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PION AND KAON VECTOR FORM FACTORS*

J.E. Palomar, E. Oset

Departamento de Física Teórica e IFIC
Centro Mixto Universidad de Valencia-CSIC
46100 Burjassot (Valencia), Spain

and J.A. Oller

Forschungszentrum Jülich, Institut für Kernphysik (Theorie)
D-52425 Jülich, Germany

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The pion and kaon coupled-channel vector form factors are described by making use of the resonance chiral Lagrangian results together with a suitable unitarization method in order to take care of the final state interactions. A very good reproduction of experimental data is accomplished for the vector form factors up to $\sqrt{s} \leq 1.2$ GeV and for the $\pi\pi$ P-wave phase shifts up to $\sqrt{s} \leq 1.5$ GeV.

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1. Unitarization

Using an appropriate unitarization method we take into account the final state interaction corrections to the tree level amplitudes calculated from the lowest order χPT [1] and from the inclusion of explicit resonance fields in a chiral symmetry fashion as given in Ref. [2]. A similar procedure has already been used in the scalar sector to describe the scalar form factor associated with the strange-change scalar current $\overline{u}s$ in Ref. [3]. Starting from the unitarity of the S-matrix and the introduction of the electromagnetic meson form factor $F_{MM'}(s)$:

$$\langle\gamma(q)|T|M(p)M'(p')\rangle = e\varepsilon_\mu(p-p')^\mu F_{MM'}(s) \tag{1}$$

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with $q^2 = s$, e the modulus of the electron charge and ε_μ the photon polarization vector, we arrive at the expression:

$$\mathrm{Im}\, F_{MM'}(s) = \sum_\alpha F^*_\alpha(s)\, \frac{p_\alpha(s)}{8\pi\sqrt{s}}\, \theta(s - 4m^2_\alpha)\, p_\alpha(s) \frac{T(s)_{\alpha,MM'}}{p_{MM'}(s)}\,, \tag{2}$$

where $\theta(x)$ is the usual Heaviside function. On the other hand $p_{MM'}(s)$ and $p_\alpha(s)$ are respectively the moduli of the three momenta of the mesons in the final and intermediate meson states, and we sum over intermediate two-meson states.

We will work in the isospin limit, with $|\pi\pi\rangle$ and $|K\bar{K}\rangle$ states (and the ρ resonance) in the $I = 1$ channel and only the $|K\bar{K}\rangle$ state (and the ω and ϕ resonances) in the $I = 0$ channel. Using a matrix notation, the P-wave amplitudes $T^I(s)$ can be written as [4]:

$$T^I = \left[1 + K^I(s)\, g(s)\right]^{-1} K^I(s)\,, \tag{3}$$

where $K^I_{ij}(s)$ are the tree level amplitudes derived from the lowest order χPT plus corresponding to the transition $i \to j$. From Eqs. (2) and (3) it follows, after some algebra, that $F^I(s)$ can be written as:

$$F^I(s) = \left[1 + \widetilde{Q}(s)^{-1}\, K^I(s)\, \widetilde{Q}(s)\, g^I(s)\right]^{-1} R^I(s)\,, \tag{4}$$

where $\widetilde{Q}_{ij}(s) = p_i(s)\delta_{ij}$ and $K^I(s)$ is the matrix collecting the tree level amplitudes between definite $\pi\pi$ and $K\bar{K}$ isospin states. $F^I(s)$ is the column matrix $F^I(s)_i = F^I_i(s)$, $R^I(s)$ is a vector made up by functions without any cut and $g^I(s)$ is the diagonal matrix given by the loop with two meson propagators:

$$g^I_i(s) = \frac{1}{16\,\pi^2}\left[-2 + d^I_i + \sigma_i(s)\, \log\frac{\sigma_i(s)+1}{\sigma_i(s)-1}\right]\,, \tag{5}$$

where $\sigma_i(s) = \sqrt{1 - 4m_i^2/s}$. We will label pions with 1 and kaons with 2 in the $I = 1$ case. In the $I = 0$ case we only have kaons.

In the large N_c limit loop physics is supressed and then

$$F^I(s) = R_{N_c\mathrm{leading}}(s) = F^I_t(s)\,,$$

where $F^I_t(s)$ is the tree level form factor[1]. This allows us to write:

$$F^I(s) = \left[1 + \widetilde{Q}(s)^{-1}\, K^I(s)\, \widetilde{Q}(s)\, g^I(s)\right]^{-1} \left[F^I_t(s) + R^I_{\mathrm{subleading}}(s)\right] \tag{6}$$

[1] We evaluate the tree level form factors and scattering amplitudes using the lowest order χPT Lagrangian [1] plus the chiral resonance Lagrangian [2].

with $R^I_{\text{subleading}}(s)$ being of $\mathcal{O}(N_c^{-1})$. If we require that the vector form factor from Eq. (6) vanishes for $s \to \infty$ as is suggested by the experiments, we find that the subleading part of $R^I(s)$, which at first can be an arbitrary polynomial (the poles coming from the resonances are in the leading part $F_t^I(s)$), must be a constant. In order to fix the constants $R^I_{\text{subleading}}(s)$ and $d_i^I(s)$ of $g_i^I(s)$ we match our results with those of one loop χPT. We take $R^{I=1}_{\text{subleading}}(s) = 0$ in order to constrain further our approach. This can be done since we can match our results with one loop χPT by choosing appropriate values for $d_1^{I=1}$ and $d_2^{I=1}$. The values of the other constants given by the matching are:

