# SUPERSYMMETRY AND BOGOMOL'NYI EQUATIONS IN THE MAXWELL CHERN-SIMONS SYSTEMS

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We take advantage of the superspace formalism and explicitly find the $N=2$ supersymmetric extension of the Maxwell Chern–Simons model. In our construction a special form of a potential term and indispensability of an additional neutral scalar field arise naturally. By considering the algebra of supersymmetric charges we find Bogomol'nyi equations for the investigated model.

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## 1. Introduction

It has been shown [1–3] that in some models solutions can be obtained by considering the first order differential equations, which are called Bogomol'nyi equations, instead of more complicated Euler–Lagrange equations. The traditional method of obtaining such equations is based on rewriting an expression for the energy of a field configuration, in such a way, that there is a lower bound on it, which has topological nature. Field configurations, which saturate this bound satisfy Euler–Lagrange equations as well as Bogomol'nyi equations.

Another way to obtain such equations has also been pointed out [4, 5]. This method is connected with a $N=2$ supersymmetric extension of an investigated model, and Bogomol'nyi equations arise naturally during detailed analysis of the algebra of supercharges. In this case the energy of field configuration is bounded below by the central charge of the supersymmetric algebra. This method is more powerful than the previous one. As a result of this approach, we know [4] that a Bogomol'nyi bound on the energy is valid not only classically, but also quantum mechanically. Another interesting fact, indicated by this method, is that topologically non-trival field configurations of a $N=1$ supersymmetric theory must satisfy the Bogomol'nyi bound. This statement is based on the existence of the $N=2$

supersymmetric extension of the theory, which is a $N = 1$ supersymmetric and possesses a topologically conserved current [6]. This method has been successfully applied to many models.

As an example let us consider the Abelian Higgs model, which was studied in [7]. This model possesses vortex solution which has a topological charge (quantized magnetic flux). It was shown that the central charge of the $N = 2$ version of this model is in fact its topological charge. Furthermore, the special relation between coupling constants in this model, which is indispensable for the existence of Bogomol'nyi equations, appears as the necessary condition for the existence of its $N = 2$ supersymmetric extension.

There have been considerable interest in Chern–Simons systems [8]. These systems typically possess topological charge, therefore they are good candidates for investigations by the supersymmetric method. The Chern–Simons model without the Maxwell term but with a special sixth-order Higgs potential has been studied in this way [9]. It was found that the requirement of existence of the $N = 2$ SUSY version of this model leads to the special form of the previously mentioned potential. When we want to consider a more general case, we add the Maxwell term to the action. It was shown [10] that when we do so we must also add the kinetic term of a neutral scalar field to the action and considerably change the potential. This model, in fact, contains two previously mentioned models. The first one is obtained by putting coupling constant, which stays next to the Chern–Simons term, equal to zero. The second one is obtained by making suitable limit of coupling constants [10]. Our aim is to study the Maxwell Chern–Simons model by using the supersymmetric method, and find its Bogomol'nyi equations.

It is worth to notice that also the Maxwell Chern–Simons theory with an additional magnetic moment interaction was studied [11, 12]. In the first paper Bogomol'nyi equations were found by means of the supersymmetric method. Nevertheless, the results from this paper cannot be compared with ours. In the second one the $N = 2$ supersymetric extension was found via a dimensional reduction. This method is significantly different from the one used in our paper, and it is instructive to compare these two approaches.

The plan of this paper is as follows: we start our considerations from the Abelian Higgs model with the Chern–Simons term. Then we construct the $N = 1$ supersymmetric version of this model. After that we indicate the difficulties connected with construction of the $N = 2$ supersymmetric action, and we show how they can be understood and avoided. This leads to the correct form of the Maxwell Chern–Simons action. Next we find the Noether current, and construct appropriate real spinorial supercharges. Finally, we show how Bogomol'nyi equations arise from their algebra and explicitly find these equations.

## 2. Conventions

Our conventions are as follows. We use a metric with the signature $(+,-,-)$, the covariant derivative is defined as: $D_\mu = \partial_\mu - ieA_\mu$.

