DESCRIPTION OF NUCLEAR EXCITATIONS BEYOND THE MEAN FIELD*

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The properties of the coupling of mean field excitations to doorway states and to a chaotic background are discussed. Recent applications to the study of single-particle and giant resonance states in exotic nuclei and in fast rotating hot nuclei are presented.

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1. Introduction

Mean field is one of the most useful approximations in all of physics and chemistry. In it, the many-particle Schrodinger equation is replaced by a single-particle one. The many-body effects are embedded in the single-particle potential produced by the particles themselves, in particular the nuclear Hartree–Fock potential (HF). Excited states of the nuclear mean-field are (in general) collective vibrations of different multipolarity and carrying different spin–isospin quantum numbers. They are described in terms of correlated particle–hole excitations (Random Phase Approximation (RPA)). Often, use is done of effective interactions in a self-consistent approach, including the coupling to the continuum.

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The coupling between single-particle states and the excited states at the next level of complexity, the so called *doorway states* [1,2] represented by two-particle–one-hole ($2p$–$1h$) or $1p$–$2h$ states containing a collective vibration (in particular a surface vibration), renormalizes the properties of the single-particle motion. Eventually, a nucleon effective mass m^*/m and a finite lifetime (spreading width $\Gamma^{\downarrow}_{sp}$) are obtained, both determined by the "single-particle self-energy $\Sigma_\alpha(\omega)$".

Among the vibrational states are the Giant Resonances (GR), excited at energies higher than the nucleon separation energy with large cross section, close to the maximum allowed by sum rule arguments, implying that a large number of nucleons participate in a very coherent motion. Their properties as obtained in the HF+RPA approach are modified by the effects of the s.p. self-energy Σ, that is of the coupling to *doorway states* of (for the GR) $2p$–$2h$ character. In this, the coherence of the nuclear motion in the GR plays a special role in preserving the appropriate symmetries and sum rules. Thus, the spreading width $\Gamma^{\downarrow}_{\rm GR}$ is added to the fragmentation (Landau damping) and escape width $\Gamma^{\uparrow}$ to determine the total width of the GR to be compared with the experimental ones.

In the last decades, the *doorway coupling* approach was successful in describing s.p. and GR properties, including those of the Giant Dipole Resonance thermally excited on compound nuclei at high excitation energy and spin and of the double GDR (DGDR), although several problems of different type remain. This is well testified in the references (a selection) reported in [3–9]. The alert student will also find connections with concepts used in the lectures of J.-P. Blaizot, W. Cassing and V. Metag.

The success of the *doorway coupling* approach is justified by the smooth properties of the coupling to the chaotic background of many-particle-many-hole states in the compound nucleus (CN), which will not alter the main features of the strength functions obtained in the *doorway coupling* [2,10,11].

Rather than review in detail what described above, I will use my single lecture to discuss in Sect. 2 and 3, very recent calculations of the s.p. self-energy Σ in weakly bound nuclei and in deformed, very fast rotating nuclei as examples of problems on the frontier in this subfield of nuclear physics.

2. Doorway coupling in exotic nuclei

In this section, I discuss novel features of the physics discussed above in the exotic nuclei far from β-stability, (see, *e.g.*, the review articles in Ref. [12] and the lectures of W. Nazarewicz and of Y. Blumenfeld) in connection with the unusual ratio between the number of protons and neutrons and with the small binding energies of the least bound nucleons. The first feature challenges the isovector properties of the existing effective interactions, and the knowledge of the novel properties of the giant resonances excited in such exotic nuclei [13] will be crucial, as discussed in, *e.g.* Ref. [14, 15]. The second feature requires a careful handling of the continuum, as briefly discussed below.

2.1. Mean field results for neutron-rich nuclei

The isotope ^{28}O was chosen as an example of neutron-rich nucleus in Ref. [16]. Is is double magic, if we assume that the usual magic numbers are still valid, by extrapolating somehow the HF results for nuclei along the valley of stability. Indeed, a HF calculation with one of the "standard" Skyrme effective interactions, the SIII parametrization [17], results in a value of the energy of the least bound $\mathrm{d}_{5/2}$ neutron of -1.1 MeV (a number of experimental evidences point to the non existence of this isotope as a bound system [18], while essentially all mean field theories predict it to be bound). The protons are all bound by more than 30 MeV. The small value of the neutron separation energy requires to properly include, in the calculation of the excited states, transitions from bound states to particle states lying in the continuum, by means of the so-called continuum-RPA. This can be exactly performed if one uses Green's functions defined in the coordinate space [19], and the strength function $P^{\mathrm{RPA}}(\omega)$ for a given excitation operator F

$$P^{\mathrm{RPA}}(\omega) = \sum_f F_{f0}^2 \delta(\omega - \omega_0) \tag{1}$$

can be calculated.

