NON-LOCAL EFFECTS IN KAONIC ATOMS*

Matthias Lutz

Gesellschaft für Schwerionenforschung GSI
Postfach 110552, 64220 Darmstadt, Germany

and Wojciech Florkowski

H. Niewodniczański Institute of Nuclear Physics
Radzikowskiego 152, 31-342 Kraków, Poland

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Optical potentials with non-local (gradient) terms are used to describe the spectra of kaonic atoms. The strength of the non-local terms is determined from a many-body calculation of the kaon self energy in nuclear matter. We find that the non-local terms are quantitatively important and the results depend strongly on the way the gradient terms are arranged. Phenomenologically successful description is obtained for p-wave like optical potentials. It is suggested that the microscopic form of the non-local interaction terms is obtained systematically by means of a semi-classical expansion of the nucleus structure.

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Kaonic atom data provide a valuable consistency check on any microscopic theory of the K^- nucleon interaction in nuclear matter. We therefore apply the microscopic approach, developed by one of the authors in [1], to kaonic atoms. Since the typical binding energy of a K^- bound at a nucleus in a p-wave is of the order of 0.5 MeV the typical kaon momentum is expected to be roughly 20 MeV. We then expect the relevant nuclear Fermi momentum $k_{\rm F}$ to be larger than the kaon momentum $|\vec{q}| < k_{\rm F}$. For the study of kaonic level shifts it is therefore useful to introduce the effective scattering length $a_{\rm eff}(k_{\rm F})$ and the effective slope parameters $b_{\rm eff}(k_{\rm F})$ and $c_{\rm eff}(k_{\rm F})$

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$$\Pi_K(\omega,\vec{q}\,) \;=\; -\frac{8}{3\,\pi}\left(1+\frac{m_K}{m_N}\right)\left(a_{\rm eff}(k_{\rm F})\,k_{\rm F}^3+b_{\rm eff}(k_{\rm F})\,k_{\rm F}^2\,\vec{q}^{\,2}\right)$$
$$+\frac{8}{3\,\pi}\left(1+\frac{m_K}{m_N}\right)c_{\rm eff}(k_{\rm F})\,k_{\rm F}^2\left(\omega-m_K\right)+\ldots \qquad (1)$$

Here we expanded the kaon self energy for small momenta with $|\vec{q}\,| < k_{\rm F}$ and energies close to m_K.

In Fig. 1 the effective scattering length and the slope parameters are presented as extracted numerically from the self consistent calculation of [1]. The real part of the effective scattering length $a_{\rm eff}(k_{\rm F})$ changes sign as the density is increased. At large densities we find an attractive effective scattering length in qualitative agreement with previous work [2].

At first, in order to make an estimate of non-local effects in kaonic atoms, we calculate the spectra with an optical potential $U_{\rm opt}(r,\vec{\nabla})$ deduced from the kaon self energy (1) but use a phenomenological Ansatz for its non-local structure. Our starting point is the Klein–Gordon equation

$$\vec{\nabla}^2\phi(r)+\left[\left(\mu-E-\frac{i\,\Gamma}{2}-V_{\rm e.m.}(r)\right)^2-\mu^2\right]\phi(r)=2\,\mu\,U_{\rm opt}(r,\vec{\nabla})\,\phi(r)\,, \qquad (2)$$

where E and Γ are the binding energy and width of the kaonic atom, whereas μ is the reduced kaon mass in the K^- nucleus system. The electromagnetic

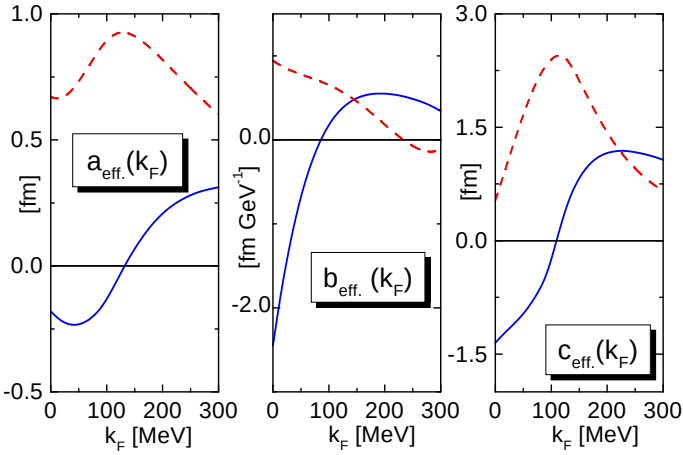

Fig. 1. The effective scattering length $a_{\rm eff}(k_{\rm F})$ and effective slope parameters $b_{\rm eff}(k_{\rm F})$ and $c_{\rm eff}(k_{\rm F})$ as defined in (1). The solid and dashed lines represent the real and imaginary parts, respectively.

potential $V_{\rm e.m.}$ is the sum of the Coulomb potential and the Uehling and Källen–Sabry vacuum polarization potentials [3] folded with the nucleus density profile. The nuclear densities are taken from [4], where they are obtained by unfolding a gaussian proton charge distribution from the tabulated nuclear charge distributions [5]. We solve the Klein–Gordon equation (2) using the computational procedure of Krell and Ericson [6].