We take Dirac matrices $(\gamma^\mu)_\alpha{}^\beta$ to be

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \qquad \gamma^2 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}. \tag{1}$$

They obey the following equation

$$\gamma^\mu \gamma^\nu = g^{\mu\nu} + i\epsilon^{\mu\nu\lambda}\gamma_\lambda. \tag{2}$$

Superspace conventions are the same as those in [13], and are briefly listed below for the reader's convenience. Spinor indices are lowered and raised by the second-rank antisymmetric symbol $C_{\alpha\beta}$ in the following way: $\psi^\alpha = C^{\alpha\beta}\psi_\beta, \psi_\alpha = \psi^\beta C_{\beta\alpha}$; $C_{\alpha\beta}$ has the form

$$(C_{\alpha\beta}) = (-C_{\beta\alpha}) = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = (-C^{\alpha\beta}). \tag{3}$$

A scalar superfield $\Phi = (\phi, \psi, F)$ is defined as

$$\Phi(x^\mu, \theta^\alpha) = \phi(x) + \theta^\alpha \psi_\alpha(x) - \theta^2 F(x), \tag{4}$$

where $\theta^\alpha$ is a real spinor, $\theta^2 = \frac{1}{2}\theta^\alpha\theta_\alpha$, and $\alpha = 0, 1$.
A vector superfield $V^\alpha = (A_\mu, \rho^\alpha)$ in the Wess–Zumino gauge reads

$$V^\alpha(x^\mu, \theta^\alpha) = i\theta^\beta(\gamma^\nu)_\beta{}^\alpha A_\nu(x) - \theta^2 2\rho^\alpha(x). \tag{5}$$

The supercovariant derivative is $D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\theta^\beta(\gamma^\mu)_{\alpha\beta}\partial_\mu$, and the gauge covariant supercovariant derivative is $\nabla_\alpha = D_\alpha - ieV_\alpha$.

## 3. The model

It was shown [10] that there are Bogomol'nyi equations in the model defined by the action

$$\begin{aligned} \mathcal{S} \; = \; & \int d^3x \bigg[ -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \kappa\varepsilon^{\mu\nu\sigma}\partial_\mu A_\nu A_\sigma + \frac{1}{2}(D_\mu\phi)^*(D^\mu\phi) \\ & + \frac{1}{2}\partial_\mu N \partial^\mu N - \frac{e^2}{2}N^2|\phi|^2 - \frac{e^2}{8}\left(-\frac{4N\kappa}{e} + |\phi|^2 - \phi_0^2\right)^2 \bigg], \end{aligned} \tag{6}$$

where $\phi$ is a complex scalar field, $N$ is a neutral real scalar field and $A_\mu$ is a gauge field.

We want to stress the fact that there are no Bogomol'nyi equations in the Abelian Higgs model, which was studied in [7], with the Chern–Simons term. The action of this model can be written as

$$\mathcal{S}' = \int d^3x \left[ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \kappa \varepsilon^{\mu\nu\sigma} \partial_\mu A_\nu A_\sigma + \frac{1}{2} (D_\mu \phi)^* (D^\mu \phi) - \lambda (|\phi|^2 - \phi_0^2)^2 \right]. \tag{7}$$

Our aim is to show, using supersymmetric formalism, that in order to obtain such equations we have to modify the action (7) to the form of the action (6). Consequently, we start our calculations from the action (7) and we are looking for its supersymmetric version.

## 4. N=1 and N=2 extensions

To obtain Bogomol'nyi equations we must find a $N = 2$ supersymmetric extension of our model. The connection between Bogomol'nyi equations and the supersymmetric form of the investigated model was explained in [5]. We will discuss it in the next section.