The large asymmetry between the neutron and proton mean fields has dramatic consequences on the multipole response, as remarked in Refs. [16, 20, 21]. Indeed, we have calculated [22] the strength functions associated with isoscalar quadrupole, octupole and isovector dipole operators in the continuum-RPA. The low-energy part, is characterized by a pronounced "threshold effect", that is, by a sudden increase of the strength function $P^{\mathrm{RPA}}(\omega)$ above ω_{th} [20–22]. On ground of simple arguments, it is expected that

$$P^{\mathrm{RPA}}(\omega) \sim (\omega - \omega_{\mathrm{th}})^{l'+1/2}, \tag{2}$$

where l' is the orbital angular momentum of the particle states contributing to $P^{\mathrm{RPA}}(\omega)$. Eventually, a well-defined bump is produced by the single-particle transitions from the $d_{3/2}$ neutron state to s- (in the case of quadrupole) or p-states (in the other two cases) in the continuum. Because of the small $d_{3/2}$ binding energy, the wave function of these neutrons is so extended that its overlap with continuum wave functions is large and this makes the matrix elements of the multipole operators quite large. On the other hand, the higher energy region of the strength functions is characterized by states to which more *ph* configurations partecipate (neutron excess states play a predominant role but neutron core states are not negligible). Transitions from proton states are completely decoupled and lie above 30 MeV.

2.2. Imaginary part of the single-particle self-energy in exotic nuclei

The analytic expression of imaginary part of the single-particle self-energy reads

$$\mathrm{Im}\,\Sigma_{lj}(r,r';\varepsilon) \;=\; \sum_{l',j',\lambda} \int d\omega_\lambda \; (-\pi) v(r) v(r') \frac{\tilde{u}_{l'j'}^{(\varepsilon-\omega_\lambda)}(r)}{r} \frac{\tilde{u}_{l'j'}^{(\varepsilon-\omega_\lambda)}(r')}{r'} \times \delta\varrho^{(\omega_\lambda)}(r) \delta\varrho^{(\omega_\lambda)}(r') \frac{\langle lj||Y_\lambda||l'j'\rangle^2}{2j+1} \,. \tag{3}$$

The quantum numbers l, j (l', j') refer to the initial (intermediate) state, $\tilde{u}(r)$ are radial wave functions of the particles, $\delta\varrho(r)$ are transition densities of $1p$–$1h$ pairs coupled to multipolarity λ (or of a collective RPA state) and $v(r)$ is the p–h interaction derived from the effective force from which the HF mean field is determined (in our case, the Skyrme force SIII). The link between Eq. (3) and the empirical imaginary part of the optical potential W is as follows,

$$W(r,r';\varepsilon) = \sum_{lj} \frac{2j+1}{4\pi} \mathrm{Im}\,\Sigma_{lj}(r,r';\varepsilon). \tag{4}$$

The real part is obtained in a similar way, or through dispersion relation techniques (see Ref. [4] and references therein).

The novel feature of Eq. (3), in comparison with what has been used in the past, is that we treat properly the continuum also at the level of the $2p$–$1h$ doorway states [23]. In the calculations performed in the seventies in well-bound nuclei like ^{208}Pb, discrete particle states were employed as the whole system was set in a box and an averaging parameter was employed in Eq. (3) to ensure the match of the initial 1p and intermediate $2p$–$1h$ energies (see Ref. [4] and references therein). This procedure is satisfactory for well-bound systems as was confirmed by making use of Eq. (3) for a test calculation in ^{208}Pb. The results of Ref. [24] were essentially reproduced. For systems with loosely bound nucleons, the use of discrete particle states is not able to reproduce the results that we illustrate below.

