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COUPLED-CHANNEL FINAL-STATE INTERACTIONS IN $B \to \pi\pi$, $K\bar{K}$ DECAYS*

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We discuss coupled-channel final-state-interaction effects for B weak decays into $\pi\pi$ and $K\bar{K}$. In particular, it is shown that in the isospin $I=0$ channel the inelastic final-state rescattering process $(\pi\pi)_{I=0} \to (K\bar{K})_{I=0}$ strongly affects the phase of the $B^0 \to (K\bar{K})_{I=0}$ amplitude.

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The Cabibbo–Kobayashi–Maskawa (CKM) mechanism of CP violation may be tested experimentally by checking whether the "observed" CKM matrix is unitary, ie. by verifying whether the "unitarity triangle" constructed from complex elements of this matrix in fact is a triangle. It is hoped that the angles of this triangle can be determined from the analysis of nonleptonic decays of B mesons. The following problem can be noticed immediately: the angles to be extracted are defined at quark level, but they are to be determined from decays in which strong final state interactions (FSI) are possible. Since these interactions may affect the extraction of standard model parameters, they must be analysed carefully.

One of the three angles of the unitarity triangle (angle β, *i.e.* phase of CKM element $V_{\rm td}$) is expected to be accessible through measurement of the time-dependent CP-violating asymmetry in $B(\overline{B}) \to J/\psi K_S$, which is proportional to $\sin 2\beta$. The angle β enters here from the B^0–$\overline{B}^0$ mixing. The $B(\overline{B}) \to J/\psi K_S$ decay itself is described by a single quark-level diagram with no quark-level phases. In addition, no troublesome (flavour exchanging) final-state interactions may appear here. Consequently, extraction of the value of β is considered fairly reliable.

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Extraction of the remaining two angles is hampered in various ways. Analysis of $B \to \pi^+\pi^-$ was suggested as a way to determine a second angle (α). The relevant tree diagram involves a $b \to u$ transition, thus introducing an additional phase (phase γ of element V_{ub}). Together with angle β entering into consideration from the B^0–$\overline{B}^0$ mixing, the relevant asymmetry becomes proportional to $\sin(2(\beta+\gamma)) = \sin(2(\pi-\alpha))$. Unfortunately, the tree diagram is not the only one important here. Contributions from the short-distance penguin diagram are estimated at the level of 20%. The net result depends on the relative size and phase of the tree and penguin amplitudes, a problem known as "penguin pollution".

Although in principle one could disentangle the tree and penguin amplitudes by analysing the decays of B into $\pi^+\pi^-$ and $\pi^0\pi^0$, experimental difficulties with measuring the latter decays make this analysis unfeasible at present. An alternative of using decays into $K^0\overline{K}^0$ in place of $\pi^0\pi^0$ and exploiting the SU(3) symmetry has been suggested. However, there still remains the question of the effect of rescattering in the final state. Importance of these final state interactions has been stressed by many authors [1]. Unfortunately, there are severe problems with their reliable treatment. Usually only some intermediate states are taken into account. Because of large B meson mass, a Regge-model-based approach is often used here to estimate high-energy interactions between decay products. Recently, a simple model of this type has been applied to the description of strong phases in $D(B) \to K\pi$ and $D(B) \to \pi\pi$, $K\bar{K}$ decays [3–5].

The model of Refs. [3–5] is based on a quasi-elastic approximation. This approximation considers rescatterings of the type: $K\pi \to K\pi$, $\pi\pi \to \pi\pi$, *etc.* All other possible final-state interactions, in particular coupled-channel effects of the type $\pi\pi \to K\bar{K}$ or $K\pi \to K\eta$, are ignored. Since inclusion of the latter processes has been shown to be very important in maintaining SU(3) symmetry predictions [2], one has to analyse their influence on the predictions of the quasi-elastic Regge approach of Ref. [4, 5].

Such an analysis was recently performed [6]. In papers [4, 5] SU(3) symmetry was broken. For technical reasons and simplicity, in the coupled-channel approach [6] we had to keep SU(3) symmetry unbroken. We checked that in the no-coupled-channels case this yields a good approximation to the results of Ref. [5]. We restricted ourselves to the analysis of coupled-channel effects in the $\pi\pi$, $K\bar{K}$, $\pi^0\eta_8$ and $\eta_8\eta_8$ channels. The latter two were included because they are needed to maintain SU(3) symmetry of the analysis.

