OPTION OF THREE PSEUDO-DIRAC NEUTRINOS*

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As an alternative for popular see–saw mechanism, the option of three pseudo-Dirac neutrinos is discussed, where $\frac{1}{2}(m^{(\mathrm{L})}+m^{(\mathrm{R})}) \ll m^{(D)}$ for their Majorana and Dirac masses. The actual neutrino mass matrix is assumed in the form of tensor product $M^{(\nu)} \otimes \begin{pmatrix} \lambda^{(\mathrm{L})} & 1 \\ 1 & \lambda^{(\mathrm{R})} \end{pmatrix}$, where $M^{(\nu)}$ is a neutrino family mass matrix ($M^{(\nu)\,\dagger} = M^{(\nu)}$) and $\lambda^{(\mathrm{L,R})} \equiv m^{(\mathrm{L,R})}/m^{(\mathrm{D})}$ with $m^{(\mathrm{L})}$, $m^{(\mathrm{R})}$ and $m^{(\mathrm{D})}$ being taken as universal for three neutrino families. It is shown that three neutrino effects (deficits of solar ν_e's and atmospheric ν_μ's as well as the possible LSND excess of ν_e's in accelerator ν_μ beam) can be nicely described by the corresponding neutrino oscillations, though the LSND effect may, alternatively, be eliminated (by a parameter choice). Atmospheric ν_μ's oscillate dominantly into ν_τ's, while solar ν_e's — into (existing here automatically) Majorana sterile counterparts of ν_e's. A phenomenological texture for neutrinos, compatible with the proposed description, is briefly presented.

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As is well known, in the popular see–saw mechanism [1] righthanded neutrinos get (by assumption) large Majorana-type masses and become practically decoupled from lefthanded neutrinos that are allowed to carry only small Majorana-type masses. On the other hand, in such a case, oscillations of three neutrinos can hardly explain all three neutrino effects (deficits of solar ν_e's and atmospheric ν_μ's as well as the possible LSND excess of ν_e's in accelerator ν_μ beam). This may suggest the existence of, at least, one extra neutrino called sterile (*i.e.*, passive to all Standard Model gauge interactions), mixing with three active neutrinos [2].

The option of three pseudo-Dirac neutrinos [3], where the Dirac mass $m^{(\mathrm{D})}$ dominates over two Majorana masses $m^{(\mathrm{L})}$ and $m^{(\mathrm{R})}$ (within their

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Majorana-type masses), is orthogonal to the see–saw mechanism with $m^{(\mathrm{R})}$ dominating over $m^{(\mathrm{L})}$ and $m^{(\mathrm{D})}$, and $m^{(\mathrm{D})}$ over $m^{(\mathrm{L})}$. Thus, in contrast to the see–saw, this option cannot guarantee automatically small Majorana-type masses for lefthanded neutrinos (their smallness must be here directly assumed). However, as will be shown in this note, oscillations of three pseudo-Dirac neutrinos are sufficient to explain all three neutrino effects without introducing any extra sterile neutrinos. This is due to the automatic existence of three conventional Majorana sterile neutrinos $\nu^{(s)}_\alpha \equiv \nu_{\alpha\mathrm{R}} + (\nu_{\alpha\mathrm{R}})^c$ which, in the pseudo-Dirac case, are not decoupled from three conventional Majorana active neutrinos $\nu^{(a)}_\alpha \equiv \nu_{\alpha\mathrm{L}} + (\nu_{\alpha\mathrm{L}})^c$ $(\alpha = e\,,\, \mu\,,\, \tau)$.

Thus, let us consider three flavor neutrinos ν_e, ν_μ, ν_τ and , approximately, assume for them the mass matrix in the form of tensor product of the neutrino family 3×3 mass matrix $\left(M^{(\nu)}_{\alpha\beta}\right)$ $(\alpha,\beta = e,\, \mu,\, \tau)$ and the Majorana 2×2 mass matrix

$$\left(\begin{array}{cc} m^{(\mathrm{L})} & m^{(\mathrm{D})} \\ m^{(\mathrm{D})} & m^{(\mathrm{R})} \end{array}\right)\;, \tag{1}$$

the latter divided by $m^{(\mathrm{D})}$ (with $m^{(\mathrm{D})}$ included into $M^{(\nu)}_{\alpha\beta}$). Then, the neutrino mass term in the Lagrangian gets the form

