross section is proportional to Γ_{π^0}, the decay width is accessible experimentally in this process. Decomposing the momentum transfer relative to the collision axis $\vec{\Delta} = (\vec{\Delta}_\perp, \Delta_{\mathrm{L}})$, one finds

$$\frac{d\sigma}{d\Omega} \sim Z^2 \frac{\vec{\Delta}_\perp^2}{(\vec{\Delta}_\perp^2 + \Delta_{\mathrm{L}}^2)^2} \Gamma_{\pi^0 \to \gamma\gamma} \,. \tag{2}$$

As a function of $\Delta_\perp$ the cross section has a pronounced peak at $\Delta_\perp = \Delta_{\mathrm{L}} = m_\pi^2/(2E_\gamma)$ (≈ 2 MeV for E_γ = 5 GeV).

3. Corrections to the Primakoff production

3.1. Strong production mechanism

Strong (non γ exchange) production $\gamma N \to \pi^0 N$ on bound nucleons has clearly to be taken into account. Since the cross sections is not precisely known, an accurate prediction for $\gamma A \to \pi^0 A$ is not possible. It can, however, be measured for large t in the experiment and extrapolated under the Coulomb peak, since it is known to vary smoothly there. A cross-check calculation in Glauber theory can then be carried out.

3.2. Final state interaction

The π^0-nucleus FSI, predominantly absorption at impact parameters $b \leq R_A$, is calculable in Glauber theory [6]:

$$F_A(\vec{\Delta}) \sim \frac{Z}{4\pi i} \int d^3x e^{-i\vec{\Delta}\vec{x}} \left[(\vec{\varepsilon}_\gamma \times \vec{n}_\gamma)\, \vec{E}(\vec{x}) \right] \exp\left(-\frac{1}{2}\sigma \int_z^\infty dz' \rho_A(\vec{b}, z') \right). \quad (3)$$

The effect caused by FSI can be cast in a modified form factor as shown in Fig. 2 for lead. The peak region is also indicated. There the effect is small, but since the strong production amplitude has to be measured accurately for large t, it is necessary to go up to higher momentum transfers in order to be able to seperate the two effects here.

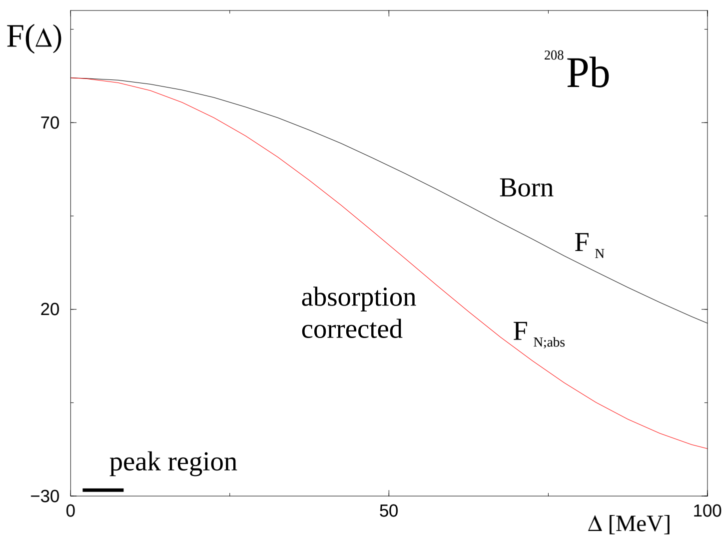

Fig. 2. The modified formfactor, including FSI.

3.3. Multiple Coulomb corrections

Both backgrounds, considered so far, are non-singular as $t \to 0$, in the region of the Coulomb peak. Due to the long range of the Coulomb interaction, this is not necessarily the case for multiple Coulomb corrections. Furthermore the factor $Z\alpha$, appearing for each additional photon-line, is not small for heavy nuclei. A related problem: Higher order Coulomb corrections to $(\pi^+\pi^-)A$ scattering can give a contribution of up to 14 % for heavy nuclei [7].