For the optical potential $U_{\rm opt}(r,\vec{\nabla})$, appearing on the right-hand side of (2), we make the following Ansatz:

$$U_{\rm opt}^{(i)} = U_{\rm opt}^{(0)} \; + \; V_i \,, \qquad 2\mu\, U_{\rm opt}^{(0)} = -4\pi \left(1+\frac{m_K}{m_N}\right) a\,[\rho(r)]\,\rho(r)\,,$$

$$\begin{aligned} 2\mu\, V_1 &= 4\pi \left(1+\frac{m_K}{m_N}\right) b[\rho(r)]\rho(r)\vec{\nabla}^2\,, \\ 2\mu\, V_2 &= 4\pi \left(1+\frac{m_K}{m_N}\right) b[\rho(r)]\vec{\nabla}\rho(r)\cdot\vec{\nabla}\,, \\ 2\mu\, V_3 &= 4\pi \left(1+\frac{m_K}{m_N}\right) b[\rho(r)][\vec{\nabla}^2\rho(r)]\,. \end{aligned} \tag{3}$$

In contrast to the approach of [7] we do not fit the spectrum. The effective scattering length $a[\rho]$ and the effective slope parameter $b[\rho]$ in (3) are determined by the expansion of the K^- self energy at small momenta (see (1)): we identify $a = a_{\rm eff}$ and $b = b_{\rm eff}/k_{\rm F}$. The optical potentials (3) follow from (1) with $\vec{q}$ replaced by the momentum operator $-i\vec{\nabla}$. Of course, this heuristic procedure is not unique and may lead to different ways of ordering of the gradients. For this reason we study three different cases separately.

Although our procedure of constructing the optical potential is not strict, it allows us to make an estimate of the magnitude of the possible non-local effects, usually neglected in other approaches. For all considered nuclei (for more details on our work we refer to the paper [8]) we observe the same type of the change in the spectrum: the effect of $U_{\rm opt}^{(1)}$ is small, $U_{\rm opt.}^{(2)}$ leads to a further (desired) increase of the widths, whereas $U_{\rm opt}^{(3)}$ makes the energy level shifts negative — in no way comparable to the data. In contrast to $U_{\rm opt}^{(1)}$, the effects of both $U_{\rm opt}^{(2)}$ and $U_{\rm opt}^{(3)}$ are very large. They change the binding energies and widths by hundreds of eV.

The best results are obtained for the optical potential $U_{\rm opt.}^{(2)}$. The non-local part of this potential leaves the energy shift almost unchanged but the widths become substantially larger as compared to the local potential $U_{\rm opt}^{(0)}$. This is an effect leading towards the proper spectrum. However, in our case the strength of b is too small to obtain a completely successful agreement with data.

A systematic derivation of the non-local part of the optical potential can be achieved by a semi-classical expansion of the nucleus structure. The starting point of this approach is the Klein–Gordon equation

$$\left[\vec{\nabla}^2 - \mu^2 + \Big(\omega - V_{\rm e.m.}(r)\Big)^2\right]\phi(\vec{r}\,) = 2\,\mu \int d^3r'\, U_{\rm opt}(\omega,\vec{r},\vec{r}\,')\,\phi(\vec{r}\,') \qquad (4)$$

with the non-local K^- nuclear optical potential $U_{\rm opt}(\vec{r},\vec{r}\,')$ and $\omega = \mu - E - \frac{i\,\Gamma}{2}$. For the details of the evaluation of $U_{\rm opt}(\omega,\vec{r},\vec{r}\,')$ we refer the reader to Ref. [8]. Here we can summarize our results. The semi-classical potential leads to substantial changes (100–250 eV) of the energy shifts and widths. The results differ from those obtained with the phenomenological potentials, indicating that a correct ordering of the gradients is crucial. The semi-classical potential is not able to reproduce the empirical level shifts satisfactorily. Thus, we conclude that an improved microscopic input for the many-body calculation of the kaon self energy is required. This is supported by the recent chiral SU(3) analysis of the kaon-nucleon scattering data which includes for the first time also p-wave effects systematically [9]. It is found that the subthreshold kaon-nucleon scattering amplitudes differ strongly from those of [2, 10] and that p-wave effects in the isospin one channel are strong. An evaluation of the many-body effects based on the improved chiral SU(3)-dynamics is in progress [11].

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