INFLUENCE OF THE STARK SHIFT AND KERR-LIKE MEDIUM ON THE EVOLUTION OF FIELD ENTROPY AND ENTANGLEMENT IN TWO-PHOTON PROCESSES

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We have investigated the evolution of the field quantum entropy and the entanglement of the atom-field in the two-photon process, taking into account the level shifts produced by Stark effect with an additional Kerr-like medium for one mode. The exact results are employed to perform a careful investigation of the temporal evolution of the entropy. A factorization of the initial density operator is assumed, with the privileged field mode being in a coherent state. We invoke the mathematical notion of maximum variation of a function to construct a measure for entropy fluctuations. The effect of both the Stark shift and the presence of a Kerr-like medium on the entropy is analyzed. It is shown that the addition of the Kerr medium and the Stark shift has an important effect on the properties of the entropy and entanglement. The results show that, the effect of the Kerr medium and the Stark shift changes the quasiperiod of the field entropy evolution and entanglement between the atom and the field. The general conclusions reached are illustrated by numerical results.

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1. Introduction

One of the curious features of quantum mechanics is that it is a theory in which probabilities play a most central role and yet, from a foundational point of view, the concept of entropy is conspicuously absent. Entropy appears only later as an auxiliary quantity to be used only when a problem is sufficiently complicated that clean deductive methods have failed and one is forced to use inference methods. This is curious indeed because once the use of the notion of probability has been accepted, the issue of whether or not quantum mechanics is a theory of inference has been unequivocally settled. Quantum theory should be regarded as a set of rules for reasoning in situations where even under optimal conditions the information available to predict the outcome of an experiment may still turn out to be insufficient. In such a theory entropy, as a measure of the amount of information [1], should play a central role.

In recent years much attention has been focused on the properties of the entanglement between the field and the atom and in particular the entropy of the system [2–11]. The authors in [2–4] have shown that entropy is a very useful operational measure of the purity of the quantum state, which automatically includes all moments of the density operator. The time evolution of the field (atomic) entropy reflects the time evolution of the degree of entanglement between the atom and the field. The higher the entropy, the greater the entanglement. An expression for the field entropy for the entangled state of a single two-level atom interacting with a single electromagnetic field mode in an ideal cavity with the atom undergoing either a one or a two-photon transition has been studied [10]. However, these results are obtained for the case where the Stark shift is ignored. To make the two-photon processes closer to the experimental realization, the effect of the dynamic Stark shift in the evolution of the field entropy, which is necessary and interesting is added [11]. Furthermore, we examine the effect of the dynamic Stark shift in the evolution of the field entropy and entanglement in the presence of a Kerr-like medium. This model consists of a single two-level atom undergoing a two-photon processes in a single-mode field surrounded by a nonlinear Kerr-like medium contained inside a very good quality cavity. The cavity mode is coupled to the Kerr medium as well as to the two-level atom. The Kerr medium can be modeled as an anharmonic oscillator with frequency ω. Physically this model may be realized as if the cavity contains two different species of Rydberg atoms, of which one behaves like a two-level atom undergoing two-photon transition and the other behaves like an anharmonic oscillator in the single-mode field of frequency $\omega_\circ$ [12,13]. Such a model is interesting by itself as another exactly solvable quantum model [14] that gives nontrivial results, but we can also think of its possible applications. This Hamiltonian is natural for local modes in molecular physics or for a nonlinear Jahn–Teller effect, although long-time behavior in either case might be obscured by omnipresent damping. There may also be optical applications, since this type of nonlinearity may be realized by letting the electromagnetic radiation pass through a nonlinear Kerr medium [15]. One can think of an experiment with a Rydberg atom in a nonlinear Kerr-like cavity. We will keep in mind this last situation throughout this paper.

The material of this paper is arranged as follows: In Section 2, we introduce the model and write the expressions for the final state vector at any time $t > 0$ and the field entropy calculation when the Stark shift and the

Kerr-like medium effects are included. By a numerical computation, we examine the influence of the Stark shift and the Kerr-like medium on the field entropy evolution and entanglement of the atom and the field for a coherent field input in Section 3. Finally, conclusions are presented in Section 4.