For the π^0 production it is convenient to introduce $t = \Delta^2 = -\vec{\Delta}^2 - \Delta_{\rm L}^2$ and $\Delta_{\rm L} = \sum \Delta_{{\rm L},i} = m_\pi^2/2E_\gamma$ (right part of Fig. 1). In the high energy limit, *i.e.* $\Delta_{{\rm L},i} = \mathcal{O}(1/E_\gamma) \approx 0$, we find

$$A = A_{\rm Born}\left(1 - {\rm const}(Z\alpha)^2\Delta_\perp^2 R_\pi^2 \log\left(\frac{1}{-R_A^2 t}\right)\right), \tag{4}$$

where R_π is closely related to the charge radius of the pion. There is a potential enhancement of the correction by the $\log(1/(-R_A^2 t))$ singularity. At moderately high energies one must also take care of finite $\Delta_{{\rm L},i}$. The scale for the masses of the intermediate states is set by the typical sequence $\gamma \to \pi^0 \to \rho^0, \omega^0 \to \pi^0$, therefore $\Delta_{\rm L1} = m_\pi^2/(2E_\gamma) \ll \Delta_{\rm L2} = (m_\rho^2 - m_\pi^2)/(2E_\gamma)$.

At the Coulomb peak ($\vec{\Delta}^2 \approx \Delta_{\rm L1}^2$) the correction is only proportional to $\log\left(1/(R_A^2\Delta_{\rm L2}^2)\right)$. In the CEBAF energy range 4.6–5.7 GeV, $R_A^2\Delta_{\rm L2}^2 \sim 1$, hindering the logarithmic enhancement of multiple Coulomb corrections. Numerical results for lead are shown in the left part of Fig. 3. There are two major effects: (1) the smallness of $\Delta_{\rm L1}$ leads to a pronounced peak in the Born amplitude and (2) large $\Delta_{\rm L2}$ and $\Delta_{\rm L3}$ give a strong suppression of multiple photon exchange. Thus we find only a negligible correction for the π^0 Primakoff production. The production process $\gamma A \to \eta A$ is very similar

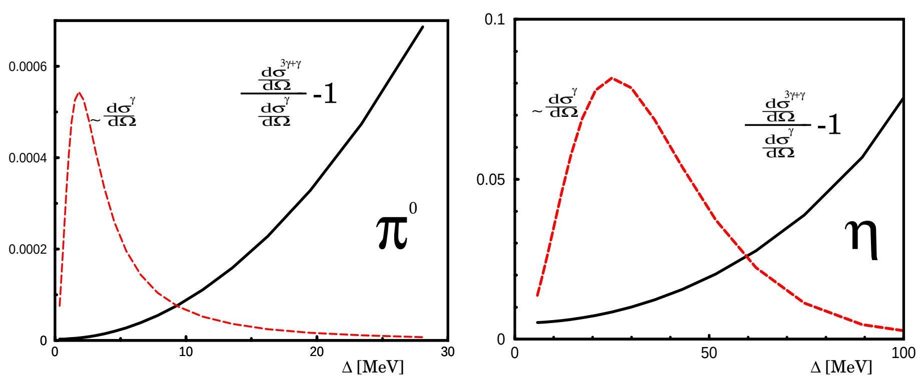

Fig. 3. The change to $\frac{d\sigma}{d\Omega}$ due to three photon exchange is shown. The Coulomb peak is indicated with arbitrary normalisation.

in structure. The correction, however, is enhanced in comparison to the π^0 case for two reasons: (1) the Born amplitude is suppressed by a factor of $(m_\eta/m_\pi)^4 \approx 250$ and (2) $\Delta_{\rm L2}, \Delta_{\rm L3}$ are smaller, thus the $\log(1/(R^2\Delta_{{\rm L},2;3}^2))$ gives a larger contribution (right part of Fig. 3).

4. Conclusions

Multiple Coulomb corrections are a potentially dangerous background in the peak region, but were shown to be numerically small in the π^0 case. For the η, however, the correction amounts to ≈ 1 % and should be taken into account in possible Primakoff lifetime measurements. The background from strong interaction and $\pi^0 A$ FSI can either be eliminated in the experiment or calculated reliably. Thus corrections to the Primakoff technique are under firm theoretical control and from that point of view per mille accuracy measurements of Γ_{π^0} are indeed possible.

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