MODEL-INDEPENDENT CONFIRMATION OF THE σ-MESON BELOW 1 GeV and INDICATION FOR THE $f_0(1500)$ GLUEBALL* **

Yu.S. Surovtsev

Bogoliubov Laboratory of Theoretical Physics
Joint Institute for Nuclear Research
Dubna 141 980, Moscow Region, Russia

D. Krupa and M. Nagy

Institute of Physics, Slovak Academy Sciences
Dúbravská cesta 9, 842 28 Bratislava, Slovakia

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On the basis of a simultaneous description of the isoscalar s-wave channelof the $\pi\pi$ scattering (from the threshold up to 1.9 GeV) and of the $\pi\pi \to K\overline{K}$ process (from the threshold to $\sim$ 1.4 GeV) in the model-independent approach, a confirmation of the σ-meson at $\sim$ 660 MeV and an indication for the glueball nature of the $f_0(1500)$ state are obtained.

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1. We are going to demostrate in the model-independent approach that the large background (*e.g.*, that happens in analyzing $\pi\pi$ scattering), can hide low-lying states, even such important for theory as a σ-meson. Recent new analyses of the old and new experimental data found a possible candidate for that state [1]. However, their results are model-dependent. A model-independent information on multichannel states can be obtained on the basis of the first principles (analyticity

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and unitarity) immediately applied to analyzing experimental data. The way of realization is a consistent allowance for the nearest singularities on all sheets of the Riemann surface of the S-matrix [2]. Here we restrict ourselves to a 2-channel simultaneous consideration of the processes $\pi\pi \to \pi\pi, K\overline{K}$, *i.e.*, we have the S-matrix determined on the 4-sheeted Riemann surface. The matrix elements $S_{\alpha\beta}$, where $\alpha, \beta = 1(\pi\pi), 2(K\overline{K})$, have (on the real axis of the s-plane) the right-hand cuts, starting at $4m_\pi^2$ and $4m_K^2$, and the left-hand cuts, starting at $s = 0$ for S_{11} and at $4(m_K^2 - m_\pi^2)$ for S_{22} and S_{12}. We number the Riemann-surface sheets according to the signs of analytic continuations of the channel momenta $k_1 = (s/4 - m_\pi^2)^{1/2}$, $k_2 = (s/4 - m_K^2)^{1/2}$ as follows: signs $(\mathrm{Im}k_1, \mathrm{Im}k_2) = ++, -+, --, +-$ correspond to sheets I, II, III, IV. To elucidate the resonance representation on the Riemann surface, we express analytic continuations of the matrix elements to the unphysical sheets $S_{\alpha\beta}^L$ (L = II, III, IV) in terms of them on the physical sheet $S_{\alpha\beta}^I$. The latter have, except for the real axis, only zeros corresponding to resonances. These formulae give the resonance representation by poles and zeros on the Riemann surface. In the 2-channel case, one must discriminate between three types of resonances described by a pair of conjugate zeros on sheet I: (**a**) in S_{11}, (**b**) in S_{22}, (**c**) in each of S_{11} and S_{22}. A resonance of every type is represented by a pair of complex-conjugate clusters (of poles and zeros on the Riemann surface) of the size typical of strong interactions. The cluster kind is related to the state nature. The resonance, coupled strongly with the $\pi\pi$ channel, is described by the cluster of type (**a**); the resonance, coupled strongly with the $K\overline{K}$ and weakly with $\pi\pi$ channel (say, an $s\bar{s}$ state), by the cluster of type (**b**); the flavour singlet (*e.g.* glueball) must be represented by the pole cluster of type (**c**) [2].

2. To take account of the right-hand ($s = 4m_\pi^2$ and $s = 4m_K^2$) and left-hand ($s = 0$) branch-points, we use the uniformizing variable

$$v = \frac{m_K\sqrt{s - 4m_\pi^2} + m_\pi\sqrt{s - 4m_K^2}}{\sqrt{s(m_K^2 - m_\pi^2)}} \tag{1}$$

which maps the 4-sheeted Riemann surface onto the v-plane, divided into two parts by a unit circle centered at the origin. The sheets I (II), III (IV) are mapped onto the exterior (interior) of the unit disk on the upper and lower v-half-plane, respectively. The physical region extends from the point i on the imaginary axis ($\pi\pi$ threshold) along the unit circle clockwise in the 1st quadrant to point 1 on the real axis ($K\overline{K}$ threshold) and then along the real axis to point $b = \sqrt{(m_K+m_\pi)/(m_K-m_\pi)}$ into which $s=\infty$ is mapped on the v-plane. The intervals $(-\infty,-b],[-b^{-1},b^{-1}],[b,\infty)$ on the real axis are the images of the corresponding edges of the left-hand cut of the $\pi\pi$-scattering amplitude. The type (**a**) resonance is represented in S_{11} by two pairs of the poles on the images of the sheets II and III, symmetric to each other with respect to the imaginary axis, by zeros, symmetric to these poles with respect to the unit circle.

