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PAIRING IN FINITE NUCLEI*

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Pairing in nuclei is shortly overviewed from the perspective of mean-field theory which is the only model where particle-particle channel is uniquely defined. Attention is paid to the effects of pairing correlations on odd-even mass staggering and nuclear rotational motion. Basic theoretical concepts and effects associated with proton–neutron pairing in $N \approx Z$ nuclei are also discussed. It is pointed out that, with the present accuracy of mean-field calculations, no clear constraints can be set on spatial characteristics or density dependence of pairing interaction.

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1. Introduction

It has taken almost 50 years to understand microscopic origin of one of the most fascinating discovery of our century, the phenomenon of *superconductivity* in metals. The goal was finally accomplished in 1957 by Bardeen, Cooper and Schrieffer [1] who formulated proper trial wave function for quantal calculations of electrons moving pairwise in time-reversed states. In the BCS theory of *superconductivity* it is exclusively the property of *attractiveness* of the medium-mediated interaction at the Fermi energy which leads to energy gap, Δ, separating ground-state of a fermionic system from its low-lying elementary excitations. It was therefore soon pointed out (first by D. Pines at the 1957 Rehovot Conference) that due to attractiveness of the effective nuclear forces at the Fermi energy similar effects might also apply to nuclei. Soon after a notion of *nuclear superconductivity* was formally introduced by Bohr, Mottelson and Pines [2] and Belyaev [3] in order to explain energy gaps in low-lying spectra of even–even nuclei.

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In spite of its relatively long history rigorous microscopic theory of nuclear pairing is still lacking. Derivation of pairing interaction from the bare nucleon–nucleon force still encounters many problems [4, 5]. Hence, most of the practical applications uses phenomenological pairing interactions. Moreover, only at the level of mean-field approximation particle–particle (pairing) channel is rigorously defined and separated from the particle-hole channel. For example, within the nuclear shell-model particle–particle (p-p) and particle-hole (p-h) representations can be transformed into each other. Therefore, within the shell-model, there is no obvious procedure allowing to extract pairing interaction which constitutes the integral part of the residual shell-model interaction. The multipole decomposition technique leads to rather oversimplified pairing interaction [6]. It seems that much better insight into pairing properties can be gained by pair-structure analysis of the shell-model wave function [7].

The phenomenological pairing interactions most often used in mean-field calculations are separable in p-p channel, $\bar{v}_{\alpha\beta\gamma\delta} \propto g_{\alpha\beta} g^*_{\gamma\delta}$. The state-independent seniority or multipole-pairing interactions are the best known examples. These interactions are characterized by mean-values of gap (order) parameters and are therefore easy to interprete and handle numerically. These forces are perfectly suited for microscopic–macroscopic calculations. Within the Skyrme–Hartree–Fock–Bogolyubov (SHFB) calculation scheme the family of zero-range interactions is often used. It includes simple volume-active delta-interaction $V_V \propto \delta(\boldsymbol{r} - \boldsymbol{r}')$, density-dependent, surface-active delta-interaction (DDDI) [8, 9] $V_S \propto \delta(\boldsymbol{r} - \boldsymbol{r}')(1 - [\rho(\boldsymbol{r})/\rho_0]^\gamma)$ or certain parameterizations of Skyrme type forces [10]. Finally, the finite-range Gogny force is used consistently in both p-h and p-p channels within fully self-consistent HFB [11] or in p-p channel in relativistic Hartree–Bogolyubov (RMF) calculations [12]. The advantage of finite-range over the zero-range pairing forces is an automatic cut-off of high-momentum components. Therefore, no energetical pairing window is required in the calculations involving these forces.

Nuclear structure applications of mean-field models invoking different pairing forces will be shortly overviewed in the next Section. It appears that nuclear masses, high-spin properties, radii and isotopic shifts seem to depend only weakly on spatial character (volume or surface type) or density dependence of pairing interaction and that with present accuracy of nuclear structure calculations it is difficult to constrain on these specific features of nuclear pairing. Third Section will discuss briefly a proton–neutron (pn) pairing phenomenon. Short summary will be given in the last Section.

