Vol. 31 (2000) ACTA PHYSICA POLONICA B No 12

NUCLEON–QUARK PHASE TRANSITION IN NEUTRON STARS*

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(Received September 19, 2000)

The formation and the structure of a mixed quark–nucleon phase in neutron star cores are studied, for different models of the nuclear symmetry energy. Simple parametrizations of the nuclear matter equation of state and the bag model for the quark phase are used. For lower values of the bag constant B the properties of the mixed phase do not depend strongly on the symmetry energy. For larger B we find that a critical pressure for the first quark droplets to form in the nucleon medium is strongly dependent on the nuclear symmetry energy, but the pressure at which last nucleons disappear is independent of it.

PACS numbers: 26.60.+c, 21.65.+f, 97.60.Jd, 12.38.Mh

1. Introduction

Properties of neutron stars result from the Equation Of State (EOS) of dense nuclear matter. The present knowledge of EOS is quite uncertain, especially at densities considerably exceeding the saturation density of nuclear matter $n_0 = 0.17\ \mathrm{fm}^{-3}$. Generally, it is expected that pure nucleon matter existing at the base of a neutron star's crust changes in some new form of matter with increasing density of a core. The exact form of this matter, however, is unknown. Depending on the theoretical approach it may be *e.g.* condensate of pions or kaons, hyperonic matter or quark matter. The existence of the latter kind of matter inside neutron stars is expected as a result of the deconfinement transition which should take place when the density of nuclear matter is sufficiently high.

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* Presented by J. Niemiec at the XL Cracow School of Theoretical Physics, Zakopane, Poland, June 3–11, 2000.

Recently, Glendenning [1] has shown that the proper construction of the nucleon–quark phase transition inside neutron stars must take into account the condition that the two independent quantum numbers carried by components of the neutron star matter, namely the baryon number and the electric charge, have to be only globally conserved during the phase transition. When instead the local charge neutrality is applied to the system, general conditions of equilibrium cannot be satisfied. Imposition of the global charge neutrality allows for a nonuniform distribution of the baryon number and the electric charge in two phases creating an additional degree of freedom. Exploitation of this degree of freedom depends on the proportion of phases in equilibrium and in consequence leads to the coexistence of nucleon matter and quark matter over a finite range of pressure. This has the effect that a core, or a spherical shell, of a mixed quark–nucleon phase can exist inside neutron stars. The fraction of space occupied by quark matter smoothly increases from zero at the core boundary, where pressure reaches a critical value for the first quark droplets formation in the nucleon medium, to unity when eventually the last nucleons dissolve into quarks. In addition, nonequal division of the electric charge between two phases leads to the appearance of a geometrical structure in the mixed phase. It has been shown by Heiselberg *et al.* [2] that the structured mixed phase remains the ground state of the neutron star matter if only physically reasonable values of surface tension are assumed.

In the original construction due to Glendenning, nucleon matter was treated in the Relativistic Mean Field (RMF) model. The nuclear symmetry energy in this model increases monotonically with the baryon number density [3]. This feature is in contrast to several Variational Many-Body (VMB) calculations of the equation of state of nuclear matter [4]. These models use phenomenological nucleon–nucleon potentials and hence can be regarded as more realistic. They predict the symmetry energy to saturate and then to decrease to negative values at high baryon number densities. This discrepancy in high density behavior of the nuclear symmetry energy leads to serious uncertainty about some astrophysically important properties of the neutron star matter. Here we study the consequences of this uncertainty for the formation and structure of a mixed quark–nucleon phase in neutron star cores.

2. Mixed quark–nucleon phase

In the construction of the phase transition the neutron star matter is assumed to be cold, β-stable and globally charge neutral. The geometry of droplets is neglected in these calculations since inclusion of finite-size effects results in only small corrections to the equation of state, which do not change considerably the results of this simple approach. We discuss these effects in our paper [5], where the reader can find more details of the calculation method and the analysis of the properties of the phase transition is also given there. Some most important results regarding finite-size effects will be shortly discussed at the end of the paper.