$$\begin{aligned} -\mathcal{L}_{\mathrm{mass}} &= \frac{1}{2}\sum_{\alpha\beta}\left(\overline{\overset{\circ}{\nu}}{}^{(a)}_\alpha\,,\,\overline{\overset{\circ}{\nu}}{}^{(s)}_\alpha\right) M^{(\nu)}_{\alpha\beta}\left(\begin{array}{cc}\lambda^{(\mathrm{L})} & 1\\ 1 & \lambda^{(\mathrm{R})}\end{array}\right)\left(\begin{array}{c}\overset{\circ}{\nu}{}^{(a)}_\beta\\ \overset{\circ}{\nu}{}^{(s)}_\beta\end{array}\right)\\ &= \frac{1}{2}\sum_{\alpha\beta}\left(\overline{\left(\overset{\circ}{\nu}_{\alpha\mathrm{L}}\right)^c},\,\overline{\overset{\circ}{\nu}_{\alpha\mathrm{R}}}\right) M^{(\nu)}_{\alpha\beta}\left(\begin{array}{cc}\lambda^{(\mathrm{L})} & 1\\ 1 & \lambda^{(\mathrm{R})}\end{array}\right)\left(\begin{array}{c}\overset{\circ}{\nu}_{\beta\mathrm{L}}\\ \left(\overset{\circ}{\nu}_{\beta\mathrm{R}}\right)^c\end{array}\right) + \mathrm{h.c.}\,, \end{aligned} \tag{2}$$

where

$$\overset{\circ}{\nu}{}^{(a)}_\alpha \equiv \overset{\circ}{\nu}_{\alpha\,\mathrm{L}} + \left(\overset{\circ}{\nu}_{\alpha\,\mathrm{L}}\right)^c\,,\;\overset{\circ}{\nu}{}^{(s)}_\alpha \equiv \overset{\circ}{\nu}_{\alpha\,\mathrm{R}} + \left(\overset{\circ}{\nu}_{\alpha\,\mathrm{R}}\right)^c \tag{3}$$

and $\lambda^{(\mathrm{L,R})} \equiv m^{(\mathrm{L,R})}/m^{(\mathrm{D})}$. Here, $\overset{\circ}{\nu}{}^{(a)}_\alpha$ and $\overset{\circ}{\nu}{}^{(s)}_\alpha$ are the conventional Majorana active and sterile neutrinos of three families as they appear in the Lagrangian before diagonalization of neutrino and charged-lepton family mass matrices. Due to the relation $\overline{\nu^c_\alpha}\nu_\beta = \overline{\nu^c_\beta}\nu_\alpha$, the neutrino family mass matrix $M^{(\nu)} = M^{(\nu)\,\dagger}$, when standing at the position of $\lambda^{(\mathrm{L})}$ and $\lambda^{(\mathrm{R})}$ in Eq. (2), reduces to its symmetric part $\frac{1}{2}(M^{(\nu)} + M^{(\nu)\,T})$ equal to its real part $\frac{1}{2}(M^{(\nu)} + M^{(\nu)\,*}) = \mathrm{Re}\,M^{(\nu)}$. We will simply assume that (at least approximately) $M^{(\nu)} = M^{(\nu)\,T} = M^{(\nu)\,*}$, and hence for neutrino family diagonalizing matrix

$U^{(\nu)} = U^{(\nu)\,*} = \left(U^{(\nu)\,-1}\right)^T$. Then, CP violation for neutrinos does not appear if, in addition, for charged-lepton diagonalizing matrix $U^{(e)} = U^{(e)\,*}$. Further on, we will always assume that $0 < \frac{1}{2}(\lambda^{(\mathrm{L})} + \lambda^{(\mathrm{R})}) \quad (\equiv \lambda^{(\mathrm{M})}) \ll 1$ (the pseudo-Dirac option, in contrast to the see–saw mechanism, where $\lambda^{(\mathrm{L})} \ll 1 \ll \lambda^{(\mathrm{R})}$).

