SINGLE PARTICLE NUCLEAR LEVELS IN EXTENDED THOMAS–FERMI POTENTIALS* **

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Single particle nuclear levels are calculated on the basis of the potentials derived from the Extended Thomas–Fermi type Skyrme models for the Skyrme forces SkIII, SkM* and SLy4.

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1. Introduction

Single particle nuclear energy levels are still the basic ingredients of the macroscopic-microscopic methods used in studies of nuclear properties. They help to understand the properties of nuclei and the nuclear structure.

Single particle levels are the semi-observables. The procedure of extracting them from experimentally known *total* nuclear levels is highly uncertain. In order to do such an extraction one usually assumes some intrinsic mechanisms responsible for the observed nuclear levels like *e.g.*, the coupling schemes of single particle levels to collective nuclear excitations, the pairing forces *etc*. The shell model potentials of Nilsson or Saxon–Woods type are adjusted to fit the single particle experimental levels. The uncertainties mentioned remain in all the calculations following them as *e.g.*, ground state properties of nuclei and the properties of the excited states. This is especially true in the calculations for exotic nuclei close to the neutron or proton drip lines as well as in the case of superhaevy nuclei.

Therefore it seems to be usefull to find the more reliable way of adjusting the shell model single particle potentials. An effort of doing this in the framework of Extended Thomas-Fermi method and the Skyrme forces was

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undertaken in paper [1]. Very similar study of the single particle potentials of this type was done in the case of relativistic mean field model (RMF) for NL1 and NL2 parametrizations in [2]. The results were also discussed in [3] and in [4] for the case of NL3 and NLSH parametrizations. The potentials were succesfully used in [5] to describe the pseudospin symmetry and the pseudospin degeneracy mechanism in nuclear single particle levels.

The ETF method [6,7] determines the stucture of a given nucleus selfconsistently by a density variational calculation [6] particularly in connection with effective interactions of the Skyrme type. It has been proven to be extremely efficient in the precise description of average nuclear properties. The by-product of the selfconsistent procedure are the single particle central, spin-orbit and Coulomb potentials and the effective mass for a given nucleus.

The main difference between the conventional shell model and the present one is in the effective nucleon mass. In the former case the mass is constant free nucleon mass $m \approx 940$ MeV whereas in the latter case it depends on the coordinate $\vec{r}$. It is approximatelly equal to $0.6m$ inside the nucleus and approaches free nucleon mass in the far perifery of the nucleus.

In the present paper we calculate the single particle spectra in ETF potentials represented by Saxon–Woods type functions. The nucleon mass depends on the radial coordinate r. The investigations are limited to the two spherical nuclei ^{208}Pb and ^{132}Sn for which the approximate experimental single particle levels are known. The spectra of a deformed nuclei can be calculated by taking into consideration the typical deformation dependence of the potentials as *e.g.*, in the standard Saxon–Woods shell model calculations. Three sets of parameters coresponding to the three types of the Skyrme interactions: SkIII [7], SkM* [8] and the SLy4 force [9] are given in [1].

The paper is organized as follows. Section 2 is devoted to the question, whether it is possible to replace the ETF potentials by the simpler potentials of the Saxon–Woods type. The potentials and their parameters are shown. In section 4 we discuss the results of comparison of the single particle levels calculated in different parametrizations as given in [1] and for the parametrization at constant mass [10]. The results are compared to the experimental data.

2. Potentials and mass parametrization

The ETF procedure is explained *e.g.*, in [11]. This section asks the following question. Is it possible to parametrize the ETF form factors of the potentials and the effective mass in terms of Saxon–Woods functions such that the single particle spectra basically do not change? The answer on this

question is positive and is given in [1] (the answer is the same in the case of relativistic mean field theory, see [2–4]).

The central potential is assumed in the Saxon–Woods form

$$V(r) = \frac{V_c}{1 + \exp(\frac{r-R}{a^v})} \tag{1}$$

where the parameters V_c, R and a^v are adjusted by a least-square fitting procedure independently for neutrons and protons for nine spherically symmetric nuclei separately.

For the Skyrme spin-orbit potential V^{so} which enters the expression

$$\vec{W}(\vec{r}) = \vec{\nabla} V^{so}(\vec{r}) \tag{2}$$

one assumes the same shape

$$V^{so}(\vec{r}) = \frac{V_c^{so}}{1 + \exp(\frac{r-R^{so}}{a^{so}})}\,. \tag{3}$$

The effective mass is parametrized in the simillar fashion. The formula reads

$$\frac{m^*(\vec{r})}{m} = 1 - \frac{m_c}{1 + \exp(\frac{r-R^m}{a^m})}\,. \tag{4}$$

The energy of the Coulomb interaction, is assumed in the form of the potential generated by the sharp spherical density distribution of Z protons with adjustable radius R_C and an additional modifier c

$$V^C(r) = \begin{cases} \frac{e^2 Z}{2R^C} c \left(3 - \left(\frac{r}{R^C}\right)^2\right) & \text{if } r < R^C \\ \frac{e^2 Z}{r} c & \text{if } r \geq R^C \end{cases}\,. \tag{5}$$

All the parameters of the nuclear Saxon–Woods potentials, Coulomb potential and the effective mass depend on the nucleus considered. The detailed Z- and N-dependence of the parameters is shown in the following subsections.