In the case where geometrical effects are neglected the equilibrium conditions are those for bulk systems. The coexistence of the quark matter with the nucleon medium under given pressure p requires equality of chemical potentials in both phases. In the case of the neutron star matter there are two independent chemical potentials representing two globally conserved quantum numbers. Both are functions of pressure which is chosen as an independent variable. Hence, the equilibrium conditions are

$$\mu_N^n(p) \;=\; \mu_N^q(p)\,, \tag{1}$$
$$\mu_P^n(p) \;=\; \mu_P^q(p)\,, \tag{2}$$

where μ_N^n and μ_N^q are the neutron chemical potentials in the nucleon and the quark phase, respectively. Similarly, μ_P^n and μ_P^q are the proton chemical potentials in respective phases. The coexistence conditions become complete by adding the β-equilibrium equation

$$\mu_N^i - \mu_P^i = \mu_e\,, \qquad i = n, q\,, \tag{3}$$

where μ_e is the electron chemical potential, and n and q refer to the nucleon and the quark phases, respectively. From Eq. (3) and the formula for the electron chemical potential

$$\mu_e = (3\pi^2 n_e)^{1/3} \tag{4}$$

we obtain the density of electrons n_e. We assume them to be uniformly distributed throughout both phases.

Solutions of Eqs. (1)–(3) provide the densities of protons n_P in the nucleon phase and the electric charge density of quarks n_q in units of e. The global charge neutrality condition requires that

$$V n_e = V_n n_P + V_q n_q\,, \tag{5}$$

where V_n is the volume occupied by nucleons and V_q is the volume of quarks. Since the total available volume is $V_n + V_q = V$, we can define a quantity $\alpha = V_n/V$, which is the fraction of space containing nucleons. From Eq. (5) we obtain α in the form

$$\alpha = \frac{n_e - n_q}{n_P - n_q}\,. \tag{6}$$

The quarks occupy a complementary fraction, $1-\alpha$, of the volume.

At a sufficiently low pressure free quarks are absent in the neutron star matter and $\alpha = 1$. The first quark droplets form at the lower critical pressure p_i. It corresponds to α starting to deviate from unity for the first time. With increasing pressure, more space is filled with the quark matter and $\alpha < 1$. Quarks coexist with the nucleon matter up to the upper critical pressure p_f at which nucleons finally disappear and $\alpha(p_f) = 0$.

3. Nuclear and quark matter models

To implement the above construction in practical calculations specific models of the nucleon matter and the quark matter have to be used. The nuclear matter equation of state yielded by variational many-body calculations can be simply parametrized as a function of the baryon number density n and the proton fraction $x = n_P/n$ [6]. The energy per particle is expressed as

$$E(n,x) = T(n,x) + V_0(n) + (1-2x)^2 V_2(n)\,, \tag{7}$$

where $T(n,x)$ is the kinetic energy contribution and $V_0(n)$ and $V_2(n)$ functions represent the interaction energy contributions. From Eq. (7) the pressure $p(n,x)$ and chemical potentials of neutrons and protons $\mu_N^n(n,x)$ and $\mu_P^n(n,x)$ can be calculated. The function $V_2(n)$ is the most important component in this expression as it is responsible for the high density behavior of the nuclear symmetry energy,

$$E_{\rm sym}(n) = \tfrac{5}{9} T\left(n, \tfrac{1}{2}\right) + V_2(n)\,, \tag{8}$$

which influence on the mixed phase properties we study.

As an example of variational many-body calculations, we use the EOS with the UV14+TNI interactions from Ref. [4]. The corresponding function $V_2(n)$ is presented in Fig. 1. One should note that with this $V_2(n)$ the symmetry energy Eq. (8) reproduces the empirical value, $E_{\rm sym}(n_0) = 34 \pm 4$ MeV [7]. At higher densities, $E_{\rm sym}(n)$ saturates and then decreases, reaching negative values for $n > 1.0~{\rm fm}^{-3}$.