Then, diagonalizing the neutrino mass matrix, we obtain from Eq. (2)

$$-\mathcal{L}_{\mathrm{mass}} = \frac{1}{2}\sum_i \left(\overline{\nu}_i^{\mathrm{I}}\,,\ \overline{\nu}_i^{\mathrm{II}}\right)\ m_{\nu_i} \begin{pmatrix} \lambda^{\mathrm{I}} & 0 \\ 0 & \lambda^{\mathrm{II}} \end{pmatrix} \begin{pmatrix} \nu_i^{\mathrm{I}} \\ \nu_i^{\mathrm{II}} \end{pmatrix}, \tag{4}$$

where

$$\left(U^{(\nu)\,\dagger}\right)_{i\,\alpha} M^{(\nu)}_{\alpha\beta} U^{(\nu)}_{\beta\,j} = m_{\nu_i}\delta_{ij} \quad , \quad \lambda^{\mathrm{I,II}} \simeq \mp 1 + \lambda^{(\mathrm{M})} \simeq \mp 1 \tag{5}$$

$(i,\, j = 1,2,3)$ and

$$\nu_i^{\mathrm{I,II}} \simeq \sum_i \left(U^{(\nu)\,\dagger}\right)_{i\,\alpha} \frac{1}{\sqrt{2}} \left(\overset{\circ}{\nu}{}^{(a)}_{\alpha} \mp \overset{\circ}{\nu}{}^{(s)}_{\alpha}\right) = \sum_i V_{i\,\alpha} \frac{1}{\sqrt{2}} \left(\nu^{(a)}_{\alpha} \mp \nu^{(s)}_{\alpha}\right) \tag{6}$$

with $V_{i\,\alpha} = \left(U^{(\nu)\,\dagger}\right)_{i\,\beta} U^{(e)}_{\beta\alpha}$ describing the lepton counterpart of the Cabibbo–Kobayashi–Maskawa matrix. Here,

$$\nu^{(a,s)}_{\alpha} \equiv \nu_{\alpha\,\mathrm{L,R}} + (\nu_{\alpha\,\mathrm{L,R}})^c = \sum_\beta \left(U^{(e)\,\dagger}\right)_{\alpha\beta} \overset{\circ}{\nu}{}^{(a,s)}_{\beta} \simeq \sum_i \left(V^{\dagger}\right)_{\alpha\,i} \frac{1}{\sqrt{2}} \left(\pm\nu_i^{\mathrm{I}} + \nu_i^{\mathrm{II}}\right) \tag{7}$$

and

$$\left(U^{(e)\,\dagger}\right)_{\alpha\gamma} M^{(e)}_{\gamma\delta} U^{(e)}_{\delta\beta} = m_{e_\alpha}\delta_{\alpha\beta}\ , \tag{8}$$

where $\left(M^{(e)}_{\alpha\beta}\right)$ $(\alpha,\beta = e,\, \mu,\, \tau)$ is the mass matrix for three charged leptons e^-, μ^-, τ^-, giving their masses m_e, m_μ, m_τ after its diagonalization is carried out. Now, $\nu^{(a)}_\alpha$ and $\nu^{(s)}_\alpha$ are the conventional Majorana active and sterile flavor neutrinos of three families, while ν_i^{I} and ν_i^{II} are Majorana massive neutrinos carrying masses $m_{\nu_i}\lambda^{\mathrm{I}}$ and $m_{\nu_i}\lambda^{\mathrm{II}}$ (phenomenologically, they get nearly degenerate masses $|m_{\nu_i}\lambda^{\mathrm{I}}|$ and $|m_{\nu_i}\lambda^{\mathrm{II}}|$).

If CP violation for neutrinos does not appear or can be neglected, the probabilities for oscillations $\nu^{(a)}_\alpha \to \nu^{(a)}_\beta$ and $\nu^{(a)}_\alpha \to \nu^{(s)}_\beta$ are given by the following formulae (in the pseudo-Dirac case):

$$\begin{aligned} P\left(\nu^{(a)}_\alpha \to \nu^{(a)}_\beta\right) &= |\langle \nu^{(a)}_\beta | e^{iPL} | \nu^{(a)}_\alpha \rangle|^2 = \delta_{\beta\alpha} - \sum_i |V_{i\,\beta}|^2 |V_{i\,\alpha}|^2 \sin^2\left(x_i^{\mathrm{II}} - x_i^{\mathrm{I}}\right) \\ &- \sum_{j>i} V_{j\,\beta} V^*_{j\,\alpha} V^*_{i\,\beta} V_{i\,\alpha} \big[\sin^2\left(x_j^{\mathrm{I}} - x_i^{\mathrm{I}}\right) + \sin^2\left(x_j^{\mathrm{II}} - x_i^{\mathrm{II}}\right) \\ &+ \sin^2\left(x_j^{\mathrm{II}} - x_i^{\mathrm{I}}\right) + \sin^2\left(x_j^{\mathrm{I}} - x_i^{\mathrm{II}}\right)\big] \end{aligned} \tag{9}$$