2. Effect of pairing correlations on nuclear properties

Although the effect of pairing correlations on nuclear masses is modest, these correlations strongly modify gross nuclear properties. The odd–even mass staggering (OES), moments of inertia, alignments, electromagnetic and β-decay rates, particle or α emission rates *etc.* are all strongly modified inside paired medium. The BCS theory allows for qualitative understanding of all these phenomena within simple, intuitive framework. However, quantitative description of these phenomena is far more difficult due to coupling between single-particle field, pairing field, and the effects going beyond mean-field. This will be demonstrated in the following two subsections where odd–even mass staggering and pairing related high-spin phenomena will be shortly overviewed.

2.1. Odd–even mass staggering

The odd–even mass staggering (OES) of nuclear binding energies is usually directly related to pairing. Indeed, within the BCS approximation the quantity:

$$\Delta^{(3)}(N) \equiv \frac{\pi_N}{2}\left[B(N-1) - 2B(N) + B(N+1)\right] \approx \frac{\partial^2 B(N)}{\partial N^2} \tag{1}$$

can be interpreted as a measure of empirical pairing gap. However, because of strong contribution due to nuclear symmetry energy ($\propto (N-Z)^2$) indicator (1) is usually replaced by the average:

$$\Delta^{(4)}(N) \equiv \frac{1}{2}\left[\Delta^{(3)}(N) + \Delta^{(3)}(N+1)\right], \tag{2}$$

which leads to the commonly used estimate $\Delta = 12/\sqrt{A}\,$MeV for the empirical pairing gap. In the above formulas $B(N)$ is the (negative) binding energy of a system of N particles of number-parity $\pi_N = (-1)^N$.

According to the Strutinsky-energy theorem [13], nuclear binding energy can be written as a sum of macroscopic and shell correction energies. The indicator (2) properly separates out empirical gap provided that not only macroscopic energy but also shell correction $\delta E_{\text{shell}} = E_{\text{shell}} - \widetilde{E}_{\text{shell}}$ [$E_{\text{shell}} = \sum_{\text{occup}} e_i$ is a single-particle (s.p.) shell energy and $\widetilde{E}_{\text{shell}}$ denotes Strutinsky-averaged s.p. energy] smoothly varies with particle number [14]. These smoothness criteria do not in fact apply. Indicator (2) gives systematic values of the order of few hundred keV when applied to *single-particle* energies calculated using Skyrme–Hartree–Fock model independently on number-parity, see Fig. 1 in Ref. [15]. On the contrary, indicator (1) gives single-particle OES:

$$\Delta^{(3)}(N)_{sp} = \frac{1}{2}\delta e \equiv \frac{1}{4}(1+\pi_N)(e_{n+1} - e_n) \tag{3}$$

which can be traced back to the deformed single-particle field which lifts spherical degeneracies leaving only two-fold Kramers degeneracy. This single-particle mechanism behind OES is well recognized in metallic clusters [16]. In fact, similarities between OES pattern in light nuclei and small Na clusters that emerged in calculations of Ref. [17] led them to conclude that OES in light nuclei is a mere deformation effect rather than a consequence of pairing. Closer examination shows, however, that OES in light nuclei is rather democratically shared between shape and pairing effects. Both effects can be separated from each other to large extent because contributions to (1) due to macroscopic symmetry energy ($\sim 38/A\,$MeV) and Strutinsky-averaged energy ($\sim -36/A\,$MeV) nearly cancel each other [15]. Consequently, $\Delta^{(3)}(N=2n+1)$ can be interpreted as empirical measure of the pairing gap while $\Delta^{(3)}(N=2n)$ strongly mixes shape and pairing effects. Similar interpretation and conclusions can be essentially drown based on seniority model as well as on pairing-plus-quadrupole and equidistant level models [18].

The new way of extracting pairing has far going consequences particularly for light nuclei. The $\Delta = 12/\sqrt{A}\,$MeV estimate strongly overshoots the data particularly in *sd*- and *pf*-shell nuclei. In fact, *new* experimental gaps rather smoothly decrease with mass indicating much weaker mass dependence of the average gap than $\sim A^{-1/2}$, see Fig. 1. Apart of empirical data Fig. 1 shows average neutron pairing gaps calculated using Gogny–HFB method [19]. The agreement between calculations and the data is surprisingly good.