2. Basic equation

The model considered here consists of a single-mode with an effective two-level atom when the dipole forbidden transition is replaced by a two-photon one. We consider the degenerate case, in which pairs of photons with the same frequency are created or absorbed and the quantized radiation field in the rotating wave approximation in an ideal cavity ($Q = \infty$) filled with a nonlinear Kerr-like medium. We also assume that the cavity mode interacts with both the atom and the Kerr-like medium. However, a real cavity cannot be ideal. But in Ref. [16] the influence of a cavity with finite bandwidth at nonzero temperature T was studied and it was shown that for new available experimental values of $Q = 2^{10}$ and $T = 0.5K$ the effect of the bandwidth and the temperature are negligible until the time $t \sim 10^{-3}(\lambda t = 30)$ from the start of the interaction. The excited and ground states of the atom will be designated by $\mid e\rangle$ and $\mid g\rangle$, respectively. We assume that these states have identical parity, whereas the intermediate states, labeled $\mid j\rangle (j = 3, 4, ...)$, are coupled to $\mid e\rangle$ and $\mid g\rangle$ by a direct dipole transition and so located as to give rise to a significant Stark shift. The intensity dependent Stark effect can be employed in quantum nondemolition measurements [17–19]. Kerr effects can be observed by surrounding the atom by a non-linear medium inside a high Q-cavity [20]. The effective Hamiltonian of the model under consideration in this paper in the rotating-wave approximation can be written as [21],

$$\begin{aligned} \hat{H}_{\text{eff}} \;=\; & \omega_c \hat{a}^\dagger \hat{a} + \tfrac{1}{2}\omega_a \hat{\sigma}_z + \hat{a}^\dagger \hat{a}(\beta_2 \mid e\rangle\langle e \mid \\ & +\beta_1 \mid g\rangle\langle g \mid) + \chi \hat{a}^{\dagger 2} \hat{a}^2 + \lambda(\hat{a}^{\dagger 2}\hat{\sigma}_- + \hat{a}^2 \hat{\sigma}_+)\,, \end{aligned} \tag{1}$$

where ω_c is the field frequency and ω_a is the transition frequency between the excited and ground states of the atom, $\hat{a}$ and $\hat{a}^\dagger$, are the annihilation and the creation operators of the cavity field respectively, β_1 and β_2 are parameters describing the intensity-dependent Stark shifts of the two levels that are due to the virtual transitions to the intermediate relay level, λ is the effective coupling constant, σ_z and $\sigma_\pm$ are the atomic pseudo-spin operators. We denote by χ the dispersive part of the third-order nonlinearity of the Kerr-like medium, with the detuning parameter $\Delta = \omega_a - 2\omega_c$.

The initial state of the total atom-field system can be written as

$$\mid \psi_{\text{AF}}(0)\rangle = \sum_n^\infty q_n \mid n; e\rangle\,, \tag{2}$$

where the field is assumed to be initially in a coherent state where $q_n = e^{(-\bar{n}/2)}\alpha^n/\sqrt{n!}$, $\alpha =\mid \alpha \mid e^{i\phi}$ and $\bar{n} =\mid \alpha \mid^2$ is the mean photon number of the coherent field. However, at any time $t > 0$ the atom-field state is described by the entangled state

$$\mid \psi(t)\rangle = \sum_n^{\infty} \Big(A_n(t) \mid n; e\rangle + B_n(t) \mid n+2; g\rangle \Big) , \tag{3}$$

where the coefficients $A_n(t)$ and $B_n(t)$ are given by

$$\begin{aligned}
A_n(t) &= q_n e^{-i\lambda t G_n} \left(\cos \lambda t F_n - i W_n \frac{\sin \lambda t F_n}{F_n} \right) , &(4)\\
B_n(t) &= -i q_n V_n e^{-i\lambda t G_n} \frac{\sin \lambda t F_n}{F_n} , &(5)\\
G_n &= [n + r^2(n+2)]/2r + \chi n^2/\lambda , &(6)\\
F_n &= \sqrt{W_n^2 + V_n^2} , &(7)\\
W_n &= \Delta/2\lambda + [n - r^2(n+2)]/2r - \chi n/\lambda , &(8)\\
r &= \sqrt{\beta_1/\beta_2}, \quad V_n = \sqrt{(n+1)(n+2)} . &(9)
\end{aligned}$$