and

$$P\left(\nu_\alpha^{(a)} \to \nu_\beta^{(s)}\right) = |\langle \nu_\beta^{(s)}|e^{iPL}|\nu_\alpha^{(a)}\rangle|^2 = \sum_i |V_{i\,\beta}|^2 |V_{i\,\alpha}|^2 \sin^2\left(x_i^{\rm II} - x_i^{\rm I}\right)$$
$$- \sum_{j>i} V_{j\,\beta} V^*_{j\,\alpha} V^*_{i\,\beta} V_{i\,\alpha} \left[\sin^2\left(x_j^{\rm I} - x_i^{\rm I}\right) + \sin^2\left(x_j^{\rm II} - x_i^{\rm II}\right)\right.$$
$$\left. - \sin^2\left(x_j^{\rm II} - x_i^{\rm I}\right) - \sin^2\left(x_j^{\rm I} - x_i^{\rm II}\right)\right] , \tag{10}$$

where $P|\nu_i^{\rm I,II}\rangle = p_i^{\rm I,II}|\nu_i^{\rm I,II}\rangle$, $p_i^{\rm I,II} = \sqrt{E^2 - (m_{\nu_i}\lambda^{\rm I,II})^2} \simeq E - (m_{\nu_i}\lambda^{\rm I,II})^2/2E$ and

$$x_i^{\rm I,II} = 1.27\frac{(m^2_{\nu_i}\lambda^{\rm I,II})^2 L}{E} \;\; , \;\; (\lambda^{\rm I,II})^2 = 1 \mp 2\lambda^{\rm (M)} \simeq 1 \tag{11}$$

with m_{ν_i}, L and E expressed in eV, km and GeV, respectively (L is the experimental baseline). Here, due to Eqs. (11),

$$x_i^{\rm II} - x_i^{\rm I} = 1.27\frac{4m^2_{\nu_i}\lambda^{\rm (M)} L}{E} \tag{12}$$

and for $j > i$

$$x_j^{\rm I} - x_i^{\rm I} \simeq x_j^{\rm II} - x_i^{\rm II} \simeq x_j^{\rm II} - x_i^{\rm I} \simeq x_j^{\rm I} - x_i^{\rm II} \simeq 1.27\frac{(m^2_{\nu_j} - m^2_{\nu_i})L}{E} \, . \tag{13}$$

Then, the bracket [ ] in Eq. (9) and (10) is reduced to $4\sin^2[1.27\,(m^2_{\nu_j} - m^2_{\nu_i})L/E]$ and 0, respectively. The probability sum rule

$$\sum_\beta \left[P\left(\nu_\alpha^{(a)} \to \nu_\beta^{(a)}\right) + P\left(\nu_\alpha^{(a)} \to \nu_\beta^{(s)}\right)\right] = 1$$

follows readily from Eqs. (9) and (10).

Notice that in the case of lepton Cabibbo–Kobayashi–Maskawa matrix being nearly unit, $(V_{i\,\alpha}) \simeq (\delta_{i\,\alpha})$, the oscillations $\nu_\alpha^{(a)} \to \nu_\beta^{(a)}$ and $\nu_\alpha^{(a)} \to \nu_\beta^{(s)}$ are essentially described by the formulae

$$P\left(\nu_\alpha^{(a)} \to \nu_\beta^{(a)}\right) \;\simeq\; \delta_{\beta\alpha} - P\left(\nu_\alpha^{(a)} \to \nu_\beta^{(s)}\right) \; ,$$
$$P\left(\nu_\alpha^{(a)} \to \nu_\beta^{(s)}\right) \;\simeq\; \delta_{\beta\alpha}\sin^2\left(1.27\frac{4m^2_{\nu_\alpha}\lambda^{\rm (M)} L}{E}\right) \tag{14}$$

corresponding to three maximal mixings of $\nu_\alpha^{(a)}$ with $\nu_\alpha^{(s)}$ ($\alpha = e,\, \mu,\, \tau$). Of course, for a further discussion of the oscillation formulae (9) and (10), in particular those for appearance modes $\nu_\alpha^{(a)} \to \nu_\beta^{(a)}$ $(\alpha \neq \beta)$, a detailed knowledge of $(V_{i\,\alpha})$ is necessary.

