STRONG DECAYS OF MESONS IN THE $^3P_0$ MODEL USING SOPHISTICATED WAVE FUNCTIONS *

B. Silvestre-Brac and R. Bonnaz

Institut des Sciences Nucléaires
53 Avenue des Martyrs, F38026 Grenoble Cedex, France

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We apply the $^3P_0$ model to the strong decay of a meson into two mesons. We modify the traditional vertex by allowing a momentum and flavor dependent form. Moreover the meson wave functions are obtained using realistic quark–antiquark potentials including finite size for the quarks, instanton effects, spin–orbit and tensor forces. The results are in very good agreement with the data.

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1. Introduction

It is no doubt that Quantum Chromodynamics (QCD) is presently a good theory for strong interactions, but the formalism is complicated and the results that can be obtained in the non perturbative regime are not easy to be derived from first principles.

On the other hand, mesonic resonances are very abundant and they are the simplest systems that can be used as a probe for exploring the non perturbative properties of QCD. The description of mesons can be done in a number of theoretical frameworks, but we present here only a formulation based on a non relativistic potential model. One can find potentials that allow a very good description of the spectra. But spectra are not very stringent observables to test the wave functions. We must rely on other observables, such as static or dynamical properties.

In this paper, we focus our attention on the strong decay of a meson into two mesons $A \to B + C$, that results from a $q\bar{q}$ pair creation in the so-called $^3P_0$ model. We consider the case where the mesons in the final state are stable or have a narrow width. We present the results with the best realistic wave functions and dynamical vertex.

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2. The model

To describe the decay, several models have been proposed; they are all phenomenological. We study here the $^3P_0$ model in which the pair is created with the quantum numbers of the vacuum. Written in second quantization, the corresponding transition operator has the following form:

$$\begin{aligned} T &= -3\sum_{\mu}\int d\boldsymbol{p}d\boldsymbol{p}'\delta(\boldsymbol{p}+\boldsymbol{p}')\gamma_\mu\left(\frac{|\boldsymbol{p}-\boldsymbol{p}'|}{2}\right) \\ &\quad\times\left[\mathcal{Y}_1\left(\frac{\boldsymbol{p}-\boldsymbol{p}'}{2}\right)b^\dagger_\mu(\boldsymbol{p})d^\dagger_\nu(\boldsymbol{p}')\right]^{C=1,I=0,S=1,J=0} . \end{aligned} \tag{1}$$

The summation runs over the various flavors, $b^\dagger_\mu(\boldsymbol{p})$ is the quark creation operator and $d^\dagger_\nu(\boldsymbol{p'})$ the antiquark creation operator. The coupling makes transparent the $^3P_0$ structure. The dynamical vertex $\gamma_\mu\left(\frac{|\boldsymbol{p}-\boldsymbol{p}'|}{2}\right)$ is chosen usually as pure constant. We give up this prescription, and comment this point later.

The meson wave functions are also written in second quantized form and in momentum representation because this is more suitable in the $^3P_0$ model. The Jacobi coordinates are defined as usual. The total momentum occurs in a plane wave, whereas the relative momentum $\boldsymbol{p}$ appears in the intrinsic wave function $\varphi(\boldsymbol{p})$, solution of the Schrödinger equation.

This last equation is solved with two different potentials DNR and NRAL that have been presented elsewhere ([1, 2]). They both include central, hyperfine part, as well as instanton effects and allow for a finite size for the quark. In addition, NRAL takes into account spin–orbit and tensor forces. This is the first time that such sophisticated wave functions are employed in the calculation of strong decays.

In the final state, the wave functions for mesons B and C are coupled to adequate quantum numbers. Then the transition operator is sandwiched between the initial state and the final state. The practical calculation is made with help of Wick's theorem. Among the 4 possible contractions, two are OZI forbidden and must be discarded, while, in most cases, only one remains after a suitable labelling of the final mesons.

The transition amplitude is given by:

$$M_{A\to BC}(k) = \frac{1}{\sqrt{1+\delta_{BC}}}\,\mathcal{I}(A\to BC)\ \sum\mathcal{C}(A\to BC)\mathcal{E}(A\to BC,k)\,. \tag{2}$$

TABLE I

Square root of the partial decay width of a meson into two stable mesons in the final channel. Two potentials DNR and NRAL used for obtaining the meson wave functions are considered as well as two prescriptions for the dynamical vertex.