With the wave function $\mid \psi(t)\rangle$ calculated, any property related to the atom or the field can be calculated. The reduced density matrix of the field of the system can be written as $\rho_f(t) = \mathrm{Tr}_{\mathrm{atom}} \mid \psi(t)\rangle\langle\psi(t) \mid$,

$$\begin{aligned}
\rho_f(t) = \sum_{n,m=0}^{\infty} & [A_n(t) A_m^*(t) \mid n\rangle\langle m \mid + B_n(t) B_m^*(t) \\
& \times \mid n+2\rangle\langle m+2 \mid = \mid C\rangle\langle C \mid + \mid S\rangle\langle S \mid ,
\end{aligned} \tag{10}$$

where

$$\mid C\rangle = \sum_{n=0}^{\infty} A_n(t) \mid n\rangle, \quad \mid S\rangle = \sum_{n=0}^{\infty} B_n(t) \mid n+2\rangle .$$

Employing the reduced field density operator given by Eq. (10), we investigate the properties of the entropy. The quantum dynamics described by the Hamiltonian (1) leads to an entanglement between the field and the atom. In this paper, we use the field entropy as a measurement of the degree of entanglement between the field and the atom of the system under consideration. In order to derive a calculation formalism of the field entropy, we must obtain the eigenvalues and eigenstates of the reduced field density operator given by Eq. (10). Knight and co-workers [4] have developed a general method to calculate the various field eigenstates in a simple way. By using this method we obtain the eigenvalues and eigenstates of the reduced density operator,

$$\begin{aligned} \lambda_f^{\pm}(t) &= \langle C \mid C \rangle \pm \exp[\mp\theta] \mid \langle C \mid S \rangle \mid \\ &= \langle S \mid S \rangle \pm \exp[\pm\theta] \mid \langle C \mid S \rangle \mid , \end{aligned} \tag{11}$$

$$\begin{aligned} \mid \psi_f^{\pm}(t)\rangle &= \frac{1}{\sqrt{2\lambda_f^{\pm}(t)\cosh(\theta)}} \{\exp[(i\phi \pm \theta)/2] \mid C\rangle \\ &\quad \pm \exp[-(i\phi \pm \theta)/2] \mid S\rangle\} , \end{aligned} \tag{12}$$

where

$$\theta = \sinh^{-1}\left(\frac{\langle C \mid C\rangle - \langle S \mid S\rangle}{2 \mid \langle C \mid S\rangle \mid}\right). \tag{13}$$

We can express the field entropy $S_f(t)$ in terms of the eigenvalue $\lambda_f^{\pm}(t)$ of the reduced field density operator,

$$S_f(t) = -[\lambda_f^{+}(t) \ln \lambda_f^{+}(t) + \lambda_f^{-}(t) \ln \lambda_f^{-}(t)]. \tag{14}$$

It does not appear possible to express the sums in equation (10) in closed form, but for not too large $\bar{n}$, direct numerical evaluations can be performed. In what follows we shall consider the effect of both Kerr and Stark shift on dynamical behavior of the field entropy and entanglement of the system for two-photon processes.

3. Results of calculations

On the basis of the analytical solution presented in the previous section, we shall examine the temporal evolution of the field entropy. It should be emphasized that in computing all infinite series for the atomic wave function $\psi(t)$, we have invoked mathematically sound truncation criteria. To ensure an excellent accuracy the behavior of the field entropy function $S_f(t)$ has been determined with great precision. For regions exhibiting strong fluctuation a resolution of 10^3 point per unit of time has been employed. For all our plots the initial condition has been chosen, with coherence parameter α real. Its square is equal to the mean photon number. We recall that time t has been scaled; one unit of time is given by the inverse of the coupling constant λ.

We display the evolution of the field entropy for the initial coherent field with the absence of both Stark and Kerr-like medium. In our computations,

