FIRST AND SECOND-ORDER CORRECTIONS TO THE EIKONAL PHASE SHIFTS FOR THE INTERACTIONS OF TWO DEFORMED NUCLEI

Z. Metawei

Physics Department, Faculty of Science, Cairo University
Giza, Egypt

(Received October 4, 1999; Revised version received January 14, 2000)

We present first and second-order corrections to the eikonal phase shifts for the interactions of two deformed nuclei. The elastic scattering differential cross-section has been calculated for the interactions of ^{12}C–^{12}C system at energy 1016, 1449 and 2400 MeV. The two results calculated from the first and second-order corrections do not give substantial improvement between the experimental data and the theoretical calculations for the elastic scattering. The deflection function and the S-matrix have been calculated including the first and second-order corrections to the eikonal phase shift. Also, the near-side and the far-side decompositions of the scattering amplitude have been calculated including the first and second-order corrections to the eikonal phase shifts.

PACS numbers: 24.10.Cn

1. Introduction

The scattering processes have been extensively studied within the framework of the eikonal approximation method [1–7]. The correction to the eikonal phase shift has been studied by many authors. Wallace [8] has evaluated the eikonal expansion of the potential scattering T-matrix without approximation through third order in the inverse momentum. A systematic program of evaluating the first three eikonal corrections to Glauber theory has been developed and simple formulas for the eikonal phase corrections have been given. Wallace has deduced a generating function for the eikonal phase corrections of arbitrary order and also has conjectured a sum of the eikonal expansion valid in the limit of high energy and arbitrary potential strength. The correction to the eikonal phase shift has been studied for heavy ion reactions and for proton–nucleus reactions. For proton–nucleus reactions, Waxman has evaluated the first correction term to the eikonal phases in closed form for the case of scattering from a potential with a spin–orbit component. Waxman *et al.* [9] have generalized Wallace's results in the case of scattering from a potential with a spin–orbit as well as a central component. Corrections to the Glauber model have been studied for proton–nucleus elastic scattering at 1 GeV in the Glauber Model [10]. It has been found that up to momentum transfer $q \approx 2$–2.5 fm^{-1} the calculations can be performed with confidence. Going to higher values of q would require a better knowledge of the nucleon–nucleon interaction as well as more careful estimates of corrections to the Glauber amplitude. For the case of heavy ion reactions, Donnelly *et al.* [1] have calculated corrections to the lowest-order Glauber results to improve the agreement with experimental data for heavy ion scattering at intermediate and high energy. Their work is an application of some of the idea of Wallace [8] and Swift [11] to study elastic heavy ion scattering at intermediate to high energies. These corrections may be of significant value in describing heavy ion reactions, since they retain the straight line trajectory of the simple Glauber theory while folding the actual semi-classical trajectory into an effective potential along the straight line path. The validity of the eikonal approximation and its first few corrections in a low energy regime has been studied by Lombard and Carstoiu [6] to calculate the total and reaction cross-sections. Their results were coherent enough to reveal a general behavior. They have found that the eikonal approximation is a good starting point, even at 10 MeV/nucleon, to calculate σ_t and σ_R. Corrections give a result within 1% (or better) of the quantum mechanical value. The first and second order corrections to the eikonal phase shifts have been presented for heavy ion elastic scattering [7]. The eikonal phase shifts have been modified to include the deflection effect due to the Coulomb field. Including the first-and second-order corrections improves the agreement with the experimental data and the optical model result for the elastic scatterings in the ^{16}O+^{40}Ca and ^{16}O+^{90}Zr systems at $E_{\rm lab} = 1503$ MeV.

In this work, the first- and second- order corrections to the eikonal phase shifts have been presented to calculate the elastic scattering differential cross-section for ^{12}C–^{12}C system at energy 1016, 1449 and 2400 MeV. Our calculations are performed for two deformed nuclei. The deflection function and the S-matrix have been calculated for our reactions. The near-side and the far-side decompositions of the differential cross-section have been calculated by using the second-order corrections to the eikonal phase shifts. The formalism is presented in Section 2. Section 3 is devoted to the results and discussion. The conclusion is given in Section 4.