The variable v is uniformizing for the $\pi\pi$-scattering amplitude, however, the amplitudes of the $K\overline{K}\to\pi\pi, K\overline{K}$ processes do have the cuts on the v-plane which arise from the left-hand cut on the s-plane, starting at $s = 4(m_K^2-m_\pi^2)$. This left-hand cut will be neglected in the Riemann-surface structure, and the contribution on the cut will be taken into account in that part of the $K\overline{K}$ background which does not contribute to the $\pi\pi$-scattering amplitude, namely, as a pole on the real s-axis on the physical sheet in the sub-$K\overline{K}$-threshold region. For the simultaneous analysis of the experimental data on the processes $\pi\pi\to\pi\pi, K\overline{K}$ in the channel with $I^GJ^{PC}=0^+0^{++}$, we use the Le Couteur–Newton relations [2] expressing the S-matrix elements of all coupled processes in terms of the Jost matrix determinant $d(k_1,k_2)$, the real analytic function with the only branch-points at $k_i=0$. On the v-plane, the Le Couteur–Newton relations are

$$S_{11}=\frac{d(-v^{-1})}{d(v)},\quad S_{22}=\frac{d(v^{-1})}{d(v)},\quad S_{11}S_{22}-S_{12}^2=\frac{d(-v)}{d(v)} \tag{2}$$

with the d-function that on the v-plane already does not possess branch-points is taken as $d=d_Bd_{\rm res}$ with d_B taken to contain that part of the $K\overline{K}$ background which does not contribute to the $\pi\pi$-scattering amplitude: $d_B=v^{-1}(1-pv)(1+p^*v)$; $d_{\rm res}(v)$ represents the contribution of resonances, described by one of three types of the clusters:

$$d_{\rm res}=v^{-M}\prod_{n=1}^{M}(1-v_n^*v)(1+v_nv), \tag{3}$$

where M is the number of pairs of the conjugate zeros.

For a satisfactory description ($\chi^2/\text{ndf}\approx 2.2$) of the s-wave $\pi\pi$ scattering from the threshold to 1.89 GeV and $|S_{12}|$ to 1.4 GeV, three resonances turned out to be sufficient: $f_0(660)$ and $f_0(980)$ of the type (**a**), and $f_0(1500)$ of the type (**c**). A satisfactory description of the phase shift of the S_{12} element is obtained to 1.5 GeV with the parameter $p=0.993613-0.112842i$ (this corresponds to the pole on the s-plane at $s=4(m_K^2-m_\pi^2)-0.06$). In the table, the obtained poles on the corresponding sheets of the Riemann surface are cited on the complex energy plane ($\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r$). It is reasonable to report just characteristics of pole clusters which must be rather stable for various models unlike masses and widths, very model-dependent for wide resonances. We determine also the constants of the state couplings with

	$f_0(660)$		$f_0(980)$		$f_0(1500)$	
Sheet	E, MeV	Γ, MeV	E, MeV	Γ, MeV	E, MeV	Γ, MeV
II	600±14	620±26	990±5	25±10	1480±15	400±35
III	720±15	6±2	984±16	200±32	1540±25	300±35
					1530±24	400±38
IV					1500±22	300±34

the $\pi\pi$ – "1" and $K\overline{K}$ – "2" systems calculated through the residues of amplitudes at the pole on sheet II. Taking $T_{ij}^{\mathrm{res}} = \sum_r g_{ir} g_{rj} D_r^{-1}(s)$, where $D_r(s)$ is an inverse propagator ($D_r(s) \propto s - s_r$), we obtain (in GeV) for $f_0(660)$: $g_1 = 0.7376 \pm 0.12$ and $g_2 = 0.37 \pm 0.1$, for $f_0(980)$: $g_1 = 0.158 \pm 0.03$ and $g_2 = 0.86 \pm 0.09$, for $f_0(1500)$: $g_1 = 0.347 \pm 0.028$.

We calculated scattering lengths. For the $K\overline{K}$ scattering, we obtain (in $m_{\pi^+}^{-1}$.) $a_0^0(K\overline{K}) = -0.932 \pm 0.11 + (0.706 \pm 0.09)i$. For the $\pi\pi$ scattering, we obtain: $a_0^0 = 0.27 \pm 0.06$. Compare with the other experimental value 0.26 ± 0.05 [1]) and with the theoretical ones 0.16 (Weinberg [3], current algebra (non-linear σ-model)); 0.20 (Gasser, Leutwyler [3], non-linear realization of chiral symmetry); 0.26 (Volkov [3], linear realization of chiral symmetry).

In summary:

1. an existence of the low-lying state with the properties of the σ-meson and the obtained $\pi\pi$-scattering length seems to suggest the linear realization of chiral symmetry;

2. a parameterless description of the $\pi\pi$ background in the channel with $I^G J^{PC} = 0^+0^{++}$ is first given;

3. the $f_0(1500)$ state is represented by the pole cluster corresponding to a flavour singlet, *e.g.*, the glueball;

4. a minimum scenario of the simultaneous description of the processes $\pi\pi \to \pi\pi, K\overline{K}$ in the channel with $I^G J^{PC} = 0^+0^{++}$ does not require the $f_0(1370)$ resonance; therefore, if this meson exists, it must be weakly coupled with the $\pi\pi$ channel, *e.g.*, to be the $s\bar{s}$ state.

We emphasize that the obtained results are model-independent, since they are based on the first principles and on the mathematical fact that a local behaviour of analytic functions, determined on the Riemann surface, is governed by the nearest singularities on all sheets.

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