In Fig. 1 there is also shown the linear growth of the function $V_2(n)$ corresponding to the RMF approach. The energy per particle of this model can also be cast in the form (7) [8], with the function $V_2(n)$ expressed by

$$V_2(n) = \frac{1}{8}\frac{g_\rho^2}{m_\rho^2} n \tag{9}$$

in this case. The linear dependence of V_2 on the baryon number density gives the monotonical growth of the nuclear symmetry energy with n in the

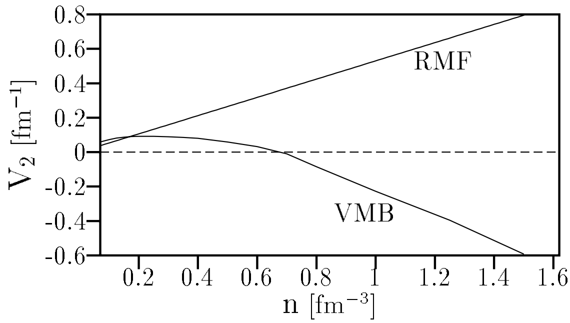

Fig. 1. The interaction energy $V_2(n)$ as a function of the baryon number density, for the VMB and RMF models.

RMF model. As we are concerned here mainly with the role of the symmetry energy, we model the energy per particle corresponding to the RMF theory using the function $V_2(n)$ in the form (9) and keeping other contributions in Eq. (7) the same as in the VMB case. We also adjust the coupling parameter g_ρ^2/m_ρ^2 so as to fit the empirical value of the nuclear symmetry energy at the saturation point.

Fig. 2 demonstrates how the discrepancy in the high density behavior of the symmetry energy predicted by both class of models implies uncertainty in the proton fraction of the neutron star matter, a crucial parameter in physics of neutron stars. For the case of the β-stable, pure nucleon matter it is fully determined by the function $V_2(n)$ [5]. As is shown in Fig. 2, the RMF model predicts that $x(n)$ monotonically increases with the density, whereas for $V_2(n)$ corresponding to the UV14+TNI interactions the proton fraction decreases with n and eventually protons disappear completely at some density.

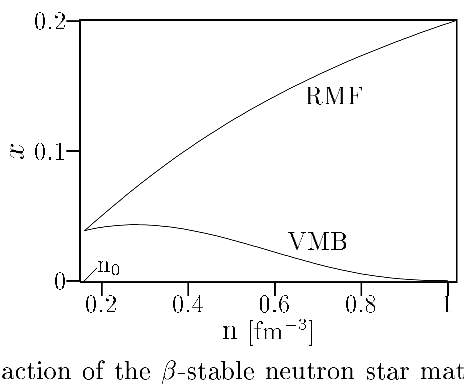

Fig. 2. The proton fraction of the β-stable neutron star matter corresponding to the interaction energy $V_2(n)$ in Fig. 1, for VMB and RMF models.

The quark matter is described by a simple bag-model equation of state. With the assumption of neglecting bare masses the energy density for three flavors is

$$\varepsilon_q = \frac{3}{4}\pi^{2/3}\left(n_u^{4/3} + n_d^{4/3} + n_s^{4/3}\right) + B\,, \tag{10}$$

where $n_i, i = u, d, s$ are the quark number densities. The exact value of the bag constant B is not well known. We treat it here as a phenomenological parameter and show results corresponding to two values of the bag constant, $B = 120$ MeV/fm^3 and $B = 200$ MeV/fm^3.

The quark chemical potentials are

$$\mu_i = \pi^{2/3} n_i^{1/3}\,, \qquad i = u, d, s\,. \tag{11}$$

Since the β equilibrium condition requires equality of the chemical potentials of down and strange flavors, for a given pressure, there are only two independent quark chemical potentials. They are explicitly related to the proton and neutron chemical potentials in the quark phase [5].

