SPONTANEOUS FISSION AND α-DECAY HALF-LIVES OF SUPERHEAVY NUCLEI* **

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Spontaneous fission and alpha-decay half-lives of even–even superheavy nuclei $112 \leq Z \leq 120$ are calculated on the basis of the deformed Woods-Saxon potential in WKB approximation by the multi-dimensional dynamical programing method in space of parameters describing the shape of nuclei $\{\beta_2, \beta_4, \beta_6\}$ and the pairing degrees of freedom $\{\Delta_p, \Delta_n\}$. The direct comparisons of the spontaneous fission and α-decay half-lives calculated by the Viola–Seaborg formula establish the regions of dominance of the spontaneous fission and the α mode of desintegration.

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1. Introduction

The last years brought many significant successes in the field of the synthesis of very heavy nuclei. Splendid works done in the GSI laboratory led to the production of nuclei with atomic numbers Z=110, 111 and 112 [1–3]. This nuclei were found almost on the edge of the *island of stability* which was for a long time unattainable for physicists.

However, the decisive work was the synthesis of the element Z=114 in JINR at Dubna [4], because it really proved the existence of this hypothetical, till now, island of stability. The lifetime of the synthesized nucleus is about one minute — very long in the scale of the lifetimes of heavy nuclei. This is interesting, that this result is almost exactly peaceable with the theoretical expectations calculated in the WKB method on the base of the single-particle Woods–Saxon potential [6,8].

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Lately the Lawrence Berkeley National Laboratory (LBNL) informed, that it succeeded to synthesize nuclei with atomic numbers Z=116 and 118 [5]. In this manner, for first time, the barrier of the *magic* number of protons Z=114 was overcome.

The above works will, of course, demand confirmations and of specifying of the received results. But, this is visible, that the attention of experimenters will be focused on the nuclei in the region of Z=112–120 and $N \sim 184$.

The subject of the present paper is to study the spontaneous-fission ($T_{\rm sf}$) and α-decay (T_α) half-lives of the even–even nuclei with proton numbers $Z = 112 \div 120$ and neutron numbers $N=152 \div 196$ in full dynamical method and in possibly optimal collective space of parameters.

There exists many theoretical papers analyzing the spontaneous fission half-lives ($T_{\rm sf}$) of nuclei in this region of atomic numbers (see for example: [7–9]). The $T_{\rm sf}$ were calculated in the static approximation in [7] and in the *combined* method [8,9] *e.g.*, by the dynamical calculations in a two-dimensional deformation space with the simultaneous minimization of the potential energy in the remaining degrees of freedom. However, more exact and many-sided theoretical investigations will permit better understanding of the spontaneous fission and allow to plan the next experiments touching the synthesis of the superheavy nuclei.

As the base of the present calculations it serves the deformed single-particle Woods–Saxon potential with the *universal* parameters [10]. As it was shown in [6,8] this parametrisation gives good results concerning fission of the heaviest nuclei.

We have taken into account three deformation degrees of freedom $(\beta_2, \beta_4, \beta_6)$, describing the shape of nuclei and two pairing degrees of freedom $(\varDelta_p, \varDelta_n)$, describing the possible changes of superfluid properties of the nucleus during the penetration of the barrier. As was shown in earlier papers [6,11] this set of collective parameters is practically sufficient for estimations of the $T_{\rm sf}$ of the superheavy nuclei.

To minimize the action integrals describing probability of spontaneous fission we have used the multi–dimensional dynamic-programming method (MDP) [6] and the WKB approximation.

The alpha-decay half-lives (T_α) are calculated by the Viola–Seaborg [12] formula with parameters modified to the new experimental data [13].

The theoretical background is described shortly in Section 2. In Section 3 the results are presented and discussed.

2. Theoretical model

We based on the single-particle Woods–Saxon potential with the *universal* set of parameters [10]. The values of 12 constants that determine the W–S potential parametrization are specified in Ref. [10]. In the present study, except of the deformation parameters (β_λ, λ=2,4,6), describing the shape of nuclei, we have included the pairing degrees of freedom: protons and neutrons gaps (Δ_p and Δ_n) as a collective coordinates. The earlier calculations have shown that the effect of the coupling of the nuclear shape parameters and the pairing vibrations plays an important role in the estimations of the fission lifetime [6].

According to the Strutinsky model, the collective potential energy V is splitted into a shell $\delta E_{\rm shell}$ and the pairing correction parts $\delta E_{\rm pair}$ and the smooth average background energy defined as the folded Yukawa plus exponential model with the standard values of its parameters [14].

