TOWARDS AN EFFECTIVE FIELD THEORY OF QED

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A procedure for reducing the functional integral of QED to an integral over bosonic gauge invariant fields is presented. Next, a certain averaging method for this integral, giving a tractable effective quantum field theory, is proposed. Finally, the current–current propagator and the chiral anomaly are calculated within this new formulation. These results are part of our programme of analyzing gauge theories with fermions in terms of local gauge invariants.

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1. Introduction

This paper is part of our programme of analyzing gauge theories in terms of physical observables (*i.e.* gauge invariants). For applications of this programme to non-Abelian Higgs models we refer to [1]. In recent years we have applied it to theories of gauge fields interacting with fermionic matter fields, see [2–4]. In [2] we have proved that the classical Dirac–Maxwell system can be formulated in a spin-rotation covariant way in terms of gauge invariant quantities. In [3] we have shown that similar constructions work on the level of the (formal) functional integral of QED, where fermion fields are treated as anticommuting (Berezian) quantities, and in [4] we have applied our procedure to the 2-dimensional Schwinger model. As a result we obtained a functional integral completely reformulated in terms of local gauge invariant quantities, which differs essentially from the effective functional integral obtained via the Faddeev–Popov procedure [5].

The present paper is a continuation of [3]. It turns out that our general procedure leads to a complicated, singular functional integral kernel. In order to make this model tractable, we propose a certain averaging procedure leading to an effective quantum field theory. This theory is characterized by a certain number of parameters, which have to be fixed by comparison with experimental data. As an application we discuss — for the massless case — the current–current propagator and the chiral anomaly within this formulation. (In principle, the massive case can be also dealt with, using an expansion in the mass parameter, but this problem will be not addressed in this paper.) A number of interesting phenomena and results comes out: A bosonization rule, similar to that in the Schwinger model appears naturally. Moreover, we get a dynamical mass generation leading to a massive spin-1 field. Physical quantities like the current–current propagator and the chiral anomaly are given as expectation values with respect to an effective non-local measure. This measure can be analyzed in terms of a power series expansion in the coupling constant, which, however, is completely different from the ordinary perturbation expansion. This is due to the fact that the above mentioned mass itself contains the bare coupling constant. Therefore, the formulae obtained suggest that automatically some resumation of the ordinary perturbation series has taken place. It is also remarkable that our formulation leads to a completely new approach to calculate the chiral (Adler–Bardeen) anomaly. In lowest order this quantity can be calculated analytically. Adjusting part of our free parameters yields the standard Adler–Bardeen anomaly with the correct coefficient. We underline that this paper contains only a first step towards a full effective theory. For that purpose one should adjust all free parameters, including some consistency checks by calculating different physical quantities. Also a systematic perturbation scheme based upon the power series expansion of the above mentioned non-local measure should be developed. Finally, it should be clarified, whether quantities like the anomalous magnetic moment can be calculated using our approach. (It is likely that for this purpose a different set of gauge invariant quantities has to be used.)

We stress, that our approach circumvents any gauge fixing and, therefore, also the Gribov problem [6]. It leads naturally to bosonization and can be viewed as a general construction scheme for effective quantum field theories. Due to the above remarks, it seems to be appropriate for the study of non-perturbative aspects, even if the approach presented here is far from being complete (*e.g.* we do not know whether different regularization methods used for our singular integral kernel preserve the essential features of the theory.) We also mention that a similar construction is possible for lattice models within the Hamiltonian framework. In this context we have discussed the charge superselection structure of QED [7].

Our method applies also to Yang–Mills theories, see [10], where the functional integral of one-flavour chromodynamics is reduced to an integral over purely bosonic invariants.