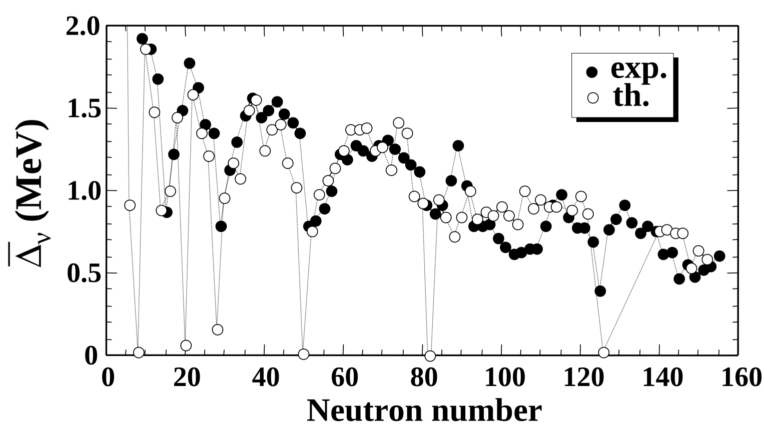

Fig. 1. The average neutron pairing gaps extracted from the empirical binding energies (full symbols) and calculated using Gogny–HFB method (open symbols).

2.2. *Effect of pairing correlations on nuclear rotational motion*

The Coriolis force acting on nucleons moving in uniformly cranked potential [20]:

$$\hat{H}_{\text{Coriolis}} \equiv -\vec{\boldsymbol{\omega}}\vec{\boldsymbol{j}} + \frac{1}{2}m(\vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{r}})^2 \tag{4}$$

can be rewritten as $[\vec{\boldsymbol{j}} = \vec{\boldsymbol{l}} + \vec{\boldsymbol{s}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}} + \vec{\boldsymbol{s}}]$:

$$\hat{H}_{\text{Coriolis}} = -\frac{1}{2m}\vec{\boldsymbol{p}}^2 + \frac{1}{2m}\left[\vec{\boldsymbol{p}} - m(\vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{r}})\right]^2 - \vec{\boldsymbol{\omega}}\vec{\boldsymbol{s}}. \tag{5}$$

The latter Hamiltonian corresponds to the Hamiltonian of particle of spin $\vec{\boldsymbol{s}}$ moving in constant magnetic field $\vec{\boldsymbol{H}}$ provided that $\vec{\boldsymbol{\omega}} \Leftrightarrow \vec{\boldsymbol{H}}$. Then $\vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{r}}$ plays a role of vector magnetic potential $\vec{\boldsymbol{A}} \equiv \vec{\boldsymbol{H}} \times \vec{\boldsymbol{r}}$. The Coriolis force will therefore try to brake nucleonic Cooper pairs in analogy to the well known Meissner effect in metallic superconductivity [21]. This bulk disappearance of nuclear pairing correlations is called the Mottelson–Valatin effect [22]. It causes steady increase of nuclear moment of inertia [3, 23]. In reality, the disappearance of pairing correlations in nuclei is non-uniform. Strong structural changes causing *back-* or *upbendings* [24] in the evolution of nuclear moment of inertia versus spin are due to the breaking of a pair of nucleons occupying high-j intruder orbitals [25]. In cranked mean-field formalism *backbending* phenomenon correspond to the rearrangement of vacuum configuration or, alternatively, to the crossing of the ground-state band with the lowest two-quasiparticle ($2qp$) band which is often called the S-band. The gross systematics of ground-band–S-band crossing frequencies is qualitatively relatively well understood and reproduced within the cranked shell-model. The details, however, depend in many cases upon delicate balance between (coupled) pairing and shape effects.