4. Results and implications for neutron stars

To solve the equilibrium conditions (1) and (2) we construct isobars for the nucleon and quark matter in the μ_P–μ_N plane [5]. The example is shown in Fig. 3 for the pressure value $p = 100$ MeV/fm^3. For the nucleon phase we parametrize isobars by the proton fraction x, whereas for the quark phase by the down quark chemical potential. The coexistence conditions are fulfilled at the crossing point of the nucleon and quark isobars. This point indicates all quantities necessary for the construction of the phase transition including also the density of the homogeneous electron background n_e, since the

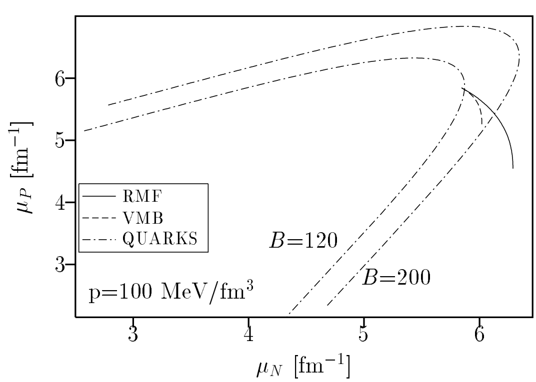

Fig. 3. VMB and RMF nucleon isobars, and quark isobars for two values of the bag constant, for pressure $p = 100$ MeV/fm^3.

β-equilibrium condition (3) is satisfied at this point with the electron chemical potential $\mu_e = \mu_N - \mu_P$. For various values of pressure the coordinates of the crossing point in the μ_P–μ_N plane, and together with them the properties of the coexisting phases, change.

The results of the construction of the nucleon–quark phase transition are displayed in Fig. 4. For VMB and RMF models and for both values of the bag constant the fraction of space occupied by nucleons smoothly changes from unity at the lower critical pressure p_i to zero at the upper critical one, p_f. The lower critical pressure p_i is a pressure at which the proton fraction of the nucleon matter at the crossing of the isobars coincides with that of the β-stable neutron star matter at this pressure. For $B = 120$ MeV/fm^3, it is $p_i = 2$ MeV/fm^3 and $p_i = 3$ MeV/fm^3 for VMB and RMF models, respectively. The lower critical pressure corresponds to the formation of the first quark droplets in the nucleon medium. At higher pressure the quark matter coexists with nucleons and the fraction of volume filled with quarks gradually increases. At the upper critical pressure p_f the last nucleon droplets immersed in quark matter finally dissolve. It has a common value for both VMB and RMF isobars. In the case of $B = 120$ MeV/fm^3, $p_f = 115$ MeV/fm^3.

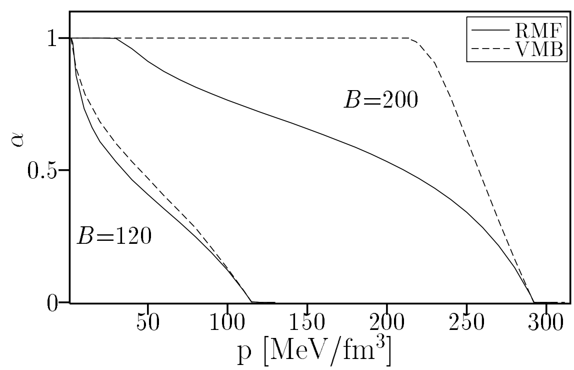

Fig. 4. The fraction of the mixed phase volume filled with nucleons as a function of pressure for VMB and RMF models and for both values of B.

For $B = 200$ MeV/fm^3, the lower critical pressure for the VMB and RMF isobars is, respectively, $p_i = 215$ MeV/fm^3 and $p_i = 35$ MeV/fm^3. The value of the upper critical pressure is $p_f = 290$ MeV/fm^3.