Further on, we will concentrate on the attractive mass hierarchy

$$m^2_{\nu_1} \ll m^2_{\nu_2} \simeq m^2_{\nu_3} \tag{15}$$

that may enable us to interpret the LSND scale $\Delta m^2_{\rm LSND}$ as $m^2_{\nu_2} - m^2_{\nu_1}$, while both smaller scales, the solar scale $\Delta m^2_{\rm sol}$ and atmospheric scale $\Delta m^2_{\rm atm}$ may be equal to $(m_{\nu_1}\lambda^{\rm II})^2 - (m_{\nu_1}\lambda^{\rm I})^2 \simeq 4m^2_{\nu_1}\lambda^{\rm (M)}$ and $m^2_{\nu_3} - m^2_{\nu_2}$, respectively.

In fact, due to Eqs. (12), (13) and (15), and the unitarity of $(V_{i\,\alpha})$, the oscillation formulae (9) imply

$$\begin{aligned} P\left(\nu_\alpha \to \nu_\alpha\right) \simeq\; & 1 - |V_{1\alpha}|^4 \sin^2\left(1.27\frac{4m_1^2\lambda^{\rm (M)}L}{E}\right) \\ & - \left(|V_{2\alpha}|^4 + |V_{3\alpha}|^4\right)\sin^2\left(1.27\frac{4m_2^2\lambda^{\rm (M)}L}{E}\right) \\ & - 4|V_{1\alpha}|^2\left(1 - |V_{1\alpha}|^2\right)\sin^2\left(1.27\frac{\Delta m^2_{21}L}{E}\right) \\ & - 4|V_{2\alpha}|^2|V_{3\alpha}|^2\sin^2\left(1.27\frac{\Delta m^2_{32}L}{E}\right) \end{aligned} \tag{16}$$

for $\alpha = e, \mu$ and

$$\begin{aligned} P\left(\nu_\mu \to \nu_\beta\right) \simeq & - |V_{1\beta}|^2|V_{1\mu}|^2\sin^2\left(1.27\frac{4m_1^2\lambda^{\rm (M)}L}{E}\right) \\ & - \left(|V_{2\beta}|^2|V_{2\mu}|^2 + |V_{3\beta}|^2|V_{3\mu}|^2\right)\sin^2\left(1.27\frac{4m_2^2\lambda^{\rm (M)}L}{E}\right) \\ & + 4|V_{1\beta}|^2|V_{1\mu}|^2\sin^2\left(1.27\frac{\Delta m^2_{21}L}{E}\right) \\ & + 4\left(|V_{2\beta}|^2|V_{2\mu}|^2 + V_{1\beta}V^*_{1\mu}V^*_{2\beta}V_{2\mu}\right)\sin^2\left(1.27\frac{\Delta m^2_{32}L}{E}\right) \end{aligned} \tag{17}$$

for $\beta = e, \tau$. Here, $m_i \equiv m_{\nu_i}$ and $\Delta m^2_{ji} \equiv m_j^2 - m_i^2$ (Eq. (15) shows that $\Delta m^2_{21} \simeq m_2^2$ and $\Delta m^2_{31} \simeq m_3^2$). Note that $\nu^{(a)}_{\alpha\rm L} \equiv \nu_{\alpha\rm L}$ and $\nu^{(s)}_{\alpha\rm L} \equiv (\nu_{\alpha\rm R})^c$.

We intend to relate Eqs. (16) with $\alpha = e$ and $\alpha = \mu$ to the experimental results concerning the deficits of solar ν_e's [4] and atmospheric ν_μ's [5], respectively, and Eq. (17) with $\beta = e$ to the possible LSND excess of ν_e's in accelerator ν_μ beam [6].

