NEUTRON–PROTON PAIRING CORRELATIONS *

S. Szpikowski

Institute of Physics UMCS
Radziszewskiego 10, 20-031 Lublin, Poland

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Recently, the neutron–proton correlations have been once again considered in rich neutron exotic nuclei with $N = Z$. In this contribution the measure of neutron–proton pairing correlations is obtained using the elementary method based on the exact group theory treatment. The transparent and simple algebraic formulas for average values of neutron, proton and neutron–proton parts of pairing energies are then applied to two examples.

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1. Formulas for $\langle S_+^n S_-^n \rangle$; $\langle S_+^p S_-^p \rangle$ and $\langle S_+^{np} S_-^{np} \rangle$

For one kind of nucleons, say neutrons, and for one j-level, the pairing Hamiltonian reads

$$H' = -GS_+^n S_-^n \tag{1}$$

where

$$S_+^n = \sum_{m>0} (-1)^{j-m} a_{jm}^+ a_{j-m}^+ ; \quad S_-^n = (S_+^n)^+ \tag{2}$$

and G is the strength parameter.

The simple group theory algebra for the group SU(2) provides diagonalisation of the H' and we can get [1]

$$E'_n = -\frac{G}{4}(n-\nu)(2\Omega + 2 - n - \nu) \,, \tag{3}$$

where $\Omega = (2j+1)/2$, ν is the seniority number and n is the number of particles. Hence, we get the simple formula for “the number of active pairs”,

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$\langle S_+^n S_-^n \rangle$ or, more exactly, the expectation value of the neutron pairing energy in $-G$ units

$$\langle S_+^n S_-^n \rangle \equiv \tfrac{1}{4}(n-\nu)(2\Omega+2-n-\nu) \tag{4}$$

which is also, in this simple case, an eigenvalue of the pairing operator.

Before considering the system of neutrons and protons let us repeat the elementary method for pairing energy [2, 3] in the above case. We divide the j-level on two-particle states with m and $-m$ each (Fig. 1).

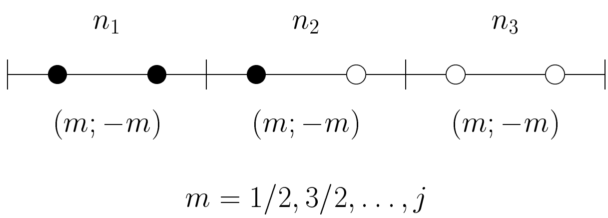

Fig. 1. The schematic picture of neutrons on the j-level: there are n_1 paired pairs, n_2 unpaired single neutrons and n_3 empty two-particle states.

From Fig. 1 we immediately get

$$n = 2n_1 + n_2\,; \qquad \nu = n_2\,; \qquad \Omega = n_1 + n_2 + n_3\,, \tag{5}$$

or

$$n_1 = \frac{n}{2} - \nu\,; \qquad n_2 = \nu\,; \qquad n_3 = \Omega - \frac{n}{2} - \frac{\nu}{2}\,. \tag{6}$$

The elementary method is based on the following scheme. The annihilation operator S_-^n while acting on a schematic state (Fig. 1) annihilates the paired pair in each of the possible $(m, -m)$ two-particle positions (n_1 possibilities) and then the operator S_+^n from the Hamiltonian (1) creates a pair on each empty state ($1 + n_3$ possibilities). Each annihilation-creation action gives the energy contribution $-G$. Hence, the pairing energy simple reads

$$E'_n = -Gn_1(1+n_3)\,. \tag{7}$$

Taking n_1 and n_3 from (5) to (6) we get exactly the pairing energy (3).

For a system with protons and neutrons the group-theoretical treatment is based on the orthogonal group SO(5) [1]. The pairing energy for the simple case with seniority $\nu = 0$ reads

$$E_{\text{pair}} = -\frac{G}{4}\left\{n\left(2\Omega + 3 - \frac{n}{2}\right) - 2T(T+1)\right\} \tag{8}$$

which is the eigenvalue of the pairing Hamiltonian

$$H = -G(S_+^n S_-^n + S_+^p S_-^p + \tfrac{1}{2} S_+^{np} S_-^{np})\,, \tag{9}$$

where

$$S_+^{np} = \sum_{m>0} (1)^{j-m} \left(a_{jm}^{p+} a_{j-m}^{n+} + a_{jm}^{n+} a_{j-m}^{p+} \right) ; \quad S_-^{np} = (S_+^{np})^+$$

and T is the isotopic spin for a system.