Therefore, a lot of effort must be devoted to optimize simultaneously both single-particle and pairing channels within mean-field to improve the agreement to the data which in turn will also improve our understanding of pairing correlations. For example, Xu an co-workers [26] investigated recently in a systematic way energetics of high-K isomers, systematics of the crossing frequencies, odd–even mass staggering effects and moments of inertia in some rare-earth nuclei. In their study they used schematic pairing interaction and phenomenological potential within the Total Routhian Surface (TRS) model of Refs. [27–29] which takes into account shape polarization effects and treats pairing effects (including blocking) self-consistently. They have demonstrated that by enlarging of seniority pairing strength by roughly ∼5–10% as compared to the value determined from the average gap method of Ref. [30] one can account for all these effects simultaneously. An elegant, systematic study has been carried out recently by Chabanat and co-workers [31] in order to optimize Skyrme forces parameterization. The family of forces which emerged from this study, the SLy-forces, is superior as compared to the other commonly used Skyrme forces.

The alternative way to study pairing is to look into the cases where pairing and shape effects are to large extend decoupled. The example are superdeformed nuclei in Hg–Pb region. Strongly elongated, stable with rotational frequency shapes, smooth processes of $\pi i_{13/2}$ and $\nu j_{15/2}$ alignment make these systems almost ideal *laboratories* to study pairing correlations. Indeed, a lot of effort was devoted lately to understand structure of these nuclei in a framework of TRS method [28, 32], Skyrme–HFB [33], Gogny–HFB [34] and RMF [35] approaches. The state dependent pairing, self-consistent treatment of pairing correlations including blocking, and proper treatment of number fluctuations appeared to be *necessary* to obtain satisfactory description of these bands. These conclusions was common for all these studies. In fact, the techniques and concepts like double-stretched quadrupole pairing [28], the surface-active density-dependent delta inter-

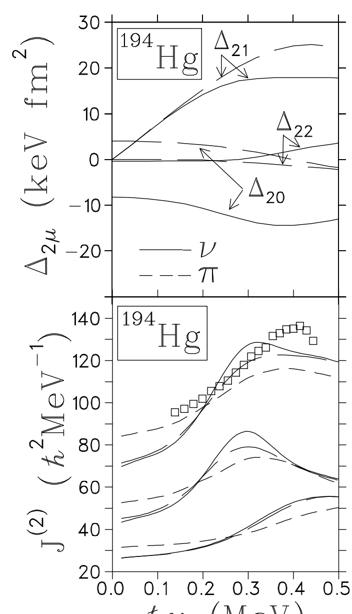

Fig. 2. The effect of time-odd $(\lambda\mu)$=(21) pairing on dynamical moment of inertia $J^{(2)}$ in SD ^{194}Hg (lower part). Solid and long–dashed line illustrates $J^{(2)}$ calculated using seniority pairing only while short–dashed line shows calculations including seniority and quadrupole pairings. The corresponding average quadrupole pairing gaps are shown in the upper part.

action [8], Lipkin–Nogami number-projection [36, 37] were applied for the first-time in large-scale calculations in $A \sim 190$ nuclei. Afterwards, following the numerous applications in this mass region, they became standard methods for large-scale calculations in high-spin physics.

Let us mention here also a particular importance of time-odd (Migdal) pairing [38]. Although energetically very modest, it strongly affects moments of inertia particularly at low-frequencies, see Fig. 2 [28]. Moreover, it clearly contributes to twinning of SD bands in odd and even nuclei reducing (too)strong effect of blocking of seniority pairing on moment of inertia [32].

3. Proton–neutron pairing

A renaissance of our interest in proton–neutron (pn) pairing is stimulated by technological development of Radioactive Ion Beams (RIB) facilities. The first RIB experiments are targeted on heavy $N \sim Z$ nuclei, where pn correlations are expected to be strongly enhanced due to large spatial overlaps between proton and neutron single-particle wave functions. Relatively large valence spaces in these nuclei and expected large deformations do rise expectations for possibility to observe coherent pn-pairing phase. In spite of theoretical and experimental efforts many problems related to pn-pairing still remain not answered. It includes fundamental questions concerning experimental fingerprints of pn-collectivity or the structure of *effective* pn-Cooper pairs.