2. The formalism

The elastic scattering differential cross-section for symmetric system is given by

$$\sigma_{\rm el} = |f(\theta) + f(\pi - \theta)|^2 \tag{2.1}$$

while for non symmetric system is given by

$$\sigma_{\rm el} = |f(\theta)|^2 \, . \tag{2.2}$$

The elastic scattering amplitude considering the Coulomb effect is given by

$$f(\theta) = f_{\rm c}(\theta) + (2ik)^{-1} \sum_{\ell} (2\ell + 1) \exp(2i\eta_\ell)(S_\ell - 1) P_\ell(\cos\theta) \, , \tag{2.3}$$

$f_{\rm c}(\theta)$ is the usual point charge Coulomb amplitude, η_ℓ is the point charge Coulomb scattering phase shift, and S_ℓ is given by

$$S_\ell = \exp(2i\delta_\ell) \, , \tag{2.4}$$

where δ_ℓ is the complex nuclear phase shift, which is obtained from [12]

$$\delta_\ell = \tfrac{1}{2}\varkappa(b) \, . \tag{2.5}$$

For potential scattering the eikonal expansion has been derived by Wallace [8] and Lombard [6]

$$\varkappa(b) = \sum_n -\frac{\mu^{n+1}}{k(n+1)!} \left(\frac{b}{k^2}\frac{\partial}{\partial b} - \frac{\partial}{\partial k}\frac{1}{k} \right)^n \int\limits_{-\infty}^{\infty} V^{n+1}(r) dZ \, . \tag{2.6}$$

Note that μ is the reduced mass and that we have set $\hbar = 1$ The zero order term in equation (2.6) gives the eikonal phase

$$\varkappa_0(b) = \frac{-\mu}{k} \int\limits_{-\infty}^{\infty} V(r) dZ \, . \tag{2.7}$$

For local potential the first and second order corrections are given, respectively, by [6]

$$\varkappa_1(b) = \frac{-\mu^2}{2k^3} \left(1 + b\frac{\partial}{\partial b} \right) \int\limits_{-\infty}^{\infty} V^2(r) dZ \, , \tag{2.8}$$

$$\varkappa_2(b) = \frac{-\mu^3}{6k^5} \left(3 + 5b\frac{\partial}{\partial b} + b^2 \frac{\partial^2}{\partial b^2} \right) \int\limits_{-\infty}^{\infty} V^3(r) dZ \, . \tag{2.9}$$

$V(r)$ is the optical potential. In our work the two nuclei are considered to have a static quadrupole deformation, so, one can write $V(r,\beta_1,\beta_2)$ as [13]

$$V=\sum_{\ell_1,\ell_2}V(\ell_1,\ell_2)\,,$$

where

$$\begin{aligned}
V(0,0) &= \frac{2}{\pi}\int_0^\infty dkk^2 j_0(kr)\tilde{t}'(e,k)A_{00}'^{(1)}(k)A_{00}'^{(2)}(k)\,,\\
V(0,2) &= \frac{2\sqrt{5}}{\pi}\int_0^\infty dkk^2 j_2(kr)\tilde{t}'(e,k)[A_{00}'^{(1)}(k)A_{20}'^{(2)}(k)P_2(\cos\beta_2)\\
&\quad +A_{20}'^{(1)}(k)A_{00}'^{(2)}(k)P_2(\cos\beta_1)]\,,\\
V(2,2) &= \sum_{\ell=0,2,4}\frac{10}{\pi}i^{-\ell}(2\ell+1)\begin{pmatrix}2&2&\ell\\0&0&0\end{pmatrix}\\
&\quad\times\int_0^\infty dkk^2 j_\ell(kr)\tilde{t}'(e,k)A_{20}'^{(1)}(k)A_{20}'^{(2)}(k)\\
&\quad\times\sum_{m=-2}^{2}\begin{pmatrix}2&2&\ell\\m&-m&0\end{pmatrix}d^2_{m0}(\beta_1)d^2_{-m0}(\beta_2)
\end{aligned}\tag{2.10}$$

and

$$A'_{\ell n}(k)=\delta_{n0}\int_0^\infty dr'r'^2\rho_{\ell 0}(r')j_\ell(kr')\,,\tag{2.11}$$

β_1,β_2 are the two Euler angles and $\begin{pmatrix}2&2&\ell\\0&0&0\end{pmatrix}$ is the $3j$-symbol. $\tilde{t}'(e,k)$ is the Fourier transform of $t'(e,y)$