The elementary method in this case [3] extended to calculations of energy-contributions gives separately the very handy *nd* simple formulas [4]

$$\begin{aligned} E_n &\equiv \langle S_+^n S_-^n \rangle = n + w\, n_1 n_4 + n_2(1+n_4) + (1-u)\, \frac{n_3}{2}(1+n_1+n_4) \\ E_p &\equiv \langle S_+^p S_-^p \rangle = n_1(1+n_2) + w\, n_1 n_4 + (1-u)\, \frac{n_3}{2}(1+n_1+n_4) \\ E_{np} &\equiv \frac{1}{2}\langle S_+^{np} S_-^{np} \rangle = n_1 + 2(1-w) n_1 n_4 + u\, n_3 (1+n_1+n_4)\,, \end{aligned} \tag{10}$$

where, in analogy to (6) we get

$$n_1 = \frac{n}{4} - \frac{T}{2}; \qquad n_2 = T_0; \qquad n_3 = T - T_0 \qquad n_4 = \Omega - \frac{n}{4} - \frac{T}{2} \tag{11}$$

and

$$u = \frac{T+T_0}{2T-1}; \qquad w = \frac{2(T^2+T+T_0^2-1)}{(2T-1)(2T+3)}\,. \tag{12}$$

Taking the sum of (10), we get

$$\langle S_+^n S_-^n \rangle + \langle S_+^p S_-^p \rangle + \frac{1}{2}\langle S_+^{np} S_-^{np} \rangle = \frac{n}{4}\left(2\Omega + 3 - \frac{n}{2}\right) - \frac{T}{2}(T+1) \tag{13}$$

which is the same exact formula as in (8).

Now, however, we can adjust any of the strength parameters G_n, G_p, G_{np} separately. Usually it is suggested that

$$G_n = G_p > G_{np}\,. \tag{14}$$

2. Two examples

We consider two applications of the formulas (10). Let us consider the dependence of three contributions (10) to the pairing energy on the third isospin component T_0. We get

$$\begin{aligned} E_n &= aT_0^2 - b\,T_0 + c\,, && (15) \\ E_p &= aT_0^2 + b\,T_0 + c\,, && (16) \\ E_{np} &= -2aT_0^2 - 2c + E_{\text{pair}}\,, \end{aligned}$$

where

$$
\begin{aligned}
a &= \frac{4n_1n_4 + (1+n_1+n_4)(2T+3)}{2(2T-1)(2T+3)} \\
b &= \frac{n_1 - n_4 - 1}{2} \\
c &= n_1 + (1+n_1+n_4)\frac{T(T-1)}{2(2T-1)} + 2n_1n_4\frac{T^2+T-1}{(2T-1)(2T+3)}
\end{aligned}
\tag{17}
$$

and $E_{\rm pair}$ is given by (13).

We take a schematic example: $\Omega = 28$; $n = 50$; $T = 21$ and then $n_1 = 2$; $n_4 = 5$

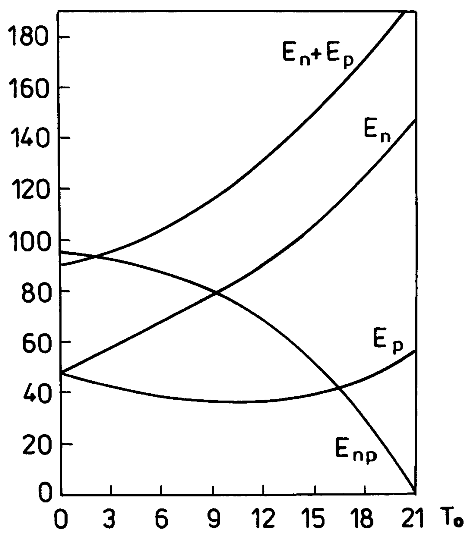

Fig. 2. Mean values of $\langle S_+S_-\rangle$ for nn, pp, and np versus $T_0; T = 21$.