Decay	$\Gamma^{\frac{1}{2}}[\mathrm{MeV}^{\frac{1}{2}}]$				
	$\gamma = A$		$\gamma = A + B\mathrm{e}^{-Cp^2}$		Exp.
	DNR	NRAL	DNR	NRAL	
$\rho \to \pi\pi$	8.57	8.61	10.19	9.72	12.28 ± 0.37
$\varphi \to K\bar{K}$	1.62	1.82	1.93	2.02	1.92 ± 0.06
$K^\star(892) \to \pi K$	4.91	5.21	5.80	5.82	7.08 ± 0.21
$f_2(1270) \to \pi\pi$	13.10	12.64	12.36	12.01	$12.53^{+0.23}_{-0.13}$
$f_2(1270) \to \eta\eta$	1.36	1.22	0.79	1.01	0.91 ± 0.10
$K_0^\star(1430) \to \pi K$	7.01	13.23	19.75	21.35	16.34 ± 1.10
$f_2'(1525) \to K\bar{K}$	8.49	9.26	8.55	9.10	8.06 ± 0.56
$K_2^\star(1430) \to \pi K$	8.78	9.03	8.69	8.81	7.01 ± 0.21
$K_2^\star(1430) \to \omega K$	1.83	1.92	1.20	1.65	1.70 ± 0.23
$a_2(1320) \to K\bar{K}$	3.06	3.02	2.29	2.29	2.29 ± 0.19
$f_2(1270) \to K\bar{K}$	2.50	2.47	2.93	2.92	2.92 ± 0.13
$f_2'(1525) \to \eta\eta$	2.67	3.15	2.76	2.80	2.80 ± 0.46
$a_2(1320) \to \eta\pi$	6.67	3.11	5.45	2.83	3.94 ± 0.19
$a_2(1320) \to \eta'\pi$	1.71	0.90	1.02	0.75	0.75 ± 0.07
$\rho_3(1690) \to \pi\pi$	10.11	9.02	8.64	7.86	6.14 ± 0.26
$\rho_3(1690) \to K\bar{K}$	2.86	2.78	2.61	2.85	1.59 ± 0.14
$\rho_3(1690) \to \omega\pi$	9.06	4.31	5.86	3.58	5.06 ± 0.96
$f_4(2050) \to \omega\omega$	7.98	5.60	7.24	4.86	7.35 ± 0.88
$f_4(2050) \to \pi\pi$	10.33	8.64	8.90	7.29	5.95 ± 0.32
$f_4(2050) \to K\bar{K}$	2.26	2.17	0.26	0.50	$1.19^{+0.30}_{-0.16}$
$f_4(2050) \to \eta\eta$	3.22	2.62	2.20	2.15	0.66 ± 0.13
$K_1(1270)_+ \to \omega K$	4.20	4.70	3.98	4.80	3.15 ± 0.45
$K_1(1400)_- \to \omega K$	3.52	5.40	3.60	5.62	1.32 ± 0.66
$K^\star(1410) \to \pi K$	2.72	5.50	1.93	4.83	3.91 ± 0.42
$K_2^\star(1430) \to \eta_- K$	5.14	5.28	1.51	1.85	$0.38^{+0.44}_{-0.13}$
$K^\star(1680) \to \pi K$	4.33	7.44	10.19	10.38	11.16 ± 1.94
$K_3^\star(1780) \to \pi K$	8.12	7.73	6.96	6.85	5.47 ± 0.39
$K_3^\star(1780) \to \eta_- K$	5.24	5.09	2.39	2.90	6.91 ± 1.56
$K_4^\star(2045) \to \pi K$	6.39	5.68	4.80	4.66	4.43 ± 0.29
χ^2	542	320	174	157	

The factor $\mathcal{I}$ depends on isospin and is merely a $6J$ symbol, the factor $\mathcal{C}$ couples spin and space and is calculated with Racah techniques, and $\mathcal{E}$ depends on space degrees of freedom and needs both the dynamical vertex and the wave functions. It is by far the most complicated term and taking into account the tensor force makes it still more difficult to compute. Some details concerning the various expressions can be found in [3].

The partial width is obtained from the amplitude by the well known golden rule formula. Nevertheless, let us stress the fact that the results are much better if one uses a relativistic phase space factor.

2.1. Results and conclusions

To determine the free parameters entering the vertex, we selected a sample of 12 transitions and made a best fit on the corresponding widths. Then, with the resulting vertex, we calculated several other transition widths. The results are presented in the Table I, where the square root of the width calculated with 2 potentials and 2 vertices (the constant one and the "constant+gaussian" one) are compared to the experimental data. It is very clear from this table that a momentum dependent vertex gives improved values as compared to a constant one. Both potentials give more or less the same quality although the meson wave functions differ sometimes appreciably. In fact, the vertex drives the quality and different wave functions participate to different sets of parameters with an overall equivalent description.

The ${}^3P_0$ model, although phenomenological, leads to very good results for the decay of a meson into 2 mesons, provided we follow essentially 2 prescriptions:

— the phase space factor is an important ingredient for a quantitative answer and a relativistic factor must be prefered as compared to a non relativistic one;

— the pair creation vertex must depend on the relative momentum of the pair. The use of a "constant+gaussian" vertex is a good compromise between quality and numerical effort.

Within this framework, with few parameters, one gets remarkable results.

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