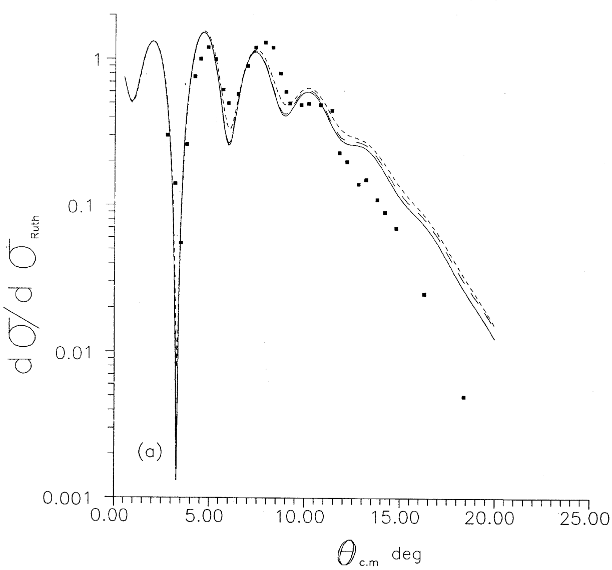
$$t'(e,y)=t(e,y)[1-C(y)]\,;\tag{2.12}$$

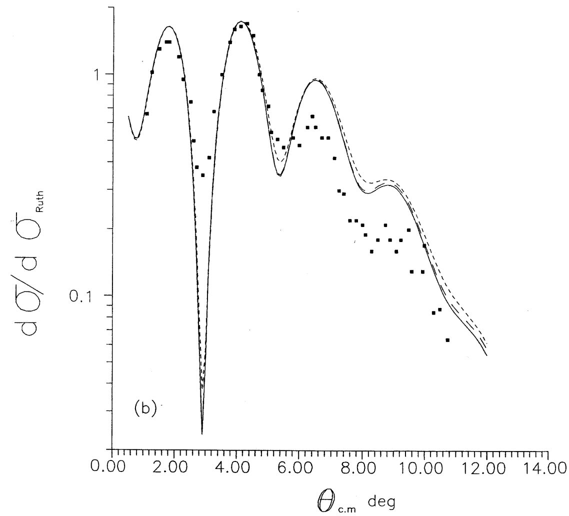
$t(e,y)$ is the energy dependent constituent-averaged two-nucleon transition amplitude obtained from scattering experiments, e is the NN kinetic energy in the c.m frame, y is the NN relative separation and $C(y)$ is the Pauli correlation function, given by

$$C(y)\approx\frac{1}{4}\exp\left(\frac{-k_F^2y^2}{10}\right)\quad\text{and}\quad K_F=1.36\ \text{fm}^{-1}\,.\tag{2.13}$$

3. Results and discussion

We have calculated the elastic scattering differential cross-section for two deformed nuclei by using the eikonal phase shift and its two higher-order corrections. These calculations have been performed for ^{12}C–^{12}C system at energies 1016, 1449 and 2400 MeV. In Fig. 1, the short-dashed curve is the result for the zero-order eikonal phase shifts, while the long-dashed curve and the solid curve are the results for the first- and second-order corrections. We can see from Fig. 1 that the difference between the long-dashed broken line, the short-dashed broken line and the solid line is considerable when compared with the experimental data [14–16]. These differences give some variations in the depth of the minimum. We can see from Fig. 1 that the two results calculated from the first- and second-order corrections improve the agreement with the observed data at large scattering angles for the three reactions considered. We can see from Fig. 1(a) that the result calculated from the second-order correction gives a satisfactory agreement with the experimental data up to $\theta_{\text{c.m.}} = 12°$. After that it gives a larger value than the experimental data. Fig. 1(b) shows that the result calculated with the second-order correction agrees with the experimental data up to $\theta_{\text{c.m.}} = 6°$. Then it gives a larger value than the experimental data. For the case of ^{12}C–^{12}C reactions at energy 2400 MeV, the calculation of the second-order correction agrees with the experimental data up to $\theta_{\text{c.m.}} = 4°$. Then it gives a larger value than the experimental data.