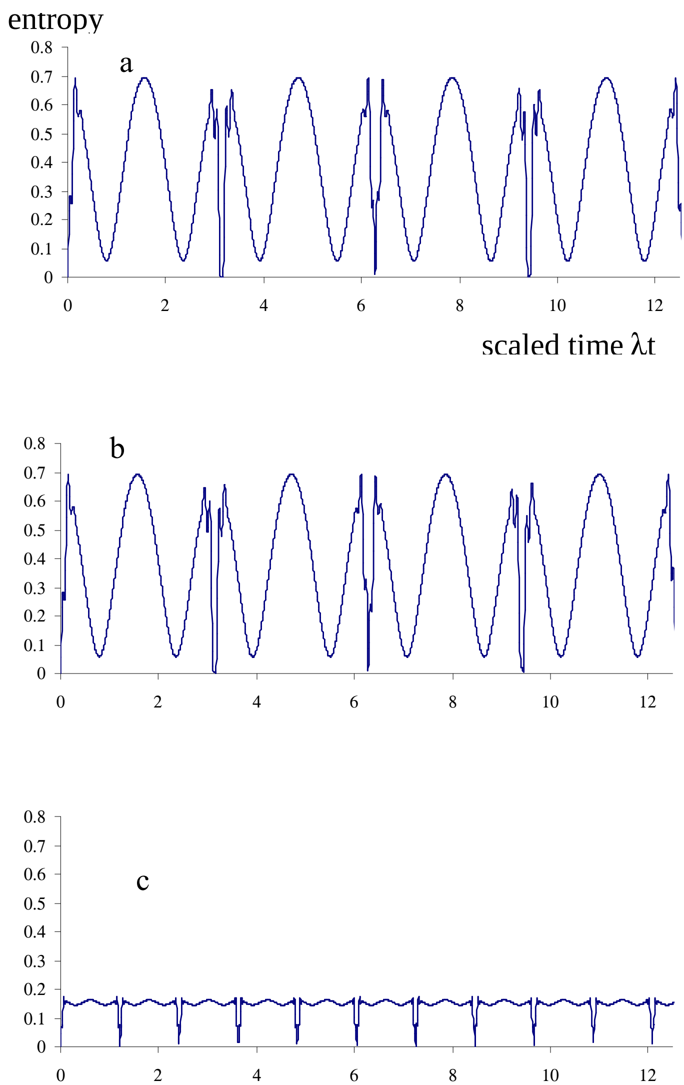

Fig. 1. The evolution of the field entropy in the two-photon process with the intensity of the initial coherent field equal to $\bar{n} = 25$, and for different values of Stark shift parameter $r = \sqrt{\beta_1/\beta_2}$, where (a) – $r = 0$, (b) — $r = 1$ and (c) — $r = 5$.

we have taken $\bar{n} = 25$. It is remarkable that the field entropy evolves periodically and in this case for the two-photon process is rather different compared with the one-photon case [4]. As seen from Fig. 1(a) in two-photon process entropy is a periodic function of time and the half of the revival time the field entropy reaches its maximum(in the case of the one-photon entropy is minimized at the half of the revival time and its behavior is rather irregular). Also, we show that the field entropy $S_f(t)$ evolved at periods π/λ, when $\lambda t = n\pi(n = 0, 1, 2, 3, ...)$, the field entropy evolves to the minimum values and the field is completely disentangled from the atom, while when $\lambda t = (n+\frac{1}{2})\pi$ the field entropy evolves to the maximum value and the field is strongly entangled with the atom. From Fig. 1(b) we show the effect of Stark shift parameter $r = 1(\beta_1 = \beta_2)$, which corresponds to the case in which the two levels of the atom are equally strongly coupled with the intermediate relay level. By comparing Fig. 1(a) and Fig. 1(b), we see that the evolution of the entropy is almost similar for both cases. This may be interpreted as follows. First, physically, this result corresponds to the fact that the Stark shift creates an effective intensity dependent detuning $\Delta_N = \beta_2 - \beta_1$[22]. When $r = 1, i.e.\ \Delta_N = 0$, in this case, the Stark shift does not affect the time evolution of the field entropy. Second, using an algebraic analysis in the high-field limit $\bar{n} >> 1$ (here $\bar{n} = 25$), the Poissonian distribution of the coherent state in the $\mid n\rangle$ representation means that the dominant contributions from q_n arise from $n \simeq \bar{n} >> 1$, and we can expand F_n which appears in Eq. (7) in powers of n^{-1}, thus, to order n^0, $F_n = \lambda(n + \frac{3}{2})$. On the other hand, if the Stark shift is ignored, we will get the same expression for $F_n = \lambda(n + \frac{3}{2})$. In Fig. 1(c), we show the case in which the two levels have unequal Stark shifts ($r > 1$, in Fig. 1(c), $r = 5$). We see that the Stark shift leads to a decreasing of the values of the maximum field entropy and the evolution period of the field entropy decreases with the parameter r.