The residual pairing interaction is treated in the BCS approximation with the pairing strength constants as in [15].

The collective mass B_{kl}, describing the inertia of a nucleus with respect to changes of its shape or superfluid properties is calculated in the adiabatic cranking model. It plays the role of a metric tensor in the multi-dimensional collective space.

The fission process of a nucleus is described as a tunnelling through the collective potential energy barrier. In the WKB approximation the spontaneous-fission half-life is inversely proportional to the probability of the penetration through the barrier:

$$T_{\rm sf} = \frac{\ln 2}{n}\frac{1}{P}\,. \tag{1}$$

Here, n is the number of *assaults* of the nucleus on the fission barrier per unit of time: $n \approx 10^{20.38}{\rm s}^{-1}$. With the use the one-dimensional WKB semi-classical approximation for the penetration probability P, the following formula can be determined:

$$P = \left(1 + {\rm e}^{S}\right)^{-1}\,, \tag{2}$$

where $S(L)$ is the action integral calculated along the fission path $L(s)$ in the multi–dimensional collective space:

$$S(L) = \int_{s_1}^{s_2} \left\{ \frac{2}{\hbar^2} B_{\text{eff}}(s)[V(s) - E] \right\}^{1/2} ds\,. \tag{3}$$

An effective inertia associated with the fission motion along the path $L(s)$ is

$$B_{\text{eff}}(s) = \sum_{k,l} B_{kl} \frac{dq_k}{ds}\frac{dq_l}{ds}\,, \tag{4}$$

In these equations ds denotes an element of the path length in the collective space. The integration limits s_1 and s_2 correspond to the classical turning points, determined by the equation $V(s) = E$, and E is the energy of the fissioning nucleus.

Dynamic calculations of (T_{sf}) mean a quest for the trajectory L_{min} which fulfills a principle of stationary action:

$$\delta[S(L)] = 0\,. \tag{5}$$

To minimize the action–integral we have used the multi-dimensional dynamic-programming method (MDP). Its application to fission was first developed by Baran *et al.* (see *e.g.*, [16] and references quoted therein).

Since the macroscopic-microscopic method is not analytical, it is necessary to calculate the potential energy and all components of the inertia tensor on a grid in the multi-dimensional space spanned by a set of collective degrees of freedom.

The finding of the least action trajectory L_{min} is equivalent to the problem of determination of the geodetic line between the turning points s_1 and s_2 with the mass parameter B_{kl} being the metric tensor of the multi-dimensional collective space $\{q_\alpha\}$.

In order to find the least action trajectory L_{min} we proceed as follows: We select the β_2 coordinate (*e.g.*, the elongation parameter). The pressumed fission path has to be monotoneous function of this coordinate. In the next step we calculate the action integral $S(L)$ along the β_2 coordinate, *passing through all* remaining grid points of $\{q_i\}$, (i=1,2,..., α–1) degrees of freedom. If we denote by n a number of grid points of β_2 coordinate and by n_i (i=1,2,..., α–1) the number of grid points on the remaining (α–1) coordinates, then the total number of trajectories examined in the MDP method will be equal to $(n_1 \cdot n_2 \cdot \ldots \cdot n_{\lambda-1})^n$. The trajectory with minimal $S(L)$ corresponds to the trajectory L_{min} searched in the least-action principle.

The method of calculating the α decay half-lives is based on the well known phenomenological formula by Viola–Seaborg [12] with refitted values of parameters as in [13].

3. Results

The multi-dimensional collective space consists of three parameters describing the shape of nuclei (β_2,β_4,β_6) and two pairing degrees of freedom (Δ_p and Δ_n). On account of computational limitations we can only perform calculations in a maximum of four dimensional collective space. The calculations of $T_{\rm sf}$ in the five-dimensional collective space $\{\beta_2, \beta_4, \beta_6, \Delta_p, \Delta_n\}$ are approximated by exact results obtained in three dimensional space $\{\beta_2, \beta_4, \beta_6\}$ and the *pairing correction* $\delta T_{\rm sf}(\Delta_p, \Delta_n)$ in the following way

$$T_{\rm sf}(\beta_2, \beta_4, \beta_6, \Delta_p, \Delta_n) = T_{\rm sf}(\beta_2, \beta_4, \beta_6) + \delta T_{\rm sf}(\Delta_p, \Delta_n)\,. \quad (6)$$

