SPIN-ORBIT PENDULUM (RELATIVISTIC EXTENSION)*

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We discuss an extension of the theory of *spin-orbit pendulum* phenomenon to relativistic approach. It is done within the so called Dirac Oscillator. Our first results, focusing on circular wave packet motion have been published recently. The scope of this paper is motion of a linear wave packet. In relativistic approach we found *Zitterbewegung* in spin-orbit motion (in Dirac representation) due to coupling to negative energy states. This effect is washed out in the Foldy-Wouthuysen representation. Another important change with respect to non-relativistic case is the loss of periodicity. The phenomenon reminds the time evolution of population inversion in Jaynes-Cummings model.

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1. Introduction

Few years ago we investigated motion of a wave packet (WP) representing a fermion in the harmonic oscillator potential with spin-orbit coupling. In this model (and non-relativistic approach) we have found the *spin-orbit pendulum* phenomenon [1]. We predicted there that spin-orbit forces creates interesting oscillations of expectation values of spin components if particle state is prepared as a well localized wave packet. Then expectation values of the spin and angular momentum components oscillate periodically. During

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one period of this time evolution, the spin collapses (at the same time the wave function initially pure in both subspaces gets maximum entanglement) then (almost) restores in the opposite direction, collapses again and restores exactly.

The aim of this paper is to extend the model to relativistic WPs, focusing on cases corresponding to linear classical trajectories of a particle (the other interesting case of circular trajectories is discussed elsewhere [2]). The relativistic harmonic oscillator has been introduced many years ago in particle physics [3] and then refreshed [4] under the name *Dirac Oscillator* (DO). More recently the behavior of the WPs in DO was studied in both Dirac and Foldy-Wouthuysen (F-W) representations in 1+1 dimensions [5].

2. Dirac Oscillator

DO is described by the time dependent equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = H_{\rm DO}\Psi = c\,[\,\boldsymbol{\alpha}\cdot(\,\boldsymbol{p} - im\omega\boldsymbol{r}\beta) + mc\beta]\;, \tag{1}$$

where $\boldsymbol{\alpha}$, β are usual Dirac matrices. One can show that both the large and small component of an DO eigenstate are proportional to $|N(ls)jm_j\rangle$ – an eigenstate of the 3d-HO with a spin-orbit coupling. The energy spectrum has the form of

$$E_{Nlj} = \pm\, mc^2\sqrt{1 + r\,A_{Nlj}} \qquad A_{Nlj} = \begin{cases} 2(N-j)+1 & \text{for } j = l+\frac{1}{2} \\ 2(N+j)+3 & \text{for } j = l-\frac{1}{2} \end{cases} \tag{2}$$

The parameter $r = \hbar\omega/mc^2$ enables, if it is small enough, a transition to the non-relativistic limit.

3. Initial form of the WP

We study the evolution of a Gaussian WP which is initially centered at $\boldsymbol{r}_0$ and has the average momentum $\boldsymbol{p}_0$ (*i.e.* a 3d-HO coherent state). Moreover, the initial WP is an eigenstate of the spin pointed at some arbitrary direction defined by two numbers α and β:

$$\Psi(\boldsymbol{r}, t=0) = \frac{1}{(2\pi)^{\frac{3}{4}}\sigma^{\frac{3}{2}}}\exp\left[\frac{(\boldsymbol{r}-\boldsymbol{r}_0)^2}{2\sigma^2} + i\frac{\boldsymbol{p}_0\cdot\mathbf{r}}{\hbar}\right]\begin{pmatrix}\alpha\\ \beta\\ 0\\ 0\end{pmatrix}, \tag{3}$$

where $\sigma = \sqrt{\hbar/m\omega}$. In the following we present the time evolution of *linear* WP (3) corresponding to the following initial conditions: $\boldsymbol{r}_0 = z_0\,\hat{z}$ and

$\boldsymbol{p}_0 = p_0\, \hat{z}$. It is possible thanks to decomposition [1] :

$$\Psi_{\rm lin}(t=0) = \sum_{N}^{\infty} \sum_{l=0(1)}^{N} \lambda_{Nl}\, |N,l,0\rangle \begin{pmatrix} \alpha \\ \beta \\ 0 \\ 0 \end{pmatrix} . \tag{4}$$

4. Time evolution

Fig. 1 shows the behavior of spin averages for the linear WP in the Dirac representation. The transition from the pure state of the spin in a well-defined direction to the entangled state is particularly clear in the case of $(\theta_\sigma = 0)$. The initial conditions are such that WP is launched along Oz axis from the center with spin parallel to $\boldsymbol{p}_0$. Notice the *Zitterbewegung* and and deviation from periodicity for the relativistic case.