$$
\begin{aligned}
d_1^{I=1} &= \frac{m_K^2}{m_K^2 - m_\pi^2}\left(\log\frac{m_\pi^2}{\mu^2} + \frac{1}{2}\log\frac{m_K^2}{\mu^2} + \frac{1}{2}\right), \\
d_2^{I=1} &= \frac{-2\, m_\pi^2}{m_K^2 - m_\pi^2}\left(\log\frac{m_\pi^2}{\mu^2} + \frac{1}{2}\log\frac{m_K^2}{\mu^2} + \frac{1}{2}\right), \\
d^{I=0} &= \frac{1}{3} + \log\frac{m_K^2}{\mu^2}, \\
R^{I=0}_{\text{subleading}} &= -\frac{m_K^2}{16\sqrt{2}\,\pi^2 f^2}\left(\frac{1}{3} + \log\frac{m_K^2}{\mu^2}\right).
\end{aligned} \tag{7}
$$

The bare masses of the resonances (which appear in the tree level quantities) are fixed by the requirements that the moduli of the $\pi\pi$ $I = 1$ and $K\overline{K}$ $I{=}0$ P-wave amplitudes have a maximum for $\sqrt{s} = M_\rho^{\text{physical}}$ MeV and for $\sqrt{s} = M_\phi^{\text{physical}}$ MeV, respectively. For the mass of the ω we take directly 782 MeV since there are no experimental data in the region of the ω and its contributions to other physical regions do not depend on such fine details since the ω is very narrow. On the other hand, the coupling of the vector resonances [2] to mesons and photons are described by two real parameters G_V and F_V respectively. We use their experimental value, $G_V{=}53$ MeV (from a study of the pion electromagnetic radii [1]) and $F_V{=}154$ MeV (from the observed decay rate $\Gamma(\rho^0 \to e^+e^-)$ [2]).

2. Results and conclusions

As can be seen in Fig. 1, we can describe in a very precise way the vector pion form factor and the P-wave $\pi\pi$ phase shifts up to about $s = 1.44$ GeV^2 (even for higher energies in the case of phase shifts).

For values of $\sqrt{s}$ higher than 1.2 GeV new effects appear: 1) the presence of more massive resonances, ρ', ω', ϕ'... 2) the effect due to multiparticle

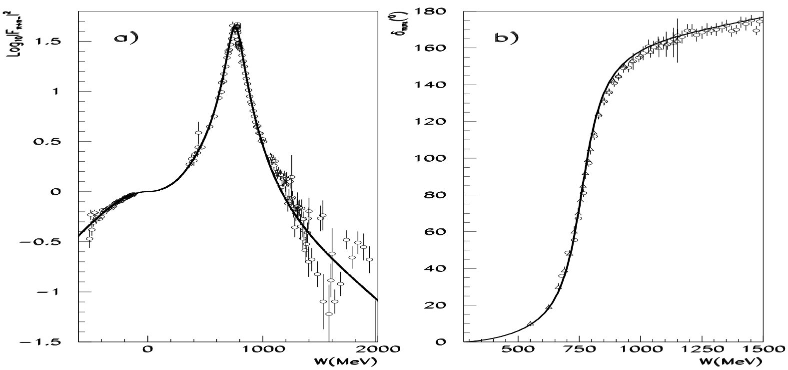

Fig. 1. W is defined as $\sqrt{s}$ for $s > 0$ and as $-\sqrt{-s}$ for $s < 0$. (a) $\pi^+\pi^-$ vector form factor. (b) $\pi\pi$ P-wave phase shifts. Both are compared with several experimental data.

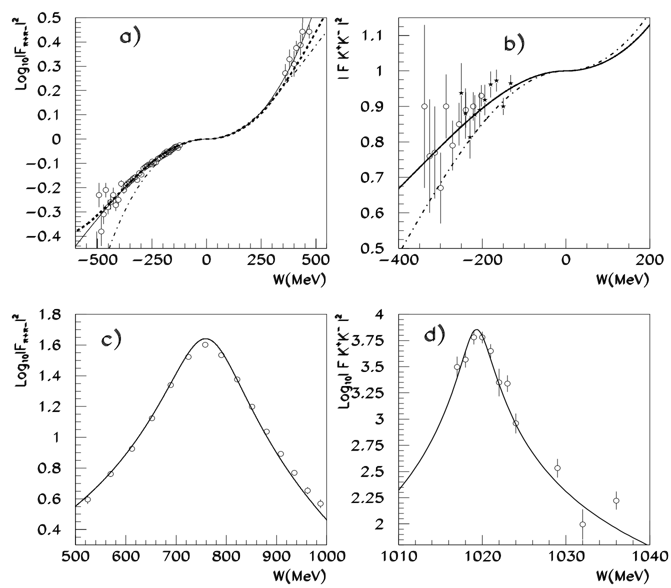

Fig. 2. W is defined as $\sqrt{s}$ for $s > 0$ and as $-\sqrt{-s}$ for $s < 0$. From left to right and top to bottom: (a) Vector pion form factor. The dashed-dotted line represents one loop χPT Ref. [1] and the dashed one the two loop χPT result Ref. [5]. (b) K^+K^- form factor. The meaning of the lines is the same as before. (c) Vector pion form factor in the ρ region. Data from tau decay. (d) K^+K^- form factor. All the results are compared with data.

states, *e.g.* 4π, $\omega\ \pi$ ... which are non negligible. In Fig. 2, we compare our results with those of χPT. We can see that the resummation of our scheme leads to a much better agreement with the two loop χPT pion vector form factor than with the one loop one. The resonance regions are also well reproduced.

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