We start our considerations from a $N = 1$ supersymmetric extension of (7). We construct the appropriate action from the complex scalar superfield $\varPhi = (\phi, \psi, F)$, the real scalar superfield $\varOmega = (N, \chi, D)$, and the vector superfield $V^\alpha = (A_\mu, \rho^\alpha)$. The $N = 1$ version of (7) reads

$$\begin{aligned} \mathcal{S}'_{N=1} = \int d^3x d^2\theta \ \Big[ & -\frac{1}{4} (\nabla^\alpha \varPhi)^* (\nabla_\alpha \varPhi) - \frac{1}{4} (D^\alpha \varOmega)^* (D_\alpha \varOmega) - \frac{\kappa}{4} V^\alpha D_\beta D_\alpha V^\beta \\ & + \frac{1}{16} (D_\beta D^\alpha V^\beta)(D_\gamma D_\alpha V^\gamma) + (2\lambda)^{\frac{1}{2}} \phi_0^2 \varOmega - (2\lambda)^{\frac{1}{2}} \varPhi^* \varPhi \varOmega \Big]. \end{aligned} \tag{8}$$

In terms of the components of the superfields it takes the form

$$\begin{aligned} \mathcal{S}'_{N=1} = \int d^3x \Big[ & -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \kappa \varepsilon^{\mu\nu\sigma} \partial_\mu A_\nu A_\sigma + \frac{1}{2} (D_\mu \phi)^* (D^\mu \phi) + \frac{1}{2} \partial_\mu N \partial^\mu N \\ & - \lambda (|\phi|^2 - \phi_0^2)^2 - 4\lambda N^2 |\phi|^2 + \frac{i}{2} \bar\psi \not{D} \psi + \frac{i}{2} \bar\rho \not\partial \rho + \frac{i}{2} \bar\chi \not\partial \chi \\ & - (2\lambda)^{\frac{1}{2}} \bar\psi \psi N + \frac{ie}{2} (\bar\psi \rho \phi - \bar\rho \psi \phi^*) - (2\lambda)^{\frac{1}{2}} (\bar\chi \psi \phi^* + \bar\psi \chi \phi) + \kappa \bar\rho \rho \Big]. \end{aligned} \tag{9}$$

The non-propagating fields $F$ and $D$ were eliminated by means of their Euler–Lagrange equations of motion. The action $\mathcal{S}'_{N=1}$ is invariant under the following $N=1$ transformations

$$\begin{aligned}
\delta\psi_\alpha &= -2(2\lambda)^{\frac{1}{2}}N\phi\eta_\alpha + i\eta^\beta(\gamma^\mu)_{\alpha\beta}D_\mu\phi,\\
\delta\phi &= \bar{\eta}\psi,\\
\delta\chi_\alpha &= -2(2\lambda)^{\frac{1}{2}}(|\phi|^2-\phi_0^2)\eta_\alpha + i\eta^\beta(\gamma^\mu)_{\alpha\beta}\partial_\mu N,\\
\delta N &= \frac{1}{2}(\bar{\eta}\chi+\bar{\chi}\eta),\\
\delta\rho^\alpha &= \frac{i}{2}\varepsilon^{\mu\nu\lambda}F_{\mu\nu}(\gamma_\lambda)^{\alpha\beta}\eta_\beta,\\
\delta A^\mu &= \frac{i}{2}(\bar{\eta}\gamma^\mu\rho-\bar{\rho}\gamma^\mu\eta),
\end{aligned}\tag{10}$$

where $\eta^\alpha$ is a real infinitesimal spinor. Evidently, when we put all fermion fields, as well as the field $N$, equal to zero, the action (9) will have the same form as the action (7). We can put the field $N$ equal to zero because its equations of motion allow us to do it. Therefore, the action (8) is in fact the $N=1$ extension of (7).

To find the $N=2$ extension of (9) we require its invariance under transformations (10) with an infinitesimal complex spinor $\xi^\alpha$ instead of the real $\eta^\alpha$. At this point, it is useful to change notation. We introduce, following [7], the spinor field $\Sigma$

$$\Sigma = \chi - i\rho. \tag{11}$$

The invariance under the $N=2$ transformations can be achived by rewriting the action (9) in the terms of $\Sigma$, $\psi$, $\phi$, N, $A_\mu$, and demanding its invariance under transformations

$$\Sigma \longrightarrow e^{-i\beta}\Sigma, \quad \psi \longrightarrow e^{-i\beta}\psi, \tag{12}$$