The pairing correction, $\delta T_{\rm sf}(\Delta_p, \Delta_n)$ is the difference between the dynamical values of $T_{\rm sf}$ calculated in 4-dimensional space $\{\beta_2, \beta_4, \Delta_p, \Delta_n\}$ (with Δ_p and Δ_n as a collective degrees of freedom) and 2-dimensional space $\{\beta_2, \beta_4\}$ (in which the proton and neutron pairing gaps are obtained in the statical BCS approximation)

$$\delta T_{\rm sf}(\Delta_p, \Delta_n) = T_{\rm sf}(\beta_2, \beta_4, \Delta_p, \Delta_n) - T_{\rm sf}(\beta_2, \beta_4)\,. \quad (7)$$

The inspection shows that this effect is important because it reduces the SF half-lives of nuclei by about 0.5–2.5 orders of magnitude. The $\delta T_{\rm sf}(\Delta_p, \Delta_n)$ is maximal for nuclei with neutron number N ∼184 and reaches $2.0 \div 2.5$ orders of magnitude.

In Figure 1 we show the spontaneous fission ($T_{\rm sf}$) and α-decay (T_α) half-lives for nuclei with atomic number Z=112, 114, 116, 118 and 120 in succession. The neutron number N changes from N=150 to N=196. Triangles denote the values of $T_{\rm sf}$ and the squares the α-decay half-lives T_α. Lifetimes, in years, are given in the logarithmic scale.

The discovery of the new element become possible if the lifetime of the nucleus is longer than 1 μs (~ -13.5 y in logharitmic scale). As it is seen from the figures this limits the nuclei to these with neutron numbers $158 \leq N \leq 190$ for Z=112, 114 and 116 and only to $168 \leq N \leq 188$ for Z=118 and 120. It is practically impossible to observe the remaining nuclei.

Two very specific effects can be observed in the demonstrated figures. One can see a two local maxima of the SF half-life curves at N ∼162 and N ∼184, separated by a minima at N ∼170. The similar behavior is also observed for the α-decay half-life, but the diminution of the half-lives at N ∼170 is considerably smaller. The enhancement in nuclear stability near the deformed shell N=162 allows for the appearance of a peninsula of the deformed metastable superheavy nuclei, while the local maximum at N=184 leads to the island of spherical superheavy nuclei.