To this end let us make the numerical conjecture that

$$1.27\frac{4m_1^2\lambda^{(\mathrm{M})}L_{\mathrm{sol}}}{E_{\mathrm{sol}}} = O(1),\; 1.27\frac{\Delta m_{32}^2 L_{\mathrm{atm}}}{E_{\mathrm{atm}}} = O(1)\,,$$
$$1.27\frac{\Delta m_{21}^2 L_{\mathrm{LSND}}}{E_{\mathrm{LSND}}} = O(1)\,, \tag{18}$$

and

$$\left(4m_2^2\lambda^{(\mathrm{M})}\right)^2 \ll \left(\Delta m_{32}^2\right)^2 \tag{19}$$

(while not necessarily $4m_2^2\lambda^{(\mathrm{M})} \ll \Delta m_{32}^2$). Then, we get from Eqs. (16) and (17) the following oscillation formulae:

$$\begin{aligned} P\left(\nu_e \to \nu_e\right) &\simeq 1- \;|V_{1e}|^4 \sin^2\left(1.27\frac{4m_1^2\lambda^{(\mathrm{M})}L_{\mathrm{sol}}}{E_{\mathrm{sol}}}\right) \\ &-\frac{1}{2}\left[|V_{2e}|^4+|V_{3e}|^4+4|V_{1e}|^2\left(1-|V_{1e}|^2\right)+4|V_{2e}|^2|V_{3e}|^2\right], \end{aligned} \tag{20}$$

$$\begin{aligned} P\left(\nu_\mu \to \nu_\mu\right) &\simeq 1- 4|V_{2\mu}|^2|V_{3\mu}|^2 \sin^2\left(1.27\frac{\Delta m_{32}^2 L_{\mathrm{atm}}}{E_{\mathrm{atm}}}\right) \\ &-2|V_{1\mu}|^2\left(1-|V_{1\mu}|^2\right) \end{aligned} \tag{21}$$

and

$$P\left(\nu_\mu \to \nu_e\right) \simeq 4|V_{1e}|^2|V_{1\mu}|^2 \sin^2\left(1.27\frac{\Delta m_{21}^2 L_{\mathrm{LSND}}}{E_{\mathrm{LSND}}}\right)\,, \tag{22}$$

$$\begin{aligned} P\left(\nu_\mu \to \nu_\tau\right) &\simeq 4\left(|V_{2\tau}|^2|V_{2\mu}|^2 + V_{1\tau}V_{1\mu}^* V_{2\tau}^* V_{2\mu}\right)\sin^2\left(1.27\frac{\Delta m_{32}^2 L_{\mathrm{atm}}}{E_{\mathrm{atm}}}\right) \\ &+2|V_{1\tau}|^2|V_{1\mu}|^2 \\ &\sim 4|V_{2\tau}|^2|V_{2\mu}|^2 \sin^2\left(1.27\frac{\Delta m_{32}^2 L_{\mathrm{atm}}}{E_{\mathrm{atm}}}\right)\,, \end{aligned} \tag{23}$$

the last step being valid for the estimate $|V_{1\mu}| \sim 0$ (compare Eqs. (27), where $|V_{1\mu}| \sim 0.07$; if the LSND effect does not exist, $|V_{1\mu}|$ ought to be distinctly smaller). From Eqs. (21) and (23) with the estimates (25) we can see that atmospheric ν_μ's oscillate dominantly into ν_τ's. Similarly, Eqs. (20) and (24) imply that $\nu_e^{(a)}$'s oscillate dominantly into $\nu_e^{(s)}$'s.

When comparing Eqs. (20), (21) and (22) with experimental estimates, we obtain for solar ν_e's (taking the global vacuum solution) [4]

$$\begin{aligned}
&|V_{1e}|^4 \leftrightarrow \sin^2 2\theta_{\rm sol} \sim 1\;,\; 4m_1^2\lambda^{(\rm M)} \leftrightarrow \Delta m^2_{\rm sol} \sim 10^{-10}\;\; {\rm eV}^2\,,\\
&\frac{1}{2}\left[|V_{2e}|^4 + |V_{3e}|^4 + 4|V_{1e}|^2\,(1-\;|V_{1e}|^2\;) + 4|V_{2e}|^2|V_{3e}|^2\right]\\
&\equiv \frac{1}{2}\left[\left(1+3|V_{1e}|^2\right)\left(1-|V_{1e}|^2\right) + 2|V_{2e}|^2|V_{3e}|^2\right] \sim 0\,,
\end{aligned} \tag{24}$$

for atmospheric ν_μ's [5]

$$\begin{aligned}
4|V_{2\mu}|^2|V_{3\mu}|^2 \leftrightarrow \sin^2 2\theta_{\rm atm} \sim 1\;\;,\;\; \Delta m^2_{32} \leftrightarrow \Delta m^2_{\rm atm} \sim 3\times 10^{-3}\;{\rm eV}^2\;,\\
|V_{1\mu}|^2\left(1-|V_{1\mu}|^2\right) \sim 0
\end{aligned} \tag{25}$$

and for LSND ν_μ's [6]

$$4|V_{1e}|^2|V_{1\mu}|^2 \leftrightarrow \sin^2 2\theta_{\rm LSND} \sim 0.02\;,\; \Delta m^2_{21} \leftrightarrow \Delta m^2_{\rm LSND} \sim 0.5\;{\rm eV}^2\;. \tag{26}$$