The results presented in Fig. 4 prove that the properties of the nucleon–quark phase transition are very sensitive to the behavior of the nuclear symmetry energy only for higher values of the bag constant. This is because, generally, the phase transition occurs at a higher pressure for higher values of B, and the VMB and RMF isobars differ much more at high values of pressure (see Figs. 5 and 6 in Ref. [5]). For low values of the bag constant, the phase transition starts at a low enough pressure for the nuclear symmetry energy not to affect the isobars significantly.

In Fig. 5 we show the proton fraction of nucleon matter coexisting at a given pressure with quark matter, as a function of the mean baryon number density $\bar{n} = \alpha n + (1-\alpha)n^Q$, where $n^Q = (n_u + n_d + n_s)/3$ is the baryon number density of quark matter. The density corresponding to the lower critical pressure p_i at which the phase transition starts is $\bar{n}_i$. For $B = 120$ MeV/fm^3, $\bar{n}_i = 0.17$ fm^{-3} is approximately the same for both nuclear models. In the case of $B = 200$ MeV/fm^3, $\bar{n}_i = 0.84$ fm^{-3} and $\bar{n}_i = 0.35$ fm^{-3}, respectively, for the VMB and RMF models. The phase transition is completed at the density $\bar{n}_f$ corresponding to the upper critical pressure p_f. It is $\bar{n}_f = 0.8$ fm^{-3} and $\bar{n}_f = 1.39$ fm^{-3} for $B = 120$ MeV/fm^3 and $B = 200$ MeV/fm^3, respectively. One can notice that nucleon matter becomes more proton rich with increasing pressure irrespective of the nuclear symmetry energy. At the upper critical pressure p_f disappearing nucleon droplets for both nuclear models are composed of symmetric nuclear matter. In Ref. [1] this increase of the proton fraction with pressure was attributed to the particular form of the symmetry energy in the RMF approach. It was suggested there that the isospin properties of this model are responsible for the existence of the mixed quark–nucleon phase. As we show here, the phase transition from nucleon to quark matter occurs irrespective of the particular form of the nuclear symmetry energy and it is the behavior of nucleon matter isobars that allows the existence of the mixed phase.

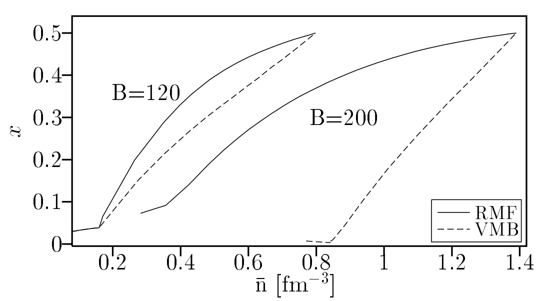

Fig. 5. The proton fraction of nucleon matter coexisting with quark matter as a function of the mean baryon number density.

In order to investigate the consequences of the existence of a mixed quark–nucleon phase for neutron stars we construct the equations of state for the models of matter with the mixed phase and for pure nucleon matter and solve models of neutron star structure. Curves presented on the plot of neutron star masses as functions of the central density (Fig. 6) correspond to

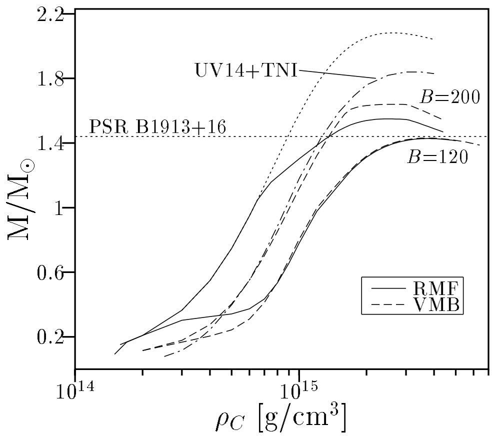

Fig. 6. Neutron star masses as functions of the central density. The dotted line corresponds to pure nucleon matter in the RMF model. The dash–dotted line is for pure nucleon VMB equation of state. Solid and dashed curves are for equations of state involving the mixed quark–nucleon phase. The horizontal line shows the empirical lower limit to the maximum neutron star mass.