where $\beta$ is defined as follows: $\xi^\alpha = e^{i\beta}\eta^\alpha$. Obviously, this requirement is equivalent to the previous one. To apply this method we rearranged the action (9) to the form

$$\begin{aligned}
\mathcal{S}'_{N=1} = \int d^3x\Bigg[&-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \kappa\varepsilon^{\mu\nu\sigma}\partial_\mu A_\nu A_\sigma + \frac{1}{2}(D_\mu\phi)^*(D^\mu\phi)\\
&+\frac{1}{2}\partial_\mu N\partial^\mu N - \lambda(|\phi|^2-\phi_0^2)^2 - 4\lambda N^2|\phi|^2 + \frac{i}{2}\bar{\psi}\not{D}\psi\\
&+\frac{i}{2}\bar{\Sigma}\not{\partial}\Sigma - (2\lambda)^{\frac{1}{2}}\bar{\psi}\psi N - \left(\frac{e}{4}+\frac{(2\lambda)^{\frac{1}{2}}}{2}\right)(\bar{\psi}\Sigma\phi+\bar{\Sigma}\psi\phi^*)\\
&+\left(\frac{e}{4}-\frac{(2\lambda)^{\frac{1}{2}}}{2}\right)(\bar{\psi}\bar{\Sigma}\phi+\Sigma\psi\phi^*) + \frac{\kappa}{2}\bar{\Sigma}\Sigma + \frac{\kappa}{4}(\bar{\Sigma}\bar{\Sigma}-\Sigma\Sigma)\Bigg].
\end{aligned}\tag{13}$$

As a consequence of the term $\frac{\kappa}{4}(\bar{\Sigma}\bar{\Sigma} - \Sigma\Sigma)$, this action is not invariant under transformations (12) even if we assume that

$$\lambda = \frac{e^2}{8}. \tag{14}$$

This relation is exactly the same as that in [7]. To obtain the $N = 2$ SUSY version of (7), we add to the action (8) the following term

$$\begin{aligned}\int d^3x d^2\theta \kappa \Omega\Omega &= \int d^3x [2\kappa ND + \kappa\chi\chi] \\ &= \int d^3x \left[2\kappa ND + \frac{\kappa}{2}\bar{\Sigma}\Sigma - \frac{\kappa}{4}(\bar{\Sigma}\bar{\Sigma} - \Sigma\Sigma)\right]. \end{aligned}\tag{15}$$

One sees that the term (15) cancel the last term of (13), but it contains a field D. As a result, this addition leads to the modification of the Higgs term in the action. The action, constructed as a sum of (8) and (15), is invariant under the $N = 2$ supersymmetric transformations if we impose condition (14) on $\lambda$ and $e$, and can be written as

$$\begin{aligned}\mathcal{S}_{N=2} &= \int d^3x \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \kappa\varepsilon^{\mu\nu\sigma}\partial_\mu A_\nu A_\sigma + \frac{1}{2}(D_\mu\phi)^*(D^\mu\phi)\right. \\ &\quad + \frac{1}{2}\partial_\mu N\partial^\mu N - \frac{e^2}{2}N^2|\phi|^2 - \frac{e^2}{8}\left(-\frac{4N\kappa}{e} + |\phi|^2 - \phi_0^2\right)^2 \\ &\quad \left. + \frac{i}{2}\bar{\psi}\not{D}\psi - \frac{e}{2}\bar{\psi}\psi N + \kappa\bar{\Sigma}\Sigma + \frac{i}{2}\bar{\Sigma}\not{\partial}\Sigma - \frac{e}{2}(\bar{\psi}\Sigma\phi + \bar{\Sigma}\psi\phi^*)\right].\end{aligned}\tag{16}$$

