THE QUADRUPOLE AND PAIRING VIBRATIONS IN RARE-EARTH NUCLEI* **

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The role of pairing collective degrees of freedom is investigated within microscopic approach based on the general collective Bohr model which includes the effect of coupling with the pairing vibrations. The excitation energies observed in transitional Gd and Er isotopes are reproduced in the frame of the calculation containing no free parameters.

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The microscopic approach to the general collective Bohr Hamiltonian (GBH) [1, 2] can be still successively used to interpret nuclear collective modes directly referring to single-particle degrees of freedom and thus introducing no adjustable parameters except those fixed for all nuclei: the single-particle potential parameters and the strength of the residual pairing interaction. However, in order to obtain in the frame of GBH (in its original form as in [2]) excitation energies comparable to the experimental data one has to enlarge mass parameters provided by the model. This situation can be improved by more careful treatment of residual forces — it seems that the renormalization of pairing strength needed in many calculations concerning collective nuclear properties is due to the pairing dynamics which should be included into description [3].

Recently we have proposed [4] the method of approximate treatment of the coupling between quadrupole and pairing vibrations which allows us to

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get the proper scale of low-lying excitation energies and moreover, to describe almost exactly collective properties of some transitional nuclei. The method was applied in the neutron deficient Te, Xe, Ba, Ce, and Nd region [4] as well as in the region of neutron rich Ru and Pd isotopes where it occurred especially efficient. In continuation we would like to extend the discussion to the rare-earth nuclei aiming to confirm the important or, at least, non negligible influence of the coupling with collective pairing degrees of freedom on nuclear movements.

The pure dynamical approach to such a coupling requires 9 dimensional solutions: we have to deal with two intrinsic variables β and γ parametrizing the nuclear shape, three Euler angles for orientation in space (denoted in short as Ω), two pairing gap parameters Δ^p and Δ^n for protons and neutrons and two corresponding gauge angles. In a given nucleus the gauge angles are constant so the "complete" hamiltonian takes the form

$$\hat{\mathcal{H}}_{\rm coll} = \hat{\mathcal{H}}_{\rm GBH}(\beta,\gamma,\Omega;\Delta^p,\Delta^n) + \hat{\mathcal{H}}_{\rm pair}(\Delta^p,\Delta^n;\beta,\gamma) + \hat{\mathcal{H}}_{\rm int}. \tag{1}$$

where (as in all following formulas) the variables placed after a semi-colon do not appear in differential operators. However, the diagonalization of (1) makes a problem because of the dimension of the appropriate basis. So we assumed instead, that the interaction term $\hat{\mathcal{H}}_{\rm int}$ can be neglected and, moreover, that only the ground state of the collective pairing hamiltonian $\hat{\mathcal{H}}_{\rm pair}$ comes into the function of a nuclear state when low-lying excitations are considered. It means that the coupling of quadrupole and pairing vibrations is realized through the inertial functions appearing in both, GBH and pairing collective Hamiltonians.

At each deformation point β,γ we can easily solve the one-dimensional eigen equation of each independent term in

$$\hat{\mathcal{H}}_{\rm pair} = \hat{\mathcal{H}}^{Z}_{\rm pair} + \hat{\mathcal{H}}^{A-Z}_{\rm pair} \tag{2}$$

which reads as [6, 7]:

$$\hat{\mathcal{H}}^{\mathcal{N}}_{\rm pair} = -\frac{\hbar^2}{2\sqrt{g(\Delta)}}\frac{\partial}{\partial\Delta}\frac{\sqrt{g(\Delta)}}{B_{\Delta\Delta}(\Delta)}\frac{\partial}{\partial\Delta} + V_{\rm pair}(\Delta), \tag{3}$$

where $\mathcal{N} = Z$, $\Delta = \Delta^p$ for protons and, respectively, $\mathcal{N} = A - Z$, $\Delta = \Delta^n$ for neutrons and $g(\Delta)$ is the appropriate determinant of the metric tensor. The collective pairing potential $V_{\rm pair}(\Delta)$ (equal to the usual BCS energy calculated for a given gap value) depends more-less parabolically on Δ while its minimum corresponds to the equilibrium pairing gap $\Delta_{\rm eq}$ obtained in the BCS formalism. But the pairing mass parameter $B_{\Delta\Delta}(\Delta)$ determined according to known formulas [7] goes up so rapidly for small Δ values that the resulting pairing vibrational ground state function is shifted towards smaller gaps. The ratio of the most probable gap value $\Delta_{\rm vib}$ to the equilibrium one is of about 0.7 on average (it grows slightly with the number of particles). This relatively small effect appears sufficient to obtain mass parameters of GBH 2–3 times larger then used to be. Assuming the Δ-shift as the main effect due to the coupling of quadrupole and pairing vibrations (at least on average) we should expect that low-lying (and low-spin) nuclear collective excitations could be quite correctly described by means of the hamiltonian [4, 5]:

$$\hat{\mathcal{H}}_{\rm coll} \approx \hat{\mathcal{H}}_{\rm GBH}(\beta,\gamma,\Omega;\Delta^p=\Delta^p_{\rm vib}(\beta,\gamma),\Delta^n=\Delta^n_{\rm vib}(\beta,\gamma))+E_{\rm pair}, \qquad (4)$$

where $E_{\rm pair}$ is the pairing vibrational ground state energy. The hamiltonian (4) has the same form as the classic one: it composes of the collective potential $V_{\rm coll}$ evaluated within the standard Strutinski macroscopic-microscopic method and of kinetic vibrational $\hat{\mathcal{T}}_{\rm vib}$ and rotational $\hat{\mathcal{T}}_{\rm rot}$ terms depending on the set of inertial functions derived microscopically in the frame of standard cranking method.

$$\begin{aligned}\hat{\mathcal{H}}_{\rm GBH} \;=\; & \hat{\mathcal{T}}_{\rm vib}(\beta,\gamma;\Delta^p_{\rm vib},\Delta^n_{\rm vib}) \\ & +\hat{\mathcal{T}}_{\rm rot}(\beta,\gamma,\Omega;\Delta^p_{\rm vib},\Delta^n_{\rm vib})+V_{\rm coll}(\beta,\gamma;\Delta^p_{\rm vib},\Delta^n_{\rm vib}), \qquad (5)\end{aligned}$$

The inertial functions appearing in (5) i.e. mass parameters and moments of inertia depend, in general, on intrinsic variables β,γ and pairing gap values. Thus approach to GBH differs from the standard one because here all inertial functions as well as the collective potential at each deformation point are calculated using the most probable values of proton and neutron gap parameters $\Delta^p_{\rm vib},\Delta^n_{\rm vib}$ instead of the BCS equilibrium ones $\Delta^p_{\rm eq},\Delta^n_{\rm eq}$.

We should mention that in all our calculations the Nilsson single-particle potential with the shell-dependent Seo parametrisation [8] and the standard estimations for the monopole pairing strength [4] were adopted.

The simple method briefed above works quite well in all nuclei considered up to now and it appears especially successful in describing collective properties of transitional triaxially deformed and/or soft nuclei [5]. Intensively studied isotopes from the rare-earth region exhibit a very rich spectroscopic structure: collective excitations of higher multipolarities as well as single particle modes should be taken into account in the complex description of their spectra. Nevertheless, in the frame of our approximation we are able to reproduce ground state and γ bands in $^{152-166}$Er and $^{148-162}$Gd isotopes (Fig. 1 and Fig. 2). Our results agree well with the experiment in spite of some discrepancies occurring mostly for nuclei with 84 neutrons only. As it is exemplified in Fig. 3 positions of levels built on the second 0^+ state are also situated rather correctly regarding the lack of hexadecapole mode in the description.

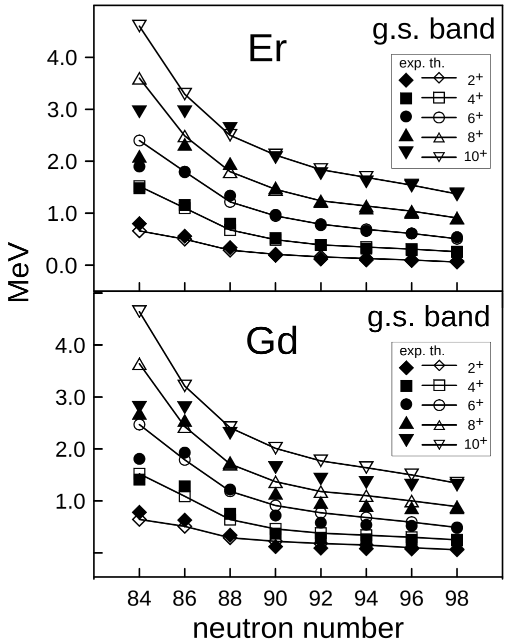

Fig. 1. The experimental [10] and the theoretical (connected with the straight lines) energies of the ground state band levels in Gd and Er isotopes.