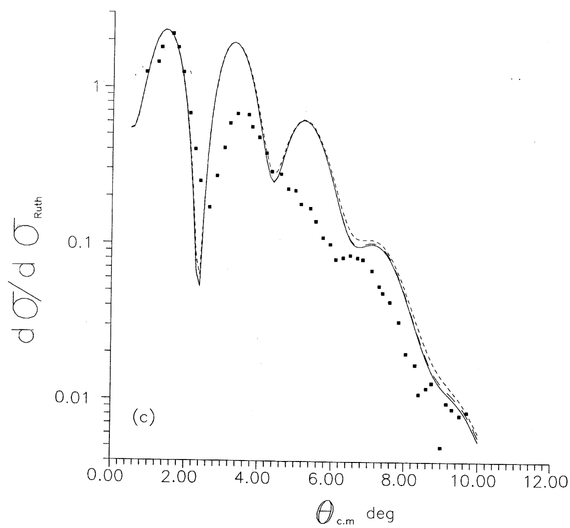


Fig. 1. The elastic scattering differential cross-section for the interactions of (a) ^{12}C–^{12}C system at energy 1016 MeV, (b) ^{12}C–^{12}C system at energy 1449 MeV and (c) ^{12}C–^{12}C system at energy 2400 MeV. The short dashed broken line represents the calculations for zero-order correction of the eikonal phase shift. The first-order correction (long dashed broken line) and the second-order correction (solid line) for the eikonal phase shift are shown in Fig. 1.

The deflection function has been calculated for the three reactions considered here along with the S-matrix. The deflection function for a real optical potential has been calculated from,

$$\theta_\ell = 2\frac{d}{d\ell}(\sigma_\ell + \Re\delta_\ell)\,, \tag{3.1}$$

where σ_ℓ is the Coulomb phase shift and δ_ℓ is the nuclear phase shift. Fig. 2 shows the deflection function for ^{12}C–^{12}C system at energy 1016 MeV.

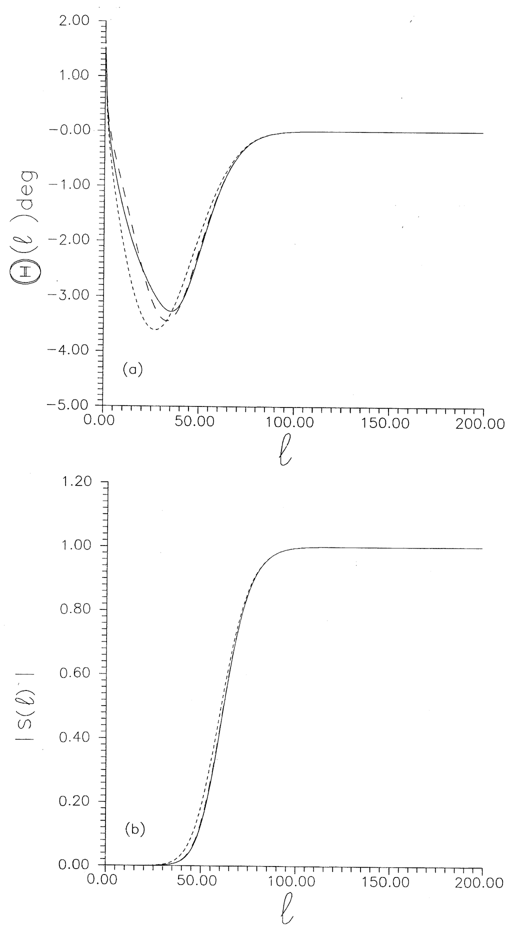


Fig. 2. The deflection function (a) along with the S-matrix (b) are calculated for ^{12}C–^{12}C system at energy 1016 MeV. The short dashed broken line represents the calculations of the zero-order correction to the eikonal phase shift. The long dashed broken line and the solid line represent the calculations for the first and second-order correction, respectively.

The short dashed broken curve represents the results of the zero-order eikonal phase shift. The long dashed broken curve and the solid curve represent the results of the first- and second-order corrections. We can see from Fig. 2 that the rainbow scattering angle value,minimum of the deflection function, decreases with increasing the order of the correction of the eikonal phase shift. The S-matrix is shifted to the right with increasing the order of the correction. Also, the deflection function is shifted to higher values of the partial wave. The deflection function and the S-matrix have been calculated also for ^{12}C–^{12}C system at energies 1449 and 2400 MeV. The deflection function and the S-matrix have the same smooth shape for the three reactions considered here. So, we put in the paper the deflection function and the S-matrix calculated for ^{12}C–^{12}C system at energy 1016 MeV only. The values of the rainbow scattering angles, the rainbow partial wave, the grazing partial wave and the values of the S-matrix at the rainbow scattering angles are shown in Table I. We can see from Table I that the rainbow scattering angle values decrease with increasing the energy. The rainbow partial wave and the grazing partial wave increase with increasing the energy. Also,the value of the S- matrix at the rainbow angle increases with increasing the energy. Concerning the strong absorption, we have found that $|S_\ell| \approx 0$ for $\ell = 34, 27$ and 23 for the interactions of ^{12}C–^{12}C system at energy 1016, 1449 and 2400 MeV, respectively. This shows that the strong absorption decreases with increasing the energy.