2. QED in terms of local gauge invariants

The functional integral of QED is given by

$$Z = \int \prod dA\, d\psi\, d\psi^*\, \mathrm{e}^{i\int d^4x\, \mathcal{L}[A,\psi,\psi^*]}\,, \tag{2.1}$$

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{mat}} \\ &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m\psi^{a*}\,\beta_{ab}\,\psi^b - \operatorname{Im}\left\{\psi^{a*}\,\beta_{ab}\,(\gamma^\mu)^b{}_c\, D_\mu\psi^c\right\}\,,\end{aligned} \tag{2.2}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu\psi^a = \partial_\mu\psi^a + ieA_\mu\psi^a$ are the electromagnetic field strength and the covariant derivative, respectively. Here $a, b, \ldots = 1, 2, \dot{1}, \dot{2}$ denote bispinor indices and $\mu, \nu, \ldots = 0, 1, 2, 3$ spacetime indices, β_{ab} denotes the Hermitean metric in bispinor space and $(\gamma^\mu)^b{}_c$ are the Dirac matrices. The anticommuting components of the bispinor field ψ^a, which can be represented by a pair of Weyl spinors $\psi^a = \begin{pmatrix} \phi^K \\ \varphi_{\dot{K}} \end{pmatrix}$, generate a Grassmann algebra of pointwise real dimension 8. The representation used for these quantities can be found in [2] and [3].

In [3] we have proposed a procedure which reduces the functional integral (2.1) to an integral over gauge invariants. It is based upon the following ideas: First, one has to analyse the algebra of Grassmann algebra valued gauge invariants, which can be built from the gauge potential A_μ and the anticommuting matter fields ψ^a. Typically, there occurs a number of identities between the invariants which, in general, cannot be solved on the algebraic level. In particular, one finds a relation which expresses the Lagrangian, or the Lagrangian multiplied by some non-vanishing element of the above algebra, in terms of invariants. In a next step one has to implement this relation under the functional integral and to reduce the original functional integral measure to a measure in terms of invariants. For that purpose we make use of the following notion of the δ-distribution on superspace (see [8] and [9])

$$\delta(u-U) = \int d\xi\, \mathrm{e}^{2\pi\mathrm{i}\xi(u-U)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\delta^{(n)}(u)U^n. \tag{2.3}$$

Here u is a c-number variable and U an element of the Grassmann algebra built from matter fields ψ and ψ^*. From this definition we get immediately

$$1 \equiv \int du\, \delta(u-U)\,. \tag{2.4}$$

Thus, by inserting identities of the form (2.4) under the functional integral, we introduce for each Grassmann algebra valued gauge invariant a c-number variable, which we call c-number mate. These mates are by definition gauge invariant. This way we are able to solve the above mentioned relations between invariants, leading to a theory reformulated in terms of gauge invariant fields. For details we refer to [3] and [10].

For QED we start with the following Grassmann algebra valued gauge invariants:

$$H := \varphi^*_K \phi^K, \tag{2.5}$$

$$B_\mu := \mathrm{Im}\left\{H^* \left(\varphi^*_K D_\mu \phi^K + \phi^K D_\mu \varphi^*_K\right)\right\}, \tag{2.6}$$

$$J^\mu := \psi^{a*}\, \beta_{ab}\, (\gamma^\mu)^b{}_c\, \psi^c, \tag{2.7}$$

$$J^\mu_5 := \psi^{a*}\, \beta_{ab}\, (\gamma^\mu)^b{}_c\, (\gamma_5)^c{}_d\, \psi^d. \tag{2.8}$$

Here H is a complex scalar field, whereas B_μ is a real-valued covector field. J^μ and J^μ_5 denote the vector and axial-vector current, respectively.

We denote the corresponding c-number mates by h, b_μ, j^μ and j^μ_5, and put $v_\mu := \frac{b_\mu}{2e\,|h|^2}$ as well as $h = |h|\mathrm{e}^{i\,\alpha}$. It was shown in [3] that our procedure yields the following functional integral:

$$Z = \int \prod_x \{dv_\mu\, dj^\mu\, dj^\mu_5\, d|h|^2\, d\alpha\, K[j^\mu, j^\mu_5, |h|^2]\}\, \mathrm{e}^{\mathrm{i}\int d^4x\, \mathcal{L}[v_\mu, j^\mu, j^\mu_5, |h|, \alpha]}, \tag{2.9}$$

where

$$\begin{aligned} K[j^\mu, j^\mu_5, |h|^2] \;=\; & \frac{1}{16\,\pi}\left\{ \frac{\delta^2}{\delta j^\mu \delta j_\mu}\frac{\delta^2}{\delta j^\nu \delta j_\nu} + 2\frac{\delta^2}{\delta j^\mu \delta j_\mu}\frac{