To visualize the influence of the Kerr-like medium in the field entropy we set different values of χ/λ, and all the other parameters are the same as in Fig. 1. The outcome is presented in Fig. 2. One can distinguish between two stages of evolution, each of which has been pictured separately. We show that weak nonlinear interaction of the Kerr-like medium with the field mode leads to increasing values of the minimum entropy and of the sustainment time of the maximum entropy. In this case, the field and the atom almost retain a strong entanglement in the time evolution process. With the increase of the nonlinear interaction of the Kerr-like medium with field mode, the value of the maximum field entropy begins to decrease. In this case the degree of entanglement between the field and the atom reduces. We note that the amplitude of the field entropy decrease as χ/λ increase. It is evident that the field and the atom are in pure states when the Kerr-like effect increases. This result corresponds to the fact that in the limit for the very strong nonlinear interaction of the Kerr-like medium with the field mode, the field and the atom are almost decoupled and the time evolution of the field is governed by the Hamiltonian $H_{\text{eff}} \simeq \chi\hat{a}^{\dagger 2}\hat{a}^2$, which preserves the field entropy's tending to zero.

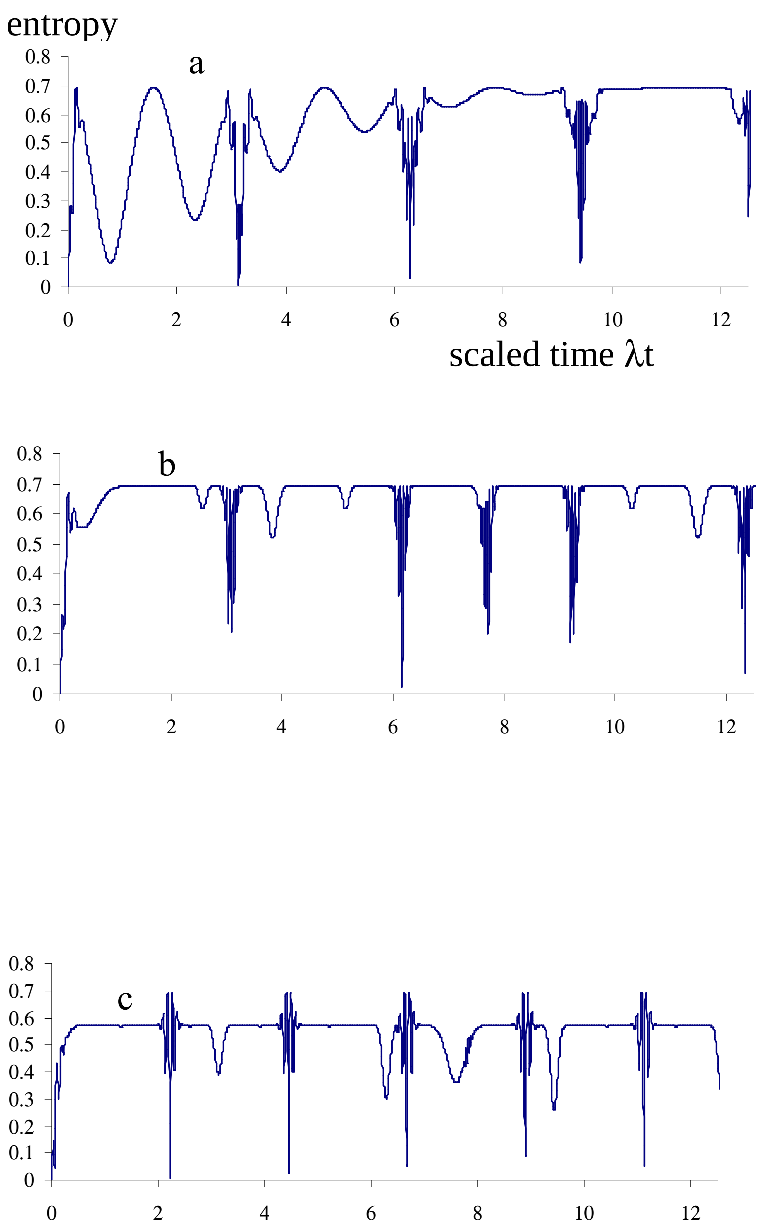

Fig. 2. The evolution of the field entropy in the two-photon process with the intensity of the initial coherent field equal to $\bar{n} = 25$, $r = 0$ and for different values of the Kerr-like medium parameter χ/λ, where (a) — $\chi/\lambda = 0.01$, (b) $- - -$ $\chi/\lambda = 0.1$, and (c) $- - -$ $\chi/\lambda = 0.5$.

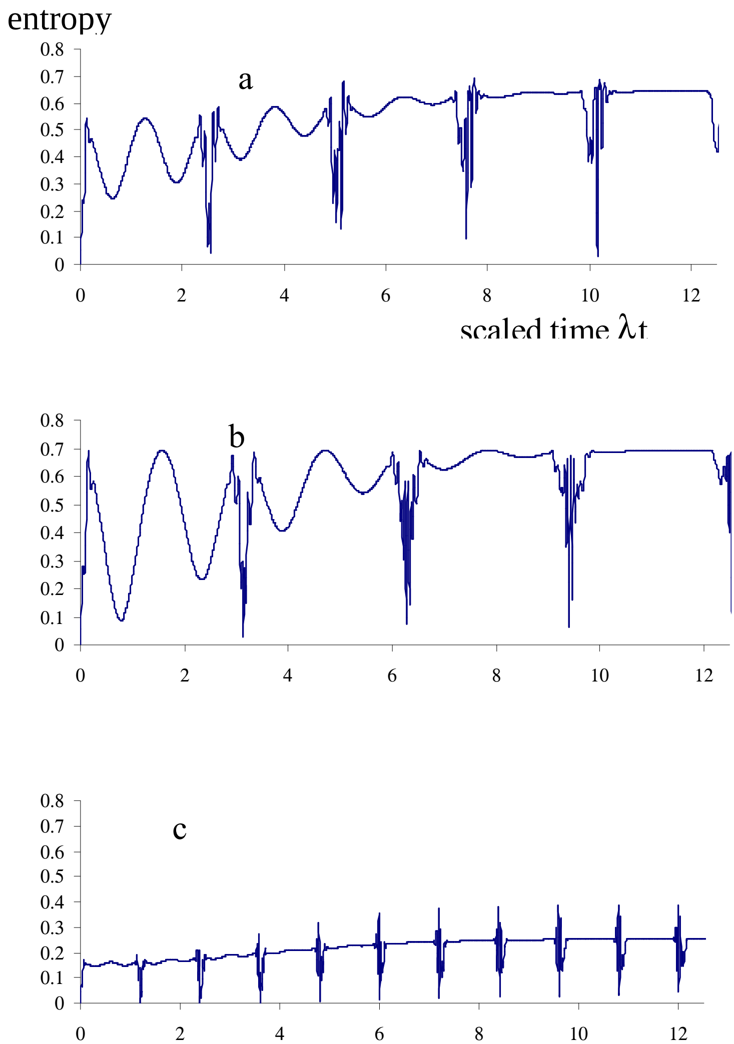

Fig. 3. The evolution of the field entropy in the two-photon process with the intensity of the initial coherent field equal to $\bar{n} = 25$, $\chi/\lambda = 0.01$, and for different values of Stark shift parameter (a) — $r = 0.2$, (b) — $r = 1$ and (c) — $r = 5$.

It is to be remarked that Stark interaction behaves like the limiting case of the Kerr interaction. This may be understood in the following way: the Kerr interaction produces two separate effects, (a) a Kerr one, which splits the field in phase space, producing a Schrödinger cat [21], and (b) a Stark interaction with the field in a cat state. The atom-field interaction when the field is initially in a cat state has been shown to be less pure than for the field in a coherent state. It has been shown that taking into account Stark shifts in the atom-field interaction agress with experimental results of micromasers [24]. Such as shifted transition lineshapes and those asymmetrically distorted. When we further take the Kerr-like medium effect through the parameter χ/λ, it is to be remarked that the amplitude of the field entropy $S_f(t)$ decreases as the Stark shift parameter r increase. The existence of the Kerr nonlinearity adds irregularity to the field entropy. It is evident that the field and the atom are in the disentangled pure state when r increase further see Fig. 3.

In conclusion, we have studied the field entropy and the entanglement of a coherent field interacting with a two-level atom when the dipole forbidden transition is replaced by a two-photon one. We consider the degenerate case, in which pairs of photons with the same frequency are created or absorbed taking into account the presence of the Stark shift and the Kerr-like medium. For small values of the Kerr-like medium, an increase of the sustainment time of the maximum field entropy, and strong entanglement of the field with the atom, while for large values, it results in a decrease of the field entropy, and the field is disentangled from the atom during the time evolution. On the other hand, the maximum field entropy and the atom-field entanglement are reduced as the Stark shift parameter r is decreased. The periodicity shown in the field entropy with Stark shift is no longer present once Kerr effect is added.

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