ENTROPY IN CLUSTER ANALYSIS OF SINGLE EVENTS IN HEAVY ION COLLISIONS

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We analyse the cluster structure of the final multihadron states resulting from heavy ion collisions using the concept of Jaynes–Shannon entropy. Further evidence for an interesting differentiation of events is provided.

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1. Introduction

The analysis of the KLM collaboration data [1] for Pb(158 GeV/nucleon) –Ag/Br collisions indicated the existence of events with "strong dynamical fluctuations". The use of factorial moments as a tool for preliminary selection of events was suggested. In our previous paper [2] we analyzed the cluster structure of events resulting from the heavy ion collision data and from the Monte Carlo (MC) generators. We investigated the dependence of the cluster distributions on the parameter defining a cluster size. The quantity we worked with was the second scaled factorial moment as a function of averaged multiplicity $\overline{n}$ with "two subtracted" (*i.e.* $\overline{n}-2$):

$$\tilde{F}_2 = \frac{\overline{(n-2)(n-3)}}{(\overline{n}-2)^2}\,. \qquad (1)$$

We gave also a motivation for the choice of such a discrimination tool. In full agreement with the suggestions following from Ref. [1] we have found two different patterns in the first four published experimental events.

In this note we would like to present some further analysis concerning the same subject. We again employ the cluster definition used in the procedure implementing the Bose–Einstein (BE) effect in MC [3] and in what follows only clusters containing at least two hadrons are taken into account.

As before, we work in a two-dimensional momentum space $(\eta - \phi)$ since in the data set [1] only the pseudorapidity η and the azimuthal angle ϕ are given. As a distance measure for clustering we use

$$\delta^2 = (\Delta\eta)^2 + (\Delta\phi)^2 \tag{2}$$

which provides also rather stable results under some coefficient change in front of these two terms defining δ^2.

In our procedure originally each particle is considered as a single cluster. We fix the value of a "cluster size parameter" ε (the upper limit of δ^2, for which two particles are joined into one cluster) and perform the clustering procedure for all pairs of particles.

Our procedure is different from the clustering procedure applied in [1]. Details are given in [2]. Here we would only like to point out that our results are significant if we have a sufficiently large number of clusters (with at least two particles). This limits the possible range of ε values, which must neither be too small nor too large. Twenty values of ε ($\varepsilon_i = 0.002 * i$) were taken for each of the events under consideration.

Thus for each value of ε for a given event we have:

- a number of particles in each cluster n_k ;
- a number of clusters with $n_k > 1$, $N(\varepsilon) \equiv N(i)$.

Obviously in the following $k = 1, \ldots, N(\varepsilon)$. With this information we may also calculate for subsequent values of ε the entropy [4]

$$S(\varepsilon) = -\sum_k p_k \ln(p_k)\,, \tag{3}$$

where the summation runs over all clusters in an event (leaving aside one particle clusters). The quantity $p_k = n_k/\sum_k n_k$ is a probability to find a particle in the k-th cluster.

As well known, the Jaynes–Shannon entropy introduced in this way is a good measure of the "amount of uncertainty" represented by a discrete probability distribution. Therefore one may expect that it will help (in a natural way) to distinguish between different heavy ion collision events. Here we do not use in our clustering procedure the principle of event entropy maximization. A further study of this interesting subject is in progress.

2. Analysis

We performed the clustering procedure for:

- the "random events" of similar multiplicities obtained by using a plain uniform random generator of (η, ϕ) points (SERENE events);
- the four KLM events presented in [1];
- the events obtained from the VENUS generator [5].

We have chosen randomly for presentation 5 out of 20 events generated for both the SERENE and VENUS classes. The results for cluster distribution are presented in Figs. 1(a)–3(a).

The number of clusters for all the SERENE events has a rather broad maximum for i between 5 and 10 ($0.01 < \varepsilon < 0.02$) with the value around 200. For VENUS events the maximum occurs at smaller i and the subsequent decrease is much faster. Two of the experimental events resemble VENUS events, the other two are more similar to SERENE events.

The striking structure seen in Figs. 1(a)–3(a) appears again if we now plot (for the same range of ε) the entropy defined in Eq. (3), as shown in Figs. 1(b)–3(b).

The maximum for all the SERENE events is very flat, whereas for all the VENUS events the values of entropy decrease rather fast for higher values of ε. The experimental events are again much more differentiated. More quantitative comparison of the events is possible if we define for each event

- the ratio ρ_c of the maximal number of clusters in each event to the number of clusters for the last 'bin' in ε in this event, *cf.* Figs. 1(a)–3(a);
- the ratio ρ_s of the maximal value of the entropy in each event to its "last bin" value, *cf.* Figs. 1(b)–3(b).

The values of these ratios are given in Table I. We checked that omitting clusters with only two particles we get similar results as shown in Table II, although the number of clusters and the values of entropy (especially for small values of ε) are strongly reduced.

The values of considered ratios for all SERENE events are very similar. For VENUS events the values are much bigger and more differentiated. For two experimental events we find the values similar to those of SERENE events and for two others the values are more similar to VENUS events.