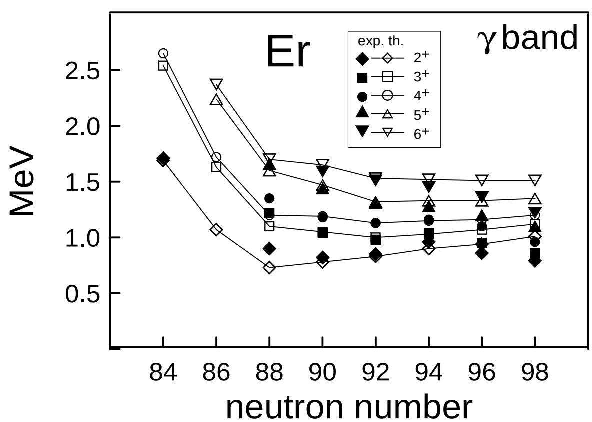

Fig. 2. The experimental [10] and the theoretical (connected with the straight line) energies of the γ band levels Er isotopes.

It should be pointed out that nuclei (Er isotopes) from the considered region were lately studied in the frame of the Triaxial Projected Shell Model [9] with the comparable success in resulting ground- and γ band positions. How-

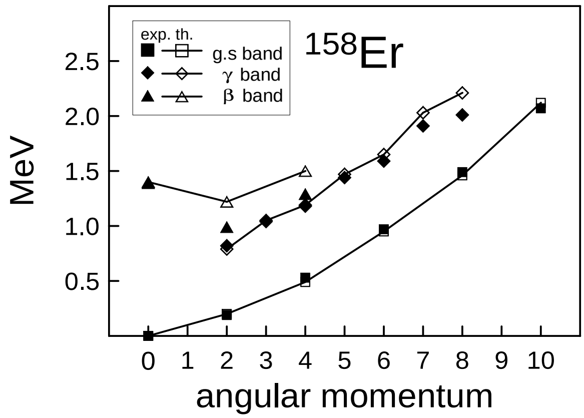

Fig. 3. The comparison of experimental [10] and the theoretical (connected with the straight lines) low-lying bands in ^{158}Er.

ever, the mentioned calculation needs some fitting procedure in order to determine the appropriate value of a triaxiality parameter while in our approximation there are no fixed shape deformations and no adjusted parameters at all.

Summarizing we can say that also in rare earths the role of coupling between quadrupole and pairing vibrations seems to be deciding for the proper balance of different collective modes. Adopting the simple approximation we are able to describe the main properties of low-lying collective excitations in most of even–even transitional nuclei.

REFERENCES

[1] M. Baranger, K. Kumar, *Nucl. Phys.* **A122**, 241 (1968).

[2] S.G. Rohoziński, J. Dobaczewski, B. Nerlo-Pomorska, K. Pomorski, J. Srebrny, *Nucl. Phys.* **A292**, 66 (1977).

[3] S. Piłat, K. Pomorski, *Nucl. Phys.* **A554**, 413 (1993).

[4] L. Próchniak, K. Zając, K. Pomorski, S.G Rohoziński, J. Srebrny, *Nucl. Phys.* **A648**, 181 (1999).

[5] K. Zając, L. Próchniak, K. Pomorski, S.G Rohoziński, J. Srebrny, *Nucl. Phys.* **A653**, 71 (1999).

[6] D.R. Bès, R.A. Broglia, R.P.J. Perazzo, K. Kumar, *Nucl. Phys.* **A143**, 1 (1970).

[7] A. Góźdź, K. Pomorski, M. Brack, E. Werner, *Nucl. Phys.* **A442**, 50 (1985).

[8] T. Seo, *Z. Phys.* **A324**, 43 (1986).

[9] J.A. Sheikh, K. Hara, *Phys. Rev. Lett.* **82**, 3968 (1999); Y. Sun, K. Hara, J.A. Sheikh, J.G. Hirsch, V. Velasquez, M. Guidry, in print.

[10] R.B. Firestone *et al.*, *Table of Isotopes* (8-th edition) John Wiley & Sons Inc. 1996.