To get some insight in the qualitative low-energy behavior of $\mathrm{Im}\,\Sigma$ (by "low-energy" we mean for values of the energy ε close to the particle emission threshold $\omega_{\rm th}$), let us consider a single term of the sum appearing in Eq. (3)

and fix $r = r'$. A plane-wave approximation for $\tilde{u}_{l'j'}^{(\varepsilon-\omega_\lambda)}(r)$ suggests that it contributes with a factor $k_{\rm part}^{l'+1/2}$ if $k_{\rm part} = \hbar^{-1}\sqrt{2m(\varepsilon-\omega_\lambda)}$ is close to zero. Adding the condition on the normalization of the radial transition densities given by

$$\left|\int dr\ r^{2+\lambda}\delta\varrho^{\omega_\lambda}(r)\right|^2 = P^{\rm RPA}(\omega),$$

with the asymptotic behaviour of the strength recalled in Eq. (2), we obtain

$$\mathrm{Im}\,\Sigma \sim \int\limits_{\omega_{\rm th}}^{\varepsilon} d\omega_\lambda\ (\varepsilon-\omega_\lambda)^{l'+1/2}(\omega_\lambda-\omega_{\rm th})^{l''+1/2}. \tag{5}$$

If ε is close to $\omega_{\rm th}$, *i.e.*, $\varepsilon = \omega_{\rm th}+\delta$ and $\omega_\lambda = \omega_{\rm th}+\delta/2$, we find that the imaginary part of the optical potential behaves approximately like $\delta^{l'+l''+2}$, that is, a fast increase just after threshold which has no counterpart in stable nuclei (where $\mathrm{Im}\,\Sigma \sim (\varepsilon-\varepsilon_F)^n$, with $1 \le n \le 2$ [3,4]) and has consequences on the *qualitative* features of scattering experiments with exotic beams. A realistically calculated $\mathrm{Im}\,\Sigma$ follows indeed this asymptotic behaviour, as shown in [23].

It is also shown that the function $W(r, r' = r; \varepsilon)$ is surface peaked, as in stable nuclei. On the other hand, the width of the peak is rather large, as can be expected if the nucleus has extended radial wave functions. It is also noticed, that for $\varepsilon = 25$ MeV a second bump, at the interior of the nucleus, shows up. This can be interpreted as follows. In the calculation of the terms of the sum appearing in Eq. (4), one performs integrals of the type $\int_{\omega_{\rm th}}^{\varepsilon} d\omega_\lambda$. If ε is large, one includes contributions from proton transitions, and these are of course not peaked on the nuclear surface, since the protons are confined in a much smaller region. In this respect, this interior peak is another example of the general statement that the core particles and excess particles are decoupled in this light, exotic isotopes.

2.3. Real part of the single-particle self-energy in exotic nuclei

Using dispersion relation techniques as in Eq. (6) of Ref. [23] with the $\mathrm{Im}\,\Sigma_{lj}(r, r'; \varepsilon')$ discussed above, results for the real part of the single-particle self-energy in ^{28}O were also obtained [23]. In particular, for the energy shift due to the coupling with the states labelled by λ, ω_λ (which are collective vibrations if ω_λ is around 15 MeV and are single-particle transitions at lower energy). This shift is given by

$$\Delta E_{nlj} = \int d\vec{r}d\vec{r}'\ \varphi^*_{nlj}(\vec{r})\ \mathrm{Re}\,\Sigma_{lj}(r, r'; \varepsilon)\ \varphi_{nlj}(\vec{r}), \tag{6}$$

where ε is usually fixed as the unperturbed (*i.e.*, HF) energy $E^{(0)}_{nlj}$ of the level under consideration. The expressions for $\mathrm{Re}\,\Sigma_{lj}$ in the case of hole states are analogous to those derived above for the case of particles. Actually, in the case of ^{28}O calculated within the SIII-HF procedure, no unoccupied particle states can be found at negative energy and the only particle resonance at positive energy is the $\mathrm{f}_{7/2}$, so the particle spectrum consists essentially only of a smooth continuum. Therefore, the shift ΔE_{nlj} for neutron hole states, in particular the loosely bound $\mathrm{d}_{3/2}$ orbital was calculated.

We find a positive shift of 330 keV, to be compared with $E^{(0)} = -1.1$ MeV. This makes the nucleus even more close to being unbound and would very much affect the strength functions of the multipoles we have considered in the previous section. The following considerations are in order: while in stable nuclei corrections of the order of less than 1 MeV to the energy of states which are usually 7–10 MeV bound are important to give the correct density of states around the Fermi energy and effective mass but do not alter the predictive power of theories like the standard RPA in which these corrections are not taken into account, results like the presents (likely to be obtained in existing exotic nuclei and including the pairing contribution as well) must force us to ask questions about the reliability of simple mean field methods to predict the single-particle energies and consequently the position of drip lines, as well as properties of the excited states.