In order to evaluate the effects of FSI in B meson decays, we have to write down short-distance amplitudes for B decays, estimate the amplitudes for the relevant transitions of the strong interaction S-matrix, and finally combine them both to obtain the FSI-corrected amplitudes for B meson decays. The short-distance amplitudes for B^0 decays may be parametrized as

$$\begin{aligned}\langle(\pi\pi)_{I=2}|w|B^0\rangle &= -\frac{1}{\sqrt{6}}(a+b)\,,\\ \langle(K\bar{K})_{I=1}|w|B^0\rangle &= \frac{1}{2}(c-e)\,, \quad \textit{etc.}\,,\end{aligned} \tag{1}$$

where the expressions for the decays to the remaining four channels ($(\pi^0\eta_8)_{I=1}$, $(\pi\pi)_{I=0}$, $(K\bar{K})_{I=0}$, $(\eta_8\eta_8)_{I=0}$) are suppressed and the quark-level amplitudes are denoted by a (tree), b (colour-suppressed), c (W-exchange), e (penguin).

Final state interactions between the pair of pseudoscalar mesons are evaluated with the help of a Regge approach. As in Refs. [4, 5], we take into account the exchange of the Pomeron and of the exchange-degenerate Reggeons ρ, f_2, ω, and a_2 (Fig. 1). The "uncrossed" diagram (1a) has phase $-\exp(-i\pi\alpha_R(t))$ with $\alpha_R(t) = 0.5 + \alpha' t$, $\alpha' \approx 1$ GeV^{-2}, while the "crossed" diagram (1b) has phase -1. We describe the method of FSI evaluation used in [6] on an example of the $I=1$ sector, with two FSI-coupled states: $(K\overline{K})_{I=1}$ and $(\pi^0\eta_8)_{I=1}$. The FSI are in this case described by two 2×2 matrices $\boldsymbol{C}_{I=1}$ and $\boldsymbol{U}_{I=1}$ of SU(3)-symmetric Regge residues, one matrix for each type of exchange (Crossed and Uncrossed).

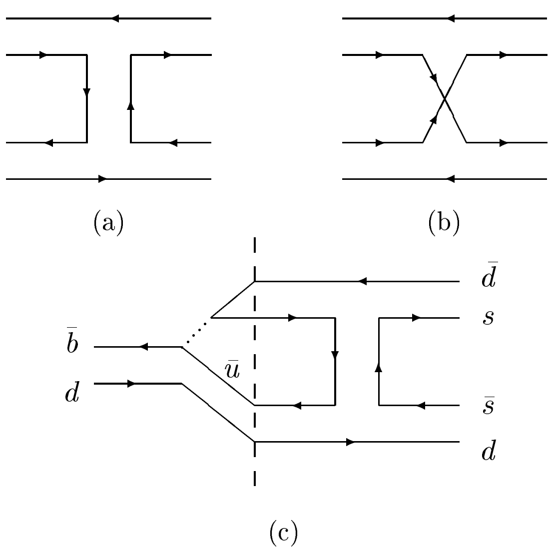

Fig. 1. Quark line diagrams: (a) Uncrossed Reggeon exchange, (b) Crossed Reggeon exchange, (c) Contribution to $(\pi\pi)_0 \to (K\bar{K})_0$ final-state interaction.

It turns out that matrices $\boldsymbol{U}_{I=1}$ and $\boldsymbol{C}_{I=1}$ commute and, consequently, their simultaneous diagonalization is possible. The relevant eigenvectors transform as an octet and a **27**-plet of SU(3); the corresponding eigenvalues may be denoted by $\lambda_{\rm U}(\boldsymbol{R})$ and $\lambda_{\rm C}(\boldsymbol{R})$, with $\boldsymbol{R} = \mathbf{8}, \mathbf{27}$. Full amplitudes $A(\boldsymbol{R})$ corresponding to Reggeon exchanges are obtained by multiplying these eigenvalues by an appropriate Regge phase and by a factor $Rs^{\alpha_R(t)}$, where R is the Regge residue. When one takes into account the Pomeron contribution, the complete amplitudes are given by:

$$A(\boldsymbol{R}) \;=\; i\beta_P s^{\alpha_P(t)} + Rs^{\alpha_R(t)}[\lambda_{\rm U}(\boldsymbol{R})\mathrm{e}^{-i\pi\alpha_R(t)} + \lambda_{\rm C}(\boldsymbol{R})]\,. \tag{2}$$

The parameters $R = -13.1$ mb and $\beta_P = 9.9$ mb are extracted from experimental total cross sections of πp, $\pi\overline{p}$, pp, $p\overline{p}$ *etc.* collisions. After projecting the amplitudes $A(\boldsymbol{R})$ onto the $l = 0$ partial wave one obtains the $l = 0$ partial wave amplitudes

$$a_0(\boldsymbol{R}) \;\propto\; \int\limits_{-\infty}^{0} dt A(\boldsymbol{R})\,. \tag{3}$$

It is the phases $\delta(\boldsymbol{R})$ of $a_0(\boldsymbol{R})$ that are of interest to us. (In the $I = 0$ sector there appears the third phase $\delta(\mathbf{1})$, corresponding to a third eigenvector — an SU(3) singlet).