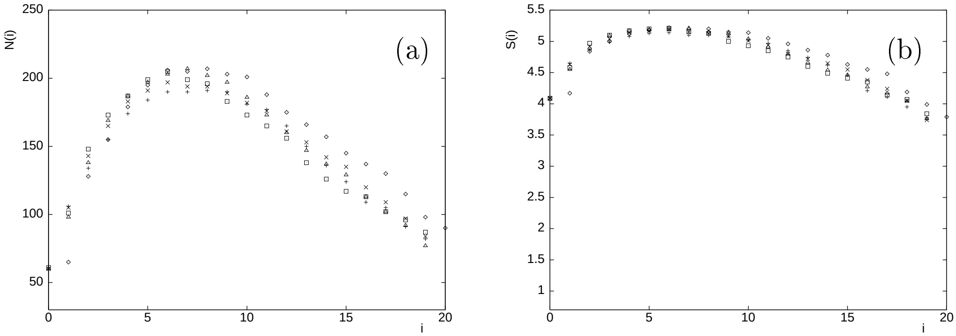

Fig. 1. (a) Number of clusters for $i = 1 \div 20$ ($\varepsilon_i = 0.002 * i$) for 5 SERENE events. Each symbol corresponds to one event. (b) Entropy for 5 SERENE events.

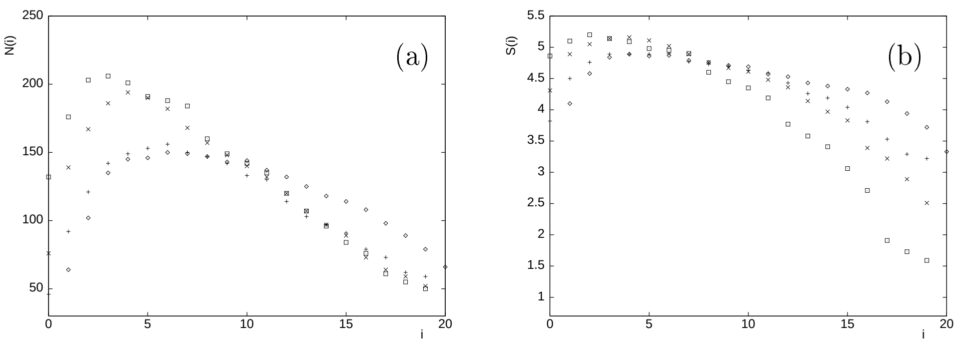

Fig. 2. (a) Number of clusters for 4 experimental events. (b) Entropy for 4 experimental events.

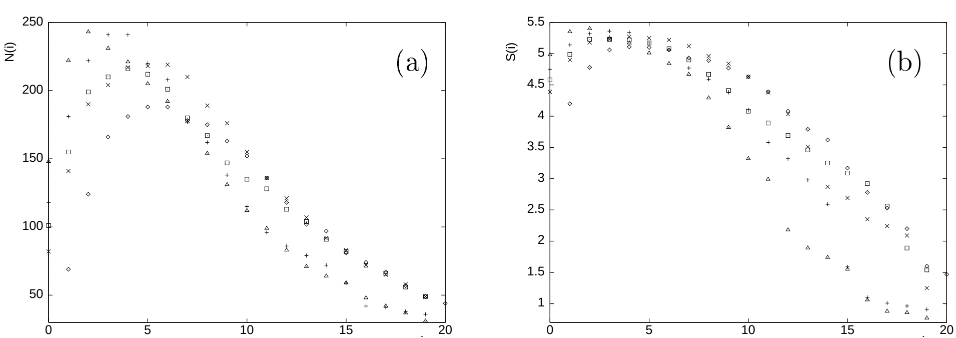

Fig. 3. (a) Number of clusters for 5 VENUS events. (b) Entropy for 5 VENUS events.

TABLE I

Values of ρ_c and ρ_s. Number of particles in a cluster > 1

SERENE		Exper.		VENUS	
ρ_c	ρ_s	ρ_c	ρ_s	ρ_c	ρ_s
2.29	1.37	2.27	1.47	4.27	3.47
2.32	1.36	2.64	1.52	6.69	5.90
2.35	1.36	4.12	3.27	4.40	3.40
2.34	1.38	3.73	2.06	4.47	4.12
2.69	1.39			7.83	7.01

TABLE II

Values of ρ_c and ρ_s. Number of particles in a cluster > 2

SERENE		Exper.		VENUS	
ρ_c	ρ_s	ρ_c	ρ_s	ρ_c	ρ_s
1.63	1.27	1.66	1.33	3.20	3.46
1.73	1.26	1.86	1.37	4.53	5.94
1.63	1.25	3.48	3.26	3.52	3.40
1.65	1.27	2.82	1.93	3.81	4.50
1.95	1.28			7.00	7.54

3. Conclusions

Comparing the results presented in Fig. 1 and Fig. 2 and the numbers given in Table I and Table II we may conclude that:

- all the SERENE events follow one ("quiet") pattern, where the fall of the number of clusters and entropy from its maximal value is rather slow; the broad entropy distribution reflects the randomness of these events' structure;
- on the contrary, all the VENUS events exhibit a completely different behaviour; the majority of particles goes into one or a few clusters for larger values of ε, which results in a fast fall of the number of clusters and entropy;
- two of the experimental events (the same as in Ref. [1] and Ref. [2]) are SERENE like, the other two resemble more the VENUS like pattern.

Thus, using a slightly different language, we confirmed our previous results [2]. The events from data seem to be more differentiated than those from the generators. Data with full momentum measurements and particle identification would be of great help in further analysis, improving our understanding of multiple production in heavy ion collisions.

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