2.1. Central potential

A Z- and N-dependence of the central potential parameters (see Eq. (9)) was assumed in the form

$$V_c = V_0(1 + \kappa^v I)\,, \tag{6}$$

$$R = r_0^v A^{1/3}(1 + \kappa_R^v I + b/A)\,, \tag{7}$$

$$a^v = a_0^v(1 + \kappa_a^v I)\,. \tag{8}$$

2.2. Spin-orbit potential

Spin-orbit potential parameters (Eq. (11)) are given by

$$V_c^{so} = V_0^{so}(1 + \kappa^{so} I)\,, \tag{9}$$

$$R^{so} = r_0^{so} A^{1/3}(1 + \kappa_R^{so} I + b_R^{so}/A)\,, \tag{10}$$

$$a^{so} = a_0^{so}(1 + \kappa_a^{so} I)\,. \tag{11}$$

2.3. Effective mass

The parameters of the effective mass (see Eq. (12)) are

$$m_c = m_0(1 + \kappa^m I)\,, \tag{12}$$

$$R^m = r_0^m A^{1/3}(1 + \kappa_R^m I + b_R^m/A)\,, \tag{13}$$

$$a^m = a_0^m(1 + \kappa_a^m I)\,, \tag{14}$$

$$m = 939\,\mathrm{MeV/c^2}\,. \tag{15}$$

2.4. Coulomb potential

The dependence of the Coulomb potential on Z and A (or N) is of the form

$$V^C(Z,N) = \begin{cases} \frac{e^2 Z}{2R^C} c \left(3 - \left(\frac{r}{R^C}\right)^2\right) & \text{if } r < R^C \\ \frac{e^2 Z}{r} c & \text{if } r \geq R^C \end{cases}\,, \tag{16}$$

where both R_C and c depend on Z, N

$$R^C = r_0^C A^{1/3}(1 + \kappa_R^C I + b_R^C/A)\,, \tag{17}$$

$$c = c_0(1 + \kappa_c^C I)\, r\,, \tag{18}$$

$$e^2 = 1.43996518\ \mathrm{MeV\,fm}\,. \tag{19}$$

2.5. Parameters

The parameters extracted for three types of Skyrme forces are shown in [1]. Here, for illustration, we list only parameters extracted from the light spherical doubly magic nuclei and SkM* force. The parameters are gathered according to the potential in question or the effective mass. The explanation of the symbols appearing here is given in the previous section. The units are the following: femtometers (fm) in the case of length (all the as and rs) and the megaelectronvolts (MeV) in the case of energy (all the Vs). All the other parameters like the κs, the bs and the cs are dimensionless.

Central potential

Protons	Neutrons
V_0 = -63.1036	V_0 = -61.4988
κ^v = 0.4320	κ^v = -0.4334
a_0^v = 0.6932	a_0^v = 0.6855
κ_a^v = 0.0705	κ_a^v = -0.0338
r_0^v = 1.1992	r_0^v = 1.2234
κ_R^v = 0.1099	κ_R^v = -0.0648
b_R^v = -0.0951	b_R^v = -0.3293

Effective mass

Protons	Neutrons
m_0 = 0.2183	m_0 = 0.2116
κ^m = 0.3827	κ^m = -0.7185
a_0^m = 0.5102	a_0^m = 0.5123
κ_a^m = 0.5134	κ_a^m = -0.2835
r_0^m = 1.1321	r_0^m = 1.1532
κ_R^m = 0.1494	κ_R^m = -0.0629
b_R^m = -0.8500	b_R^m = -1.0643

Spin-orbit potential

Protons	Neutrons
V_0^{so} = -16.2629	V_0^{so} = -16.0346
κ^{so} = 0.0331	κ^{so} = -0.4339
a_0^{so} = 0.5308	a_0^{so} = 0.5306
κ_a^{so} = 0.2573	κ_a^{so} = 0.0006
r_0^{so} = 1.1239	r_0^{so} = 1.1308
κ_R^{so} = 0.0818	κ_R^{so} = 0.0229
b_R^{so} = -1.3033	b_R^{so} = -1.3656

Coulomb potential

r_0^C = 1.1437
κ^C = -0.1014
b^C = 1.0091
c_0 = 0.4493
κ_c^C = -0.0015

3. Results and discussion

The calculations were performed in the basis of spherically symmetric shell model states $|nlj\rangle$. The single particle Hamiltonian

$$\hat{h} = -\vec{\nabla}\frac{\hbar^2}{2m^*(r)}\nabla + V(r) + \frac{1+\tau_3}{2}V^C(r) + \frac{1}{2m^2}\left(\frac{1}{r}\frac{dV^{so}(r)}{dr}\right)(\vec{l}\cdot\vec{s}), \quad (20)$$

where τ_3 is the third component of the isospin equal to -1 for neutrons and +1 for protons.