The necessary generalizations to include nn-, pp- and pn-pairing on the same footing within the mean-field approach were worked out already in the sixties [39]. The idea was to generalize Bogoliubov transformation to include mixing of particles and holes as well as protons and neutrons:

$$\hat{\alpha}_k^\dagger = \sum_{\alpha\tau>0} \left(U_{\alpha\tau,k} a_{\alpha\tau}^\dagger + V_{\tilde{\alpha}\tau,k} a_{\tilde{\alpha}\tau} + U_{\tilde{\alpha}\tau,k} a_{\tilde{\alpha}\tau}^\dagger + V_{\alpha\tau,k} a_{\alpha\tau} \right), \tag{6}$$

where index α runs over single-particle states, τ denotes third component of isospin, and k labels the quasiparticles. Unlike in the standard like-particle pairing applications, the coefficients of transformation (6) have to be complex to simultaneously include $T = 1$ and $T = 0$ pairing correlations.

Many important features of pn-pairing can be deduced from simple model assuming schematic pairing interaction:

$$H_{\text{pair}} = G^{\tau,\overline{\tau\prime}} \sum_{\alpha>0} P_{\alpha\tau,\overline{\alpha\tau\prime}}^\dagger P_{\alpha\tau,\overline{\alpha\tau\prime}} + G^{\tau,-\tau} \sum_{\alpha} P_{\alpha\tau,\alpha-\tau}^\dagger P_{\alpha\tau,\alpha-\tau} \tag{7}$$

based on pair-counting mechanism [40]. Superimposing *antilinear simplex symmetry* $S_x = \hat{P}\hat{T}\hat{R}_z$ as a self-consistent symmetry further simplifies the model. The price paid for the simplification is an absence of $T=0$ pairing component in $\boldsymbol{\alpha}\bar{\boldsymbol{\alpha}}$ channel and therefore $G^{\tau,\bar{\tau}'} \equiv G^{T=1}_{\tau\tau'}$ and $G^{\tau,-\tau} \equiv G^{T=0}$. Although this component is very important [41], lack of it in the model can be simulated by either $T=0$ pn-pairing of $\boldsymbol{\alpha}\boldsymbol{\alpha}$ type at frequency zero or by isospin-broken Hamiltonian in cranking calculations, see Ref. [40] where more details can be found.

The solution to the Hamiltonian (7) does depends on the ratio $x = G^{T=0}/G^{T=1}$. In $N=Z$ nucleus and for $x<0$ only isovector pairing exists but energetically the solution is insensitive to direction $\vec{\Delta} \equiv (\Delta_{pp}, \Delta_{nn}, \Delta_{pn}^{T=1})$. For $x=1$ energy does depend on $\Delta_{pp}^2 + \Delta_{nn}^2 + \Delta_{pn}^{T=1} + |\Delta_{pn}^{T=0}|^2$ but again is insensitive to the direction $\vec{\Delta} \equiv (\Delta_{pp}, \Delta_{nn}, \Delta_{pn}^{T=1}, \Delta_{pn}^{T=0})$. In both cases no energy is gained due to pn-pairing. Finally, for $x>1$ only $T=0$ phase exists and the nucleus gains energy. For $N \neq Z$ the $T=1$ pp- and nn-correlations coexist with $T=0$ pn-pairing, provided that its strength is larger than certain critical value $x > x_{\rm crit}$. The proton or neutron excess quenches/blocks the phase-space available for pn-pairs as shown schematically in Fig. 3. Therefore the critical parameter $x_{\rm crit}$ rapidly increases with increasing $|N-Z|$ and pn-paired solutions are possible only in the closest vicinity of $N=Z$. Similar conclusions have been reached in Ref. [42] both within schematic SO(8) model and realistic shell-model. The exclusiveness of $T=0$ and $T=1$ phases in $N=Z$ nuclei is entirely due to simplicity of the model. Already number-projection leads to mixed $T=0$ and $T=1$ solutions [40]. Also use of realistic interactions within resonable model-space gives mixed solutions [43].

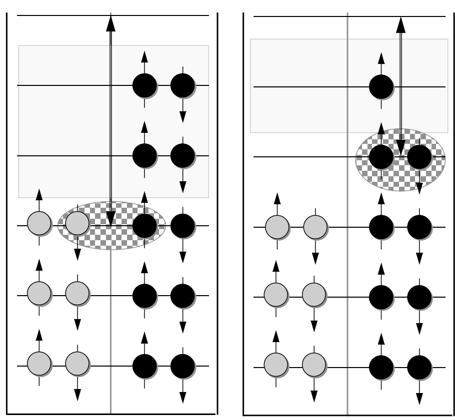

Fig. 3. Schematic illustration of blocking of pn-pairing due to (say) neutron excess (left panel) and the nn-pairing due to odd-neutron (right panel). Shaded areas shows levels unavailable for pair scattering.