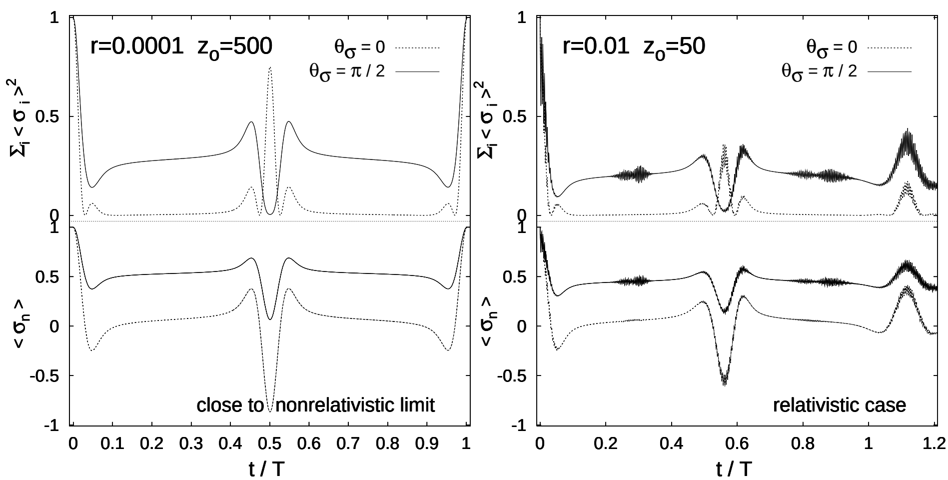

Fig. 1. Dependence of the spin averages time evolution for the linear WP on the initial spin direction. Two cases of r values are presented ($r = 0.0001$ and $r = 0.01$), $\langle \sigma_n \rangle$ is an average value of the spin projection on the initial spin direction.

An example of the spatial WP motion is shown in Fig. 2. The probability density is shown on zOy plane (there is cylindrical symmetry with respect to Oz axis). Sub-packets corresponding to states of positive and negative energies are well seen in Dirac representation, whereas only positive energy states are present in the F-W representation.

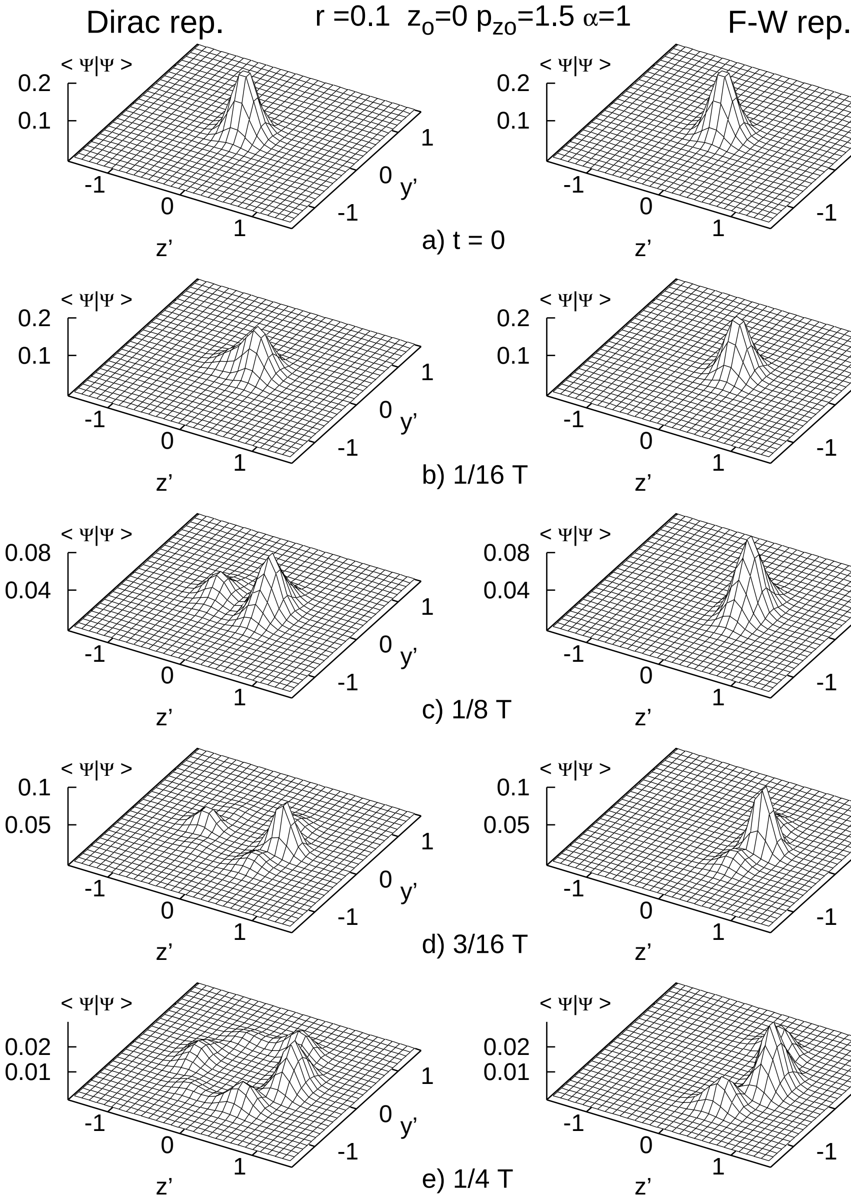

Fig. 2. Comparison of the evolution of the linear WP ($r = 0.5$) in Dirac and F-W representations. Initial spin direction is parallel to $\boldsymbol{p}_0$.

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