The $N = 2$ supersymmetric transformations read

$$\begin{aligned}\delta\psi_\alpha &= -eN\phi\xi_\alpha + i\xi^\beta(\gamma^\mu)_{\alpha\beta}D_\mu\phi, \\ \delta\phi &= \bar{\xi}\psi, \\ \delta A^\mu &= -\frac{1}{2}(\bar{\xi}\gamma^\mu\Sigma + \xi\gamma^\mu\bar{\Sigma}), \\ \delta\Sigma_\alpha &= \left(2N\kappa - \frac{e}{2}\Big(|\phi|^2 - \phi_0^2\Big)\right)\xi_\alpha + \left(\frac{1}{2}\varepsilon^{\mu\nu\lambda}F_{\mu\nu}(\gamma_\lambda)_\alpha{}^\beta - i(\gamma^\mu)_\alpha{}^\beta\partial_\mu N\right)\xi_\beta, \\ \delta N &= \frac{1}{2}\left(\bar{\xi}\Sigma + \bar{\Sigma}\xi\right).\end{aligned}\tag{17}$$

Now, if we put all fermion fields equal to zero, we can see that the requirement that the action (8) must be invariant under the $N = 2$ the transformations leads to the action that is the supersymmetric extension of the initial one (6).

## 5. Bogomol'nyi equations

Following Hlousek and Spector [5] we concisely explain how Bogomol'nyi equations arise from the algebra of supersymmetric charges. Due to the Haag–Łopuszański–Sohnius theorem [14], there are two real spinorial supercharges $q^L_\alpha$, where $L = 0, 1$ is an internal index, in our $N = 2$ supersymmetric theory. These supercharges are obtained from the Noether conserved current. They obey the algebra

$$\{q^L_\alpha, q^M_\beta\} = 2\delta^{LM}(\gamma^\mu)_{\alpha\beta}P_\mu + TC^{LM}C_{\alpha\beta}, \tag{18}$$

where if we denote energy-momentum tensor by $T_{\mu\nu}$, we have $P_\mu = \int d^2xT_{0\mu}$. $T$ is the central charge.

The Noether current $(J^\mu)^\alpha$ was found by considering variation of the action (16) under transformations (17) with space-time dependent spinorial parameter $\xi^\alpha$

$$\delta\mathcal{S}_{N=2} = \int d^3x[(J^\mu)^\alpha\partial_\mu\xi_\alpha + \text{h.c.}]. \tag{19}$$

In the present case we have

$$\begin{aligned}(J^\mu)^\alpha \; = \; & \frac{1}{2}\bar{\Sigma}^\beta(\gamma_\nu)_\beta{}^\alpha F^{\mu\nu} + \frac{i}{2}\varepsilon^{\mu\nu\lambda}\bar{\Sigma}^\beta(\gamma_\lambda)_\beta{}^\alpha\partial_\nu N + \frac{i}{4}\varepsilon^{\mu\nu\rho}F_{\nu\rho}\bar{\Sigma}^\alpha \\ & + i\kappa\bar{\Sigma}^\beta(\gamma^\mu)_\beta{}^\alpha N + \frac{1}{2}D^\mu\phi\bar{\psi}^\alpha + \frac{1}{2}\partial^\mu N\bar{\Sigma}^\alpha - \frac{ie}{2}\bar{\psi}^\beta(\gamma^\mu)_\beta{}^\alpha\phi N \\ & - \frac{ie}{4}\bar{\Sigma}^\beta(\gamma^\mu)_\beta{}^\alpha(|\phi|^2 - \phi_0^2) + \frac{i}{2}\varepsilon^{\mu\nu\lambda}\bar{\psi}^\beta(\gamma_\lambda)_\beta{}^\alpha D_\nu\phi,\end{aligned} \tag{20}$$

and $\partial_\mu(J^\mu)^\alpha = 0$.

We defined supercharges to be

$$q^1_\alpha = \int d^2x[(J^0)_\alpha + \text{h.c.}], \quad q^2_\alpha = -i\int d^2x[(J^0)_\alpha - \text{h.c.}]. \tag{21}$$