The nuclear masses show slope discontinuity at $N = Z$ line. This additional binding energy is known in the literature as a Wigner energy. Traditional mass models based on mean-field approach strongly underbind $N \sim Z$ nuclei [44–46] and a term [14, 46]:

$$E_W = W(A)|N-Z|+d(A)\delta_{NZ}\pi_{pn} \quad \text{, where} \quad \pi_{pn} = \begin{cases} 1 & \text{for odd-odd nuclei} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

has to be added to correct for this deviation. The microscopic explanation of the Wigner energy within the mean-field model is still lacking. The 'congruence' energy mechanism due to enhanced ph interaction at the $N \sim Z$ line proposed in [47] is essentially not present in traditional Skyrme–HFB calculations [46, 48] Instead strong congruence energy effects were found in time-odd channel in odd-odd $N = Z$ nuclei [49]. The isospin fluctuations do actually produce even the anti-Wigner effect. The pn-pairing scenario which naturally gives rise to $|N - Z|$-like term seems to be the most natural so far. Note, that it requires the $T = 0$ pairing to be on the average stronger than $T = 1$ but not necessarily coherent to activate generalized blocking mechanism shown in Fig. 3. There are strong experimental arguments that Wigner energy is indeed due to isoscalar interaction [46]. Similar conclusion can be drown from shell-model studies [46, 50]. However, pair-structure analysis of the shell-model wave function reveals rather complicated structure of the Wigner energy [46].

At high-spins the $T = 1$ and $T = 0$ correlations are expected to respond in different way to the Coriolis force. The traditional anti-pairing effect is expected to destroy $T = 1$ pairing and low-J, $T = 0$ correlations. However, high-J $T = 0$ correlations will survive and are expected to be even dominant at high-spins [40, 51, 52]. The shell-model calculations [53] and complex Excited VAMPIR calculations [54] provide detailed analysis of isospin and pair structure changes with increasing spin in $N = Z$ nuclei in $A \sim 80$ mass region. These calculations confirm the $T = 1 \rightarrow T = 0$ band transition observed in ^{74}Rb [55] and increasing role of high-J, $T = 0$ pairs at high spins. Important clues concerning pn-pairing at high-spin may be also gained by analyzing evolution of rotational bands beyond their standard terminating states (*e.g.* ^{48}Cr above I=16$\hbar$) [52] or moments of inertia of some SD-bands in $A \sim$80–90 mass region [56].

4. Summary

Rigorous microscopic theory of pairing correlations in finite nuclei is still lacking and phenomenological interactions are used in the applications. Available nuclear data on nuclear masses, radii and isotopic shifts, moments of inertia, crossing frequencies and alignment patterns may all be well understood within the mean-field theory. Unfortunately, these observables seem to depend only weakly on spatial characteristics or density dependence of pairing interaction and, with the present accuracy of nuclear structure mean-field calculations, it is difficult to constrain on these specific features of nuclear pairing. This rather frustrating situation calls for either systematic optimization of effective forces to improve overall agreement between theory and the data or requires new data on exotic nuclei which can provide more sensitive probes of specific parts of nuclear effective interaction. Particularly desirable are nuclei of large isospins where, for example, density-dependence of pairing force can be probed in the skin region. For review of pairing and continuum effects I refer reader to [57].

The pn-pairing correlations in $N \sim Z$ nuclei are important without any doubt. Certain evidence of $T = 1$ pn-condensate is seen in $N = Z$ odd–odd nuclei but no evidence of isoscalar coherency has been reported so far. Theoretically, physics of $N \sim Z$ nuclei, is still a challenge particularly within mean-field approach. A form of effective NN interaction, role of self-consistent symmetries, isospin and/or number-projection, the issue of congruence energy, proper treatment of residual pn interaction between valence neutron and proton are still out of control.

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