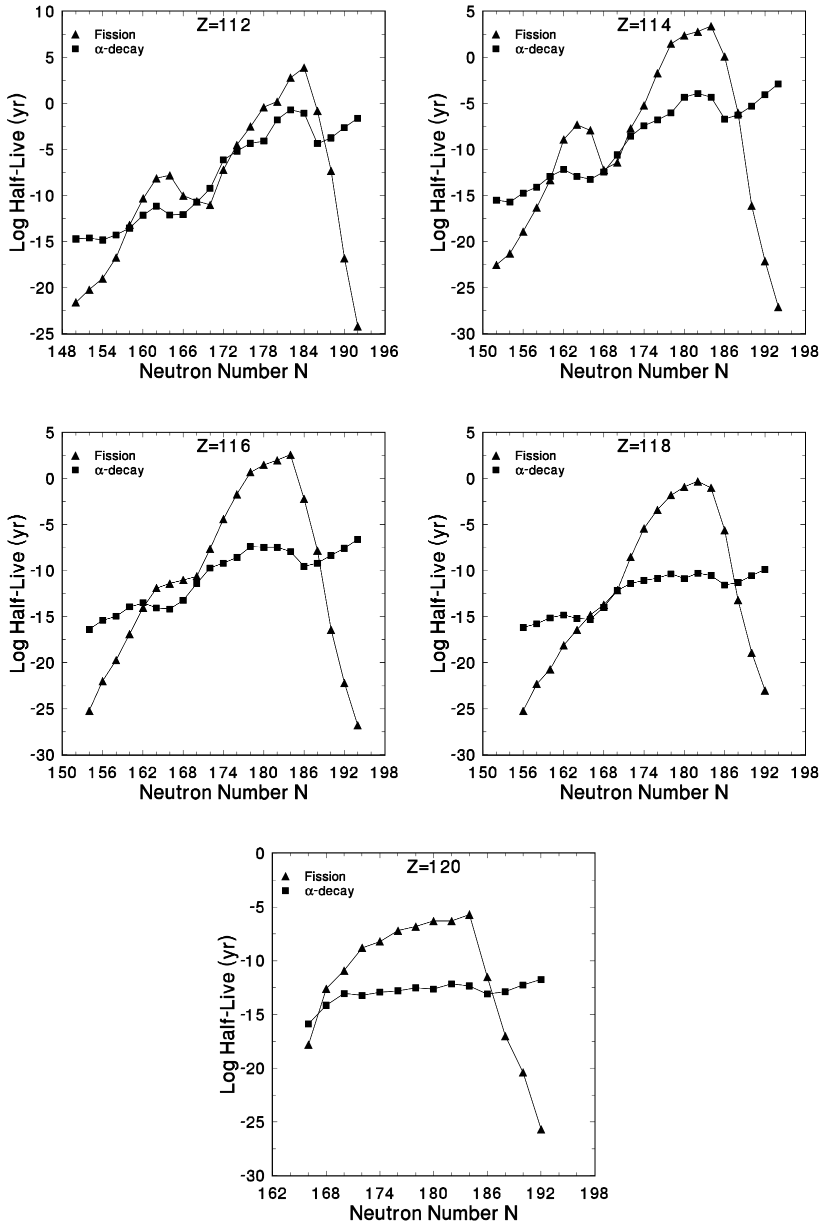

Fig. 1. Spontaneous fission ($T_{\rm sf}$) and α-decay (T_α) half-lives (given in years) of the even–even isotopes with atomic number Z=112, 114, 116, 118 and 120 plotted as a function of the neutron number N.

The value of $\sim 10^4$ y for $T_{\rm sf}$ is get for the double-magic nucleus $^{298}114_{184}$. The similar values are obtained for the nuclei Z=112 and 116 showing the closed spherical neutron shell at N=184. (The shell N=184 was also reported in recent Skyrme–Hartree–Fock–Bogoliubov calculations by Ćwiok *et. al.*, [18] and indirectly in the relativistic mean field calculations by Meng and Takigawa [19]. In their Fig. 3 one sees N=184 subshell and the large N=198 shell.)

Our results are lowered by about two orders of magnitude as compered to those obtained in [9]. It is the effect of the pairing degrees of freedom accounted in the present paper. While the effect of the inclusion of the pairing degrees of freedom reduces the spontaneous fission $T_{\rm sf}$ data and does not influence the α-decay half-lives T_α the relation between both modes of desintegration significantly changes. Let us trace this more exactly in our drawings.

In the vicinity of the deformed superheavy nuclei as well as in the region of spherical superheavy nuclei the spontaneous fission half lives are considerably longer than the corresponding α-decay half-lives. This means, that these nuclei will be desintegrated by α-decay and this is in agreement with existing experimental data [1–4]. We can also see, that the nuclei with Z=118 and 120 are practically all α-radioactive what is confirmed in the last experiment of the Berkeley group [5].

Near the neutron number N=170 one observes the opposite behaviour. Here, SF half-life values are smaller or comparable to T_α. We can say, that nuclei from this region ($N \sim$170–174) will be desintegrated by spontaneous fission rather than α-decay. This effect is very strong for Z=112 isotopes and a little more weak in the case of Z=114 and 116 nuclei.

All figures permit more exact tracing of the relations between $T_{\rm sf}$ and T_α for all considered nuclei.

In the present work we have investigated only the even–even nuclei. Unfortunatelly, the most synthesized nuclei are the odd-A or odd–odd systems. However, the approximate conclusions, relating the odd-systems can be withdrawn from the present paper and our earlier works [17,18]. For odd-A and odd–odd nuclei the $T_{\rm sf}$ increases considerably due to the effect of an unpaired nucleon [17]. Simultaneously the α-decay half-lives grows [18]. From this kind of reasoning, the above conclusions are expected to hold for odd-A and odd–odd nuclei. Howeever, it has to be confirmed by the suitable calculations for odd nuclei. This is the subject of our next paper.

4. Conclusions

The following conclusions can be drawn from our investigations:

- We have calculated the spontaneous-fission and α-decay half-lives of the even–even nuclei with proton number $Z = 112 \div 120$ and neutron number $N = 152 \div 196$ in the multi-dimensional collective space spanned by three shape parameters (β_2,β_4,β_6) and two pairing degrees of freedom (Δ_p and Δ_n).

- The pairing degrees of freedom reduce the SF half-lives by 0.5–2.5 orders of magnitude and considerably change the theoretical predictions relating the manner of disintegration of nuclei. This effect is due to the strong dependence of the nuclear inertia tensor upon the pairing gap.

- Comparison of the calculated spontaneous fission and α-decay half-lives leads to the conclusion that the α-decay mode should be dominating for nuclei around N=162 — the deformed superheavy nuclei and for N=184 — the island of the spherical superheavy nuclei.

- The nuclei in the vicinity of N=170 can desintegrate by spontaneous fission rather than α-decay process.

These conclusions we can formulate for even–even nuclei while only such systems were considered.

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