the solutions of Tolman–Oppenheimer–Volkoff equations with a given EOS. The existence of quark matter in neutron star cores makes the equation of state softer. The maximum masses of neutron stars containing the mixed phase are smaller than those with the pure nucleon matter. The effect is stronger for low B, since the mixed phase comprises more mass of the star than for higher values of the bag constant. For $B \leq 120$ MeV/fm^3 the maximum mass is below the observational limit, so that these models cannot be realized in nature. For $B = 200$ MeV/fm^3 the maximum mass safely exceeds this limit and the influence of the symmetry energy is clearly visible in this case. As the phase transition in the RMF model starts at much lower pressure than in the VMB case, the maximum mass corresponding to the RMF approach is well below that for the VMB model. Correspondingly, the mixed phase cores of neutron stars of a given mass are much larger in the RMF model as compared with the VMB case.

The above results deal with the phase transition between two bulk systems. When the finite-size effects (*i.e.* the Coulomb interaction and the surface tension) are included in the calculations, a variety of geometric structures appears in the mixed phase. They are formed by regions filled with nucleons and quarks that have opposite electric charge density [9]. Sizes and shapes of these structures change with the fraction of space occupied by nucleons, α [5]. But, as it was mentioned already, the structured mixed phase is the ground state of the neutron star matter only for sufficiently small values of surface tension. We find that the range of $\sigma < 10$ MeV/fm^2 for the VMB model and $\sigma < 150$ MeV/fm^2 for the RMF one ($B = 200$ MeV/fm^3) is allowed for the mixed phase to be energetically favored [5]. Thus, the appearance of this phase, is dependent not only on the exact value of the surface tension, but it is also very sensitive to the form of the nuclear symmetry energy.

5. Discussion and conclusions

The scenario of the phase transition from nucleon to quark matter is connected with the formation of a mixed phase. The occurrence of this phase is irrespective of the particular form of the nuclear symmetry energy, but its properties strongly depend on the nuclear matter model. Properties of neutron stars also depend on it, in particular, the size of the mixed phase core is very sensitive to the form of the symmetry energy. A neutron star of the canonical mass $M = 1.44 M_{\odot}$, for $B = 200$ MeV/fm^3 and with the RMF symmetry energy, possesses a quark–nucleon core of ~ 7 km radius, whereas its total radius is ~ 12 km. In the VMB model, the star is composed entirely of nucleons. It is because in the latter case the pressure in the center of a star, $p_{\rm c} = 120$ MeV/fm^3, is below the lower critical value $p_i = 215$ MeV/fm^3 at which the phase transition begins. Thus conclusions concerning the presence of the quark matter in neutron stars are subject to some uncertainty due to incompatible model predictions of high density behavior of the nuclear symmetry energy.

Variational many-body calculations seem to be more realistic than relativistic models. Description of the interactions between nucleons in the case of the VMB approach is based on phenomenological nucleon–nucleon potentials concluded from experiments. The allowed range of surface tension is, however, very narrow for the VMB model and unless the true value of σ is very small the quark matter will not form inside neutron stars in this case.

The observational confirmation of the existence of a mixed phase in the cores of neutron stars is difficult. A structure of such object, without any discontinuity in the density of matter, will not rather manifest itself in observational data. However, it is potentially possible to detect the phase transition to quark matter proceeding in rotating neutron stars, which are observed as pulsars. During the course of spin-down, a pulsar can develop the mixed phase core. This would lead to characteristic changes in timing properties of pulsar's signal [10]. Moreover, the mixed phase is expected to be solid. Strains appearing in such a medium lead to the quake phenomena which are probably related to glitches observed in some pulsars [11]. The existence of the mixed quark–nucleon phase inside neutron stars can also strongly influence the cooling mechanisms [11].

This research was partially supported by the Polish State Committee for Scientific Research (KBN), under Grant No. 2 P03B 112 17.

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