3. Doorway coupling in deformed, fast rotating nuclei

While much effort has been concentrated in the study of the s.p. self-energy Σ and its consequences for the structure of spherical nuclei (*cf. e.g.* Refs. [4] and the recent wotks in [25–28] at finite temperature T), little has been done concerning deformed nuclei let alone hot, rotating systems. This situation is not a particularly brilliant, if one thinks:

a) that to extract information from the decay of compound nuclei (*i.e.* systems corresponding to a statistical ensemble of shapes) which can be directly compared to model predictions one needs to know the level density as a function of temperature and of angular momentum, a quantity which depends, through the level density parameter, on the nucleon effective mass and thus on the ability the particles have to couple to surface vibration in deformed nuclei,

b) to accurately calculate the alignment of single-particle levels, a quantity needed in the study of rotating nuclei in general, and the study of the superfluid-normal nuclear phase transition as a function of the angular momentum and of temperature in particular, as well as in the study of the damping of rotational motion, one needs to calculate the coupling of single-particle motion to pairing vibrations and thus also to surface vibrations of deformed nuclei,

c) that to determine the properties of the levels of a cranked Hartree–Fock Hamiltonian entering in the calculation of the intrinsic states of rotational bands as well as in the calculation of giant resonances in general, and of the GDR in particular, one needs to know both the single-particle effective mass and width, and thus its coupling to surface vibrations of deformed nuclei.

The reason for such asymmetry in the study of the particle-vibration phenomenon (essentially no calculations exist in the literature for deformed nuclei) is quite simple and is associated with the special role spurious states play in deformed nuclei as compared to spherical nuclei, as well to the very large number of two-quasiparticle excitations (RPA roots), of the order of 10^4 for a typical medium-heavy nucleus.

In fact, within mean field theory of deformed, superfluid nuclei, there are two classes of spurious states. One, associated with the violation of rotational invariance. The other, with the violation of gauge invariance. In other words, because the BCS wave function in the Nilsson basis has neither a fixed value of the angular momentum nor of the number of particles, the system described by this wave function displays modes with vanishing frequency and divergent matrix elements of the quadrupole and of the two-particle transfer operators. These spurious states, together with that associated with the violation of translational invariance typical of the shell model, can be made orthogonal to all physical states by diagonalizing the corresponding particle-vibration coupling Hamiltonian, in the Random Phase Approximation (*cf.* *e.g.* Ref. [2] and references therein).

However, when a particle is added to the system, the spurious states, through the coupling to the odd-nucleon, mix again into the spectrum. The solution to this problem is a necessary condition to be able to carry out nuclear structure calculations in deformed, superfluid nuclei which go beyond mean field theory. While in the case of spherical nuclei, the spurious state associated with the non-conservation of the number of particles typical of superfluid nuclei does not mix , as a rule, with surface vibrational modes (quadrupole, octupole, multipole vibrations) due to angular momentum conservation, in deformed nuclei, the pairing spurious modes do mix with surface modes.

Recently, an operative and economic solution to the question of how to deal with the spurious states in superfluid, deformed nuclei, to calculate the s.p. (quasi-particle) self-energy has been discussed in two papers [29, 30] in terms of RPA linear response and complex integration techniques. All details may be found there.

Preliminary calculations were carried out based on the yrast states of ^{168}Yb. The potential energy surface of this nucleus, taking into account Nilsson–Strutinsky corrections, displays a stable minimum up to spin $I = 60\hbar$. The deformation parameters are $\epsilon = 0.262$ and $\gamma = 0^0$. The resulting quasiparticle spectrum (routhians), that is, the eigenvalues of the cranked Nilsson plus mean field pairing Hamiltonian were thus calculated and, in the basis of two quasiparticle states, the RPA linear response of the system determined with residual separable interactions describing both quadrupole $(\beta - and\gamma-)$ and octupole vibrations. Making use of these elements the self-energy of quasiparticles build on single-particle states lying around the Fermi energy, have been calculated as a function of the rotational frequency [31,32]. State dependent energy shifts $|\Delta E|$ of the order of 300–500 keV are obtained, decreasing as function of the rotational frequency because of the increasing rigidity of the nuclear surface. This means that the self- energy effects may not be described by a simple readjustement of the parameters of the cranked Nilsson potential, and fully self-consistent calculations are called for. I deeply miss the discussion with Zdzisław (Szymański) of these results.

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