Hence,

$$|V_{1e}|^2 \sim 1\;,\; |V_{1\mu}|^2 \sim 0.005 \simeq 0\;,\; 4|V_{2e}|^2|V_{3e}|^2 \sim 0\;,\; 4|V_{2\mu}|^2|V_{3\mu}|^2 \sim 1 \tag{27}$$

and

$$4m_1^2\lambda^{(\rm M)} \sim 10^{-10}{\rm eV}^2,\, m_3^2-m_2^2 \sim 3\times 10^{-3}\,{\rm eV}^2,\, m_1^2 \ll m_2^2 \simeq m_3^2 \sim 0.5{\rm eV}^2\,. \tag{28}$$

Thus, Eq. (19) requires

$$\lambda^{(M)\,2} \ll \left(\frac{3}{2}\right)^2 \times 10^{-6}\;. \tag{29}$$

On the other hand, $\lambda^{(\rm M)} \sim (1/4m_1^2)\times 10^{-10}\;{\rm eV}^2 \gg \frac{1}{2}\times 10^{-10}$.

Note that for the Chooz experiment [7] on possible deficit of reactor $\overline{\nu}_e$'s our oscillation formula (16) (with ν_e replaced by $\overline{\nu}_e$) and numerical conjecture (18) + (19) lead to

$$P(\overline{\nu}_e \to \overline{\nu}_e) \simeq 1-4|V_{2e}|^2|V_{3e}|^2 \sin^2\left(1.27\frac{\Delta m^2_{32}L_{\rm Chooz}}{E_{\rm Chooz}}\right)-2|V_{1e}|^2\left(1-|V_{1e}|^2\right)\,, \tag{30}$$

since $L_{\rm Chooz}/E_{\rm Chooz} \simeq L_{\rm atm}/E_{\rm atm}$ roughly. Thus, its negative result $P(\overline{\nu}_e \to \overline{\nu}_e) \sim 1$ implies $|V_{2e}|^2|V_{3e}|^2 \sim 0$ and $|V_{1e}|^2 \sim 1$, consistently with the estimates (27) following from solar experiments.

Concluding, with the conditions (27), (28) and (29) satisified, oscillations of three pseudo-Dirac neutrinos can nicely describe all three neutrino experimental effects, without introducing any extra sterile neutrinos. If the LSND effect does not exist, the value $|V_{1\mu}|^2 \sim 0$ ought to be distinctly smaller than 0.005.

The experimental estimates (27) are compatible with the following form [8] of neutrino family mixing matrix:

$$V^\dagger = \begin{pmatrix} c & s & 0 \\ -s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ -s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \tag{31}$$

(with $c = \cos\theta$, $s = \sin\theta$ and all phases neglected), corresponding to the maximal mixing of ν_μ and ν_τ within $\nu_i^{\mathrm{I,II}} = \sum_\alpha V_{i\alpha}\nu_\alpha^{\mathrm{I,II}}$, where $\nu_\alpha^{\mathrm{I,II}} \simeq (\nu_\alpha^{(a)} \mp \nu_\alpha^{(s)})/\sqrt{2}$ ($\alpha = e\,,\, \mu\,,\, \tau\;,\;\; i = 1,2,3$). In fact, all experimental conditions (27) are then satisfied if $c^2 \sim 0.99 \simeq 1$ and $s^2 \sim 0.01$ or

$$c \sim \sqrt{0.99} \simeq 1 \;\;,\;\; s \sim 0.1\;. \tag{32}$$

In this case,

$$V^\dagger \sim \begin{pmatrix} \sqrt{0.99} & 0.1 & 0 \\ -0.1/\sqrt{2} & \sqrt{0.99}/\sqrt{2} & -1/\sqrt{2} \\ -0.1/\sqrt{2} & \sqrt{0.99}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}\;. \tag{33}$$

If here $U^{(e)}_{\alpha\beta} \simeq \delta_{\alpha\beta}$, then $U^{(\nu)}_{\alpha i} \simeq (V^\dagger)_{\alpha i} = V^*_{i\alpha}$.