The numerical code takes into account the fact of coordinate dependent mas $m^*(r)$.

The main resuls of our calculation are presented in Fig. 1. The figure shows the proton and neutron single particle spectra in ^{132}Sn and in ^{208}Pb for the Skyrme forces SkIII, SkM* and SLy4 and for the case of *universal* (Univ.) [10] parameters of the standard Saxon Woods shell model with constant mass. The experimental data (exp) are also shown for comparison.

It is observed that the Skyrme nucleon spectra are approximately of the same density as the experimental ones for energies below the Fermi surface. The levels $2p_{1/2}$ and $1g_{9/2}$ in the case of ^{132}Sn are interchange for all of the Skyrme forces considered.

The approximate Saxon–Woods potentials give a very similar spectra to those obtained in the selfconsistent Hartree-Fock calculations.

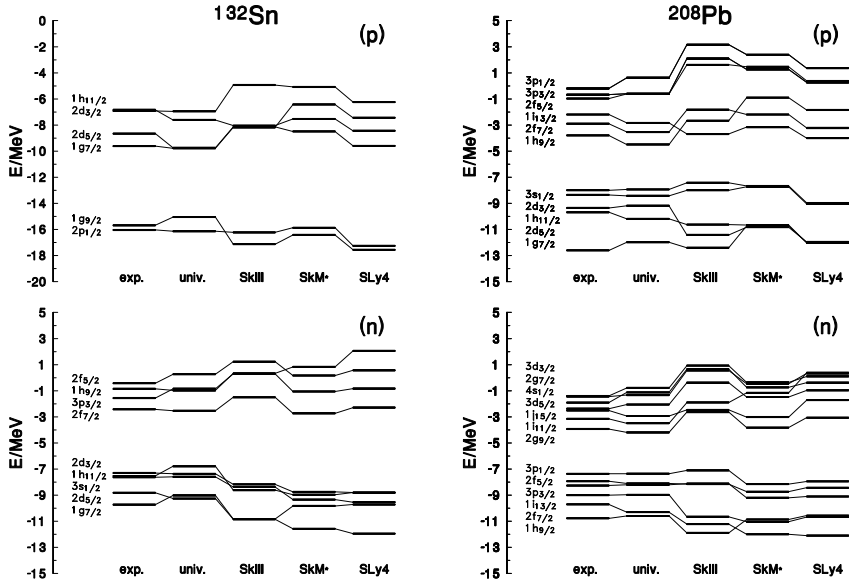

Fig. 1. Proton (p) and neutron (n) single particle spectra in ^{132}Sn and ^{208}Pb.

REFERENCES

[1] A. Baran, K. Pomorski, J. Bartel, Extended Thomas-Fermi calculations of single particle potentials, submitted to *Atom. Data and Nucl. Data Tables*, 1999.

[2] W. Koepf, P. Ring, *Z. Phys.* **A339**, 81 (1991).

[3] P. Ring, *Prog. Part. Nucl.* **37**, 193 (1996).

[4] A. Baran, Relativistic mean field calculation of single particle potentials, submitted to *Phys. Rev.* **C**, 1999.

[5] Y.K.Gambhir, J.P. Maharana, C.S. Warke, *Eur. Phys. J.* **A3**, 255 (1998).

[6] I. Zh. Petkov, M.V. Stoitsov, *Nuclear Density Functional Theory*, Oxford Science Publications, Clarendon Press, Oxford 1991.

[7] M. Beiner, H. Flocard, Nguyen van Giai, P. Quentin, *Nucl. Phys.* **A283**, 29 (1975).

[8] J. Bartel, P. Quentin, M. Brack, C. Guet, H.-B. Håkansson, *Nucl. Phys.* **A386**, 79 (1982).

[9] E. Chabanat, P. Bonche, P Haensel, J. Meyer, R. Schaeffer, *Nucl. Phys.* **A627**, 710 (1997); *Nucl. Phys.* **A635**, 231 (1998).

[10] S. Ćwiok, J. Dudek, W. Nazarewicz, J. Skalski, T. Werner, *Comput. Phys. Commun.* **46**, 379 (1987).

[11] M.J. Giannoni, P. Quentin, *Phys. Rev.* **21**, 2076 (1980).