TABLE I

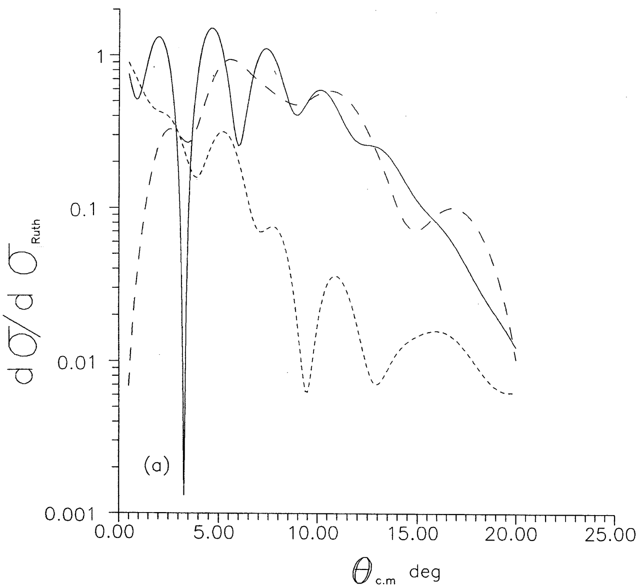
Parameters characterizing the deflection function and the S-matrix for the interactions of two deformed nuclei.

Reaction	Energy (MeV)	θ_r	L_r	L_g	$\lvert S_{L_r}\rvert$
^{12}C–^{12}C	1016	-3.28	35	62	0.006
^{12}C–^{12}C	1449	-2.22	37	70	0.017
^{12}C–^{12}C	2400	-1.45	45	87	0.027

The near-side and the far-side decompositions of the scattering amplitudes have been performed according to Fuller formalism by replacing the associated Legendre polynomial $P_\ell(\cos\theta)$ by [17]

$$\tilde{Q}_\ell^{\pm}(\cos\theta) = \frac{1}{2}\left[P_\ell(\cos\theta) \mp i\frac{2}{\pi}Q_\ell(\cos\theta)\right], \tag{3.2}$$

where Q_ℓ is a Legendre function of the second kind. Fig. 3 shows the contributions of the near-side and the far-side components to the elastic scattering cross-section along with the total differential cross-section for the


1
0.1
0.01
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(a)
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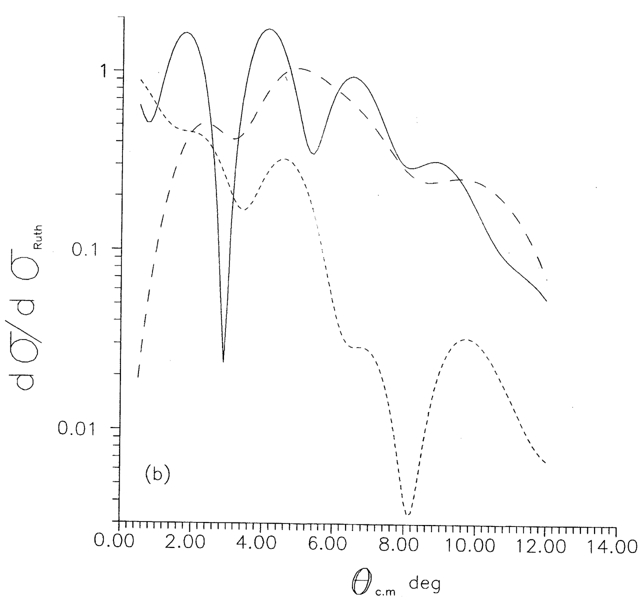
$d\sigma/d\sigma_{Ruth}$
$\theta_{c.m}$ deg


1
0.1
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10.00
12.00
14.00
$d\sigma/d\sigma_{Ruth}$
$\theta_{c.m}$ deg