Such a neutrino family diagonalizing matrix $U^{(\nu)} \simeq V^\dagger$ is related to the neutrino family mass matrix $M^{(\nu)}$ expressed through its eigenvalues m_i ($i = 1,2,3$) by the formula [see Eqs. (5)]:

$$\begin{aligned} M^{(\nu)} &= \left(\sum_i U^{(\nu)}_{\alpha i} U^{(\nu)*}_{\beta i} m_i\right) \\ &\simeq \begin{pmatrix} m_1c^2+m_2s^2 & (m_2-m_1)cs/\sqrt{2} & (m_2-m_1)cs/\sqrt{2} \\ (m_2-m_1)cs/\sqrt{2} & (m_1s^2+m_2c^2+m_3)/2 & (m_1s^2+m_2c^2-m_3)/2 \\ (m_2-m_1)cs/\sqrt{2} & (m_1s^2+m_2c^2-m_3)/2 & (m_1s^2+m_2c^2+m_3)/2 \end{pmatrix}. \end{aligned} \tag{34}$$

In the case of mass hierarchy (15) realized as $m_1 \ll m_2 \simeq m_3$ with $m_1 \sim 0$, $(m_2+m_3)/2 \sim \sqrt{0.5}$ eV and $m_3 - m_2 \sim 3\sqrt{0.5}\times 10^{-3}$ eV, Eqs. (34) and (32) give

$$M^{(\nu)}/\mathrm{eV} \sim \sqrt{0.5}\begin{pmatrix} 0.01 & \sqrt{0.005} & \sqrt{0.005} \\ \sqrt{0.005} & 0.995 & -0.0065 \\ \sqrt{0.005} & -0.0065 & 0.995 \end{pmatrix}, \tag{35}$$

since $cs/\sqrt{2} \sim \sqrt{0.005}$, $(1-c^2)/2 \sim 0.005$ and $(1+c^2)/2 \sim 0.995$. If the LSND effect does not appear, the value $|V_{1\mu}|^2 = s^2/2 \sim 0$ should be distinctly smaller than 0.005 (or s distinctly smaller than 0.1). In this case,

$$M^{(\nu)}/\mathrm{eV} \sim \sqrt{0.5} \left(\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 1 & -0.0015 \\ 0 & -0.0015 & 1 \end{array} \right) , \tag{36}$$

leading to $m_1^2 \sim 0$ and $m_{2,3}^2 \sim 0.5(1 \mp 0.003)$ eV$^2 \simeq 0.5$ eV^2 [of course, they are such by construction, both for Eqs. (35) and for (36)].

However, if there is really no LSND effect (and atmospheric ν_μ's still oscillate dominantly into ν_τ's), then, perhaps, not the three pseudo-Dirac neutrinos and mass hierarchy (15) are the most natural option, but rather the three see–saw neutrinos and popular mass hierarchy

$$m_{\nu_1}^2 \simeq m_{\nu_2}^2 \ll m_{\nu_3}^2 \, , \tag{37}$$

where now $m_{\nu_2}^2 - m_{\nu_1}^2 \leftrightarrow \Delta m_{\mathrm{sol}}^2$ and $m_{\nu_3}^2 - m_{\nu_2}^2 \leftrightarrow \Delta m_{\mathrm{atm}}^2$ (see *e.g.*, Ref. [8]).

Concluding our comments on the phenomenological texture expressed by Eqs. (33) and (35), we would like to point out that it differs from the texture model described in two last Refs [3]. There, the LSND scale $\Delta m_{\mathrm{LSND}}^2$ (if it exists) and the atmospheric scale $\Delta m_{\mathrm{atm}}^2$ are interpreted as $m_3^2 - m_2^2$ and a number $\gtrsim 4m_2^2\lambda^{(\mathrm{M})}$ (but $\ll m_3^2 - m_2^2$), respectively. In contrast, their present interpretation is $m_2^2 - m_1^2$ and $m_3^2 - m_2^2$, respectively. In both textures the hierarchy (15) holds.

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