Due to the coupled-channel effects, the FSI-corrected amplitudes $\langle(\pi\pi)_I|W|B^0\rangle$, $\langle(K\bar{K})_I|W|B^0\rangle$, etc. become linear combinations of appropriate short-distance quark-level amplitudes in Eq. (1), *i.e.*:

$$\langle(P_1P_2)_I|W|B^0\rangle = \sum_{(P_1'P_2')_I} \langle(P_1P_2)_I|D_{FSI}|(P_1'P_2')_I\rangle\langle(P_1'P_2')_I|w|B^0\rangle \tag{4}$$

with FSI described by

$$\begin{aligned}&\langle(P_1P_2)_I|D_{FSI}|(P_1'P_2')_I\rangle\\ &\quad=\sum_{\boldsymbol{R}}\langle(P_1P_2)_I|\boldsymbol{R},I\rangle\langle\boldsymbol{R},I|S_{\rm FSI}|\boldsymbol{R},I\rangle\langle\boldsymbol{R},I|(P_1'P_2')_I\rangle.\end{aligned} \tag{5}$$

In the above equation $\langle(P_1P_2)_I|\boldsymbol{R},I\rangle$ are the amplitudes for finding the $|(P_1P_2)_I\rangle$ state in the SU(3) eigenstate $|\boldsymbol{R},I\rangle$, while $\langle\boldsymbol{R},I|S_{\rm FSI}|\boldsymbol{R},I\rangle = \rho(\boldsymbol{R})\exp(i\delta(\boldsymbol{R}))$ describes SU(3)-symmetric final state interactions in the $|\boldsymbol{R},I\rangle$ state.

It is assumed that the possible renormalization of the *magnitude* of short-distance amplitudes is negligible: $\rho(\boldsymbol{R}) \approx 1$. Even with this simplifying assumption, in order to estimate how the quark-level amplitudes a, b, c

etc. add up, one has to make additional assumptions about their relative size. It is expected that the dominant contribution is provided by the tree amplitude a. Contributions from other amplitudes are expected to be much smaller [7]. We use $a = 3rb$ with $r \approx -3.0$ as appropriate for the QCD-corrected quark model [9]. Assuming furthermore e/a in the range of 0.04–0.20 (as estimated in [8]), disregarding the relative phase of e and a, and neglecting contributions from other diagrams in Eq. (1), all amplitudes are given in terms of a single parameter a. It is now straightforward to estimate the FSI phases. From Eq. (4) one then obtains the numbers given in the right-hand side of Table I.

TABLE I

Comparison of calculated phase shift values for B decays

Phase	No coupled channels			Coupled channels, Ref. [6]		
	Ref. [5]	Ref. [6]			$b = a/(3r)$, $r = -3$	
		$\epsilon = 0$	$\epsilon = 1$	$e \gg a, b$	$e = 0.2a$	$e = 0.04a$
$\delta_\pi^2 - \delta_\pi^0$	$+11° \pm 2°$	18°	18°	14°	18°	19°
δ_K^1		90°	83°	85°	85°	85°
δ_K^0		100°	103°	110°	137°	168°
$\delta_K^1 - \delta_K^0$	$-7° \pm 1°$	−10°	−20°	−25°	−52°	−83°

It is seen from Table I that phase difference between the $(\pi\pi)_{I=0,2}$ channels does not depend very strongly on the inclusion of coupled-channel effects. This is also the case for the $(K\bar{K})_1$ phase. However, the $(K\bar{K})_0$ phase changes dramatically when coupled-channel effects are included. Only for $a, b \ll e$ is the phase change induced by coupled-channel effects small. However, if e is small (*i.e.* in the "penguin pollution" case), the relevant phase change is large. The origin of this effect can be understood from Fig. 1: due to coupled-channel effects, the $(\pi\pi)_0$ state created in a short-distance decay process may be converted into a $(K\bar{K})_0$ final state. Thus, the final $(K\bar{K})_0$ state receives contribution from both the short-distance $B^0 \to (K\bar{K})_0$ decay (characterized by small amplitude e) with $(K\bar{K})_0$ elastically scattered into the final $(K\bar{K})_0$ state, as well as from the short-distance $B^0 \to (\pi\pi)_0$ decay (characterized by large amplitude a) which contributes through coupled-channel effects into the final $(K\bar{K})_0$ state. The net result is interference of the small e amplitude with some admixture coming from the large a amplitude. If the relative size of the two contributions is comparable, one can obtain a wide range of phases for their sum. This lies at the origin of large phases in the $(K\bar{K})_0$ channel of B^0 decays (Table I). Thus, in spite of a relative weakness (as compared to elastic rescattering) of strangeness-exchanging Reggeon contribution at $s = m_B^2$, the generated FSI phase may be large.

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