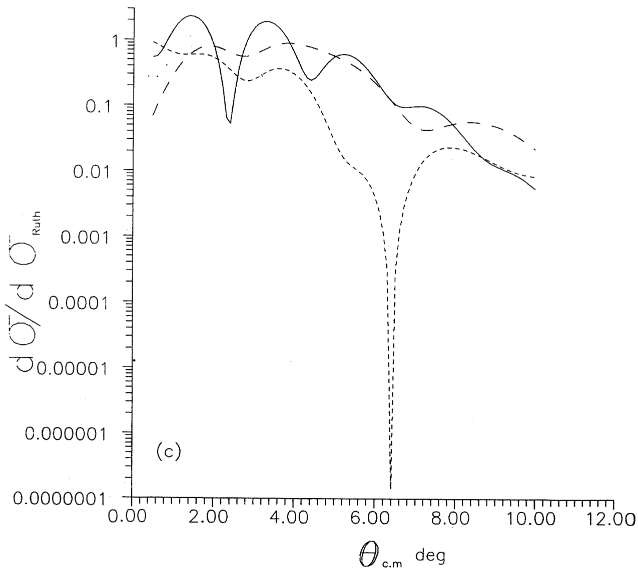

Fig. 3. The near-side (short dashed broken line) and the far-side (long dashed broken line) components of the elastic scattering along with the total differential cross-section (solid line) calculated using the second-order correction to the eikonal phase shift. The calculations are performed for (a) ^{12}C–^{12}C system at energy 1016 MeV, (b) ^{12}C–^{12}C system at energy 1449 MeV and (c) ^{12}C–^{12}C system at energy 2400 MeV.

interactions of ^{12}C–^{12}C system at energy 1016, 1449 and 2400 MeV. The near-side and the far-side decompositions of the scattering amplitude have been calculated with the second-order corrections to the eikonal phase shift. We can see from Fig. 3 that the oscillations observed at forward angles in the total differential cross-section for the three reactions are due to the strong interference between the near-side and the far-side contributions. This is because the total differential cross-section is not only the sum of the near-side and the far-side distributions but contains also the interference term representing the interference between the near-side and the far-side. The near-side and the far-side cross-over occurs at angles $\theta_{\rm c.m.} = 2.8°$, $2°$ and $1.4°$ for the interactions of ^{12}C–^{12}C system at energies 1016, 1449 and 2400 MeV, respectively. We can see from Fig. 3 that the cross-over angles decrease with increasing the energy of the interactions. The exponential fall off following the interference pattern is due to the dominance at large angles of the far-side amplitude and should thus be referred to as a far-side tail [14,18] rather than a nuclear rainbow effect. It is seen from Fig. 3 that the far-side distributions show oscillations. Oscillations will show up only if we have the beating of two waves belonging to the different negative branches of the deflection function [18]. We can notice from Table I that the nuclear rainbow, minimum of the deflection function, is shifted toward the small S-matrix element, lower L value than the grazing one. We notice also that the residual rainbow feature has moved forward in angle that it overlaps with the far/near interference pattern. This result was obtained by McVoy [19]. Our results show that the nuclear rainbow is obscured by the strong absorption [20] and cannot be recognized. The absorption for $L \leq L_r$ is strong enough to make a true rainbow pattern unobservable, but it is not strong enough to destroy the pattern of the far-side dominance due to the partial waves with L between L_r and L_g [19]. The oscillations in the near-side distributions are caused by the interference of the refractive and diffractive components of the near-side cross-section [21]. Comparing our results obtained for the near-side and far-side distributions which are calculated using the second order correction to the eikonal phase shift with that calculated for two spherical nuclei [22]. We can find that the behavior of the near-side and the far-side distributions does not change by considering both the target and projectile as deformed nuclei, also, does not change by taking the second order correction of the eikonal phase shift into consideration.

4. Conclusion

We have calculated the elastic scattering differential cross-section for the interactions of ^{12}C–^{12}C system at energies 1016, 1449 and 2400 MeV. Our calculations have been performed for the interactions of two deformed nuclei with orientation angle $\beta_1 = \beta_2 = 60^\circ$ [13]. The first and second order corrections of the eikonal phase shifts have been considered in our calculations. We have found that including the first and second order corrections to the eikonal phase shift does not give substantial improvement between the experimental data and the theoretical calculations for the elastic scattering differential cross-section. Including the first and second order corrections to the eikonal phase shift and considering the target and the projectile as deformed nuclei do not change the behavior of the near-side and the far-side distributions. The rainbow scattering angle values decrease when including the first and second order corrections to the eikonal phase shift as we can see from calculating the deflection function.

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