Fig. 2 gives the $E(T_0)$ for three cases (15). We have shown, once again, that E_{np} is of the greatest importance for T_0 around zero, in our case for $-3 \leq T_0 \leq 3$. In this region E_{np} is larger then $E_n + E_p$, especially of we assume (13).

The $T = 21$ is chosen quite arbitrarily. For other T values the ΔT_0 (around $T_0 = 0$), for which the contribution of n–p pairing is larger than the sum of n–n and p–p, can be only changed.

In the second example we consider the following problem. Assume that the initial state $|i\rangle$ with $\nu = 0$ and $T = T_0 = 0$ ($N = Z$) has energy E_1. Now add two neutrons with $\nu = 0$ and consider the ground state of a system

$|f\rangle$ with energy E_2. The question: what are the expectation values

$$\frac{E_2^{(i)} - E_1^{(i)}}{E_1^{(i)}} \equiv \frac{\Delta E^{(i)}}{E_1^{(i)}}, \tag{18}$$

where $E^{(i)}$ means the expectation value for n; p; and np separately. The answer based on the formulas (10) is astonishing:

(i) The proton part, $\frac{\Delta E^p}{E_1^p}$, increases exactly(!) by 20% for any allowed values n and Ω.

(ii) The neutron–proton part drops exactly (!) by 40% and it does not depend on n and Ω.

(iii) The neutron part can vary from +170% to −40% (!) depending on n and Ω

$$\frac{\Delta E^n}{E_1^n} = \frac{4n\Omega - n^2 - 54n + 120\Omega}{20n\Omega - 5n^2 + 30n}. \tag{19}$$

The formula (18) is illustrated in two cases

(a) $\Omega = 10$ is fixed, hence $\Delta E^n/E_1^n$ is a function of n-only, where n is the number of nucleons in the initial state (Fig. 3).

(b) $n = 20$ is fixed, hence $\Delta E^n/E_1^n$ is a function of Ω only (Fig. 4).

In Fig. 3 and Fig. 4 we see, that there are regions of n and Ω, where addition of a pair of neutrons diminishes the neutron correlations.

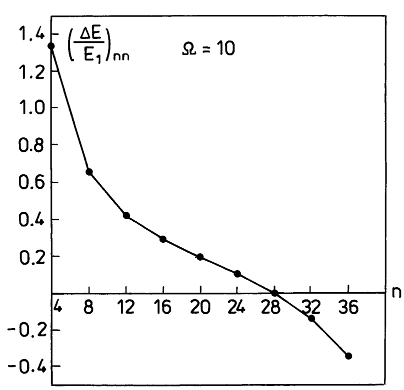

Fig. 3. Changes in neutron correlations measured by $\Delta E/E$ after a pair of neutrons is added versus n; $\Omega = 10$ is fixed.

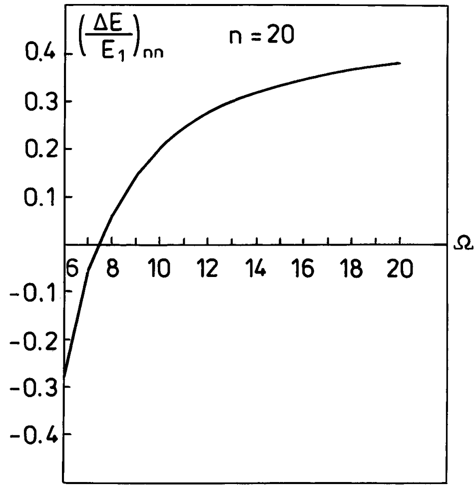

Fig. 4. The same as in Fig. 3 but versus Ω; $n = 20$ is fixed

Although the presented results are based on a simple model, their numerical part are exact. Hence, for nuclei with the shell model structure similar to considered in the paper, the pairing contribution is, at least, of the same qualitative value even in the presence of a more realistic two-body interaction.

More examples will be published soon.

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