In order to check the relation (18), we must impose canonical (anti)commutation relations on our fields. If we simplify calculations by putting all fermion fields zero after computing the anticommutator (18), we only need the following canonical anticommutation relation for the field $\Sigma$

$$\left\{\Sigma^\beta(\vec{x}), \frac{i}{2}(\gamma^0)_\alpha{}^\sigma\bar{\Sigma}_\sigma(\vec{y})\right\} = i\delta^2(\vec{x} - \vec{y})\delta_\alpha{}^\beta, \tag{22}$$

and the same relation for the field $\psi$. We do not use a special symbol for operators. It should be noticed that the relation (22) is valid when $\Sigma$ is an operator, so this equation must be understood as the operator equation. If not stated otherwise, the expressions below are the operator equations, except the case when they contain the expectation value, which is denoted by $\langle\ \rangle$. After lenghty but straightforward calculations, one obtains

$$\begin{aligned}\langle P_0\rangle &= \int d^2x\Bigg[\frac{1}{4}(F_{ij})^2+\frac{1}{2}(F_{i0})^2+\frac{1}{2}|D_0\phi|^2+\frac{1}{2}|D_i\phi|^2\\&+\frac{1}{2}(\partial_0 N)^2+\frac{1}{2}(\partial_i N)^2\\&+\frac{e^2}{2}|\phi|^2N^2+\frac{e^2}{8}\left(-\frac{4N\kappa}{e}+|\phi|^2-\phi_0^2\right)^2\Bigg],\end{aligned}\tag{23}$$

$$\langle P_i\rangle = \int d^2x\left[-F_0{}^kF_{ik}+\partial_0 N\partial_i N+\frac{1}{2}D_0\phi(D_i\phi)^*+\frac{1}{2}(D_0\phi)^*D_i\phi\right],\tag{24}$$

$$T=\int d^2x\varepsilon^{ij}\partial_j(e\phi_0^2A_i-i\phi^*D_i\phi)=e\phi_0^2\Phi,\tag{25}$$

where indices $i,j,k=1,2$. $\Phi=-\int d^2xF_{12}$, the magnetic flux, is the topological charge of the Maxwell Chern–Simons theory [10]. What's more, since the central charge $T$ is a scalar, expression (25) contains classical fields. To attain the exact form of the central charge Euler–Lagrange equations of motion for the field N have been used. We have also assumed that $D_i\phi$ tends to zero at infinity.

Now, we are ready to find Bogomol'nyi equations. Let us introduce $Q_1$ and $Q_2$ by

$$\begin{aligned}Q_1 &= \frac{1}{2}(q_1^1+iq_1^2),\\Q_2 &= \frac{1}{2}(q_2^2-iq_2^1).\end{aligned}\tag{26}$$

Hence

$$\{Q_1\pm Q_2,Q_1^\dagger\pm Q_2^\dagger\}=2\left(P_0\pm\frac{T}{2}\right),\tag{27}$$

where a unit operator next to $T$ is not written; the lower (upper) sign corresponds to a positive (negative) value of $T$. Taking the expectation value of (27), one can conclude that there is a Bogomol'nyi bound

$$\langle P_0\rangle\geq\frac{|T|}{2}.\tag{28}$$

Moreover, this bound is saturated when

$$(Q_1 \pm Q_2)|B\rangle = 0\,. \tag{29}$$

Using the relations (20), (21), (26), (29), one finds

$$\begin{aligned} F_{i0} \mp \partial_i N &= 0\,,\\ F_{12} \mp 2\left(\kappa N - \frac{e}{4}(|\phi|^2 - \phi_0^2)\right) &= 0\,,\\ (D_1 \pm iD_2)\phi &= 0\,,\\ D_0\phi \pm ie\phi N &= 0\,,\\ \partial_0 N &= 0\,, \end{aligned} \tag{30}$$

where $\phi, N, A_\mu$ are classical fields. These equations are precisely the Bogomol'nyi equations that we were looking for. They are of course the same as those in [10]. We want to emphasize that during these calculations we did not choose any particular gauge choice for the field $A_\mu$, and we did not assume that our fields are time-independent, as it was done in [7].

To summarize, the supersymmetric method of finding Bogomol'nyi equations next time turned out to be a useful tool. We saw that a special form of the potential term and an absolute necessity of the additional real neutral scalar field, in considered model, is due to existence of its $N=2$ supersymmetric extension. We also checked that the topological charge of the examined model is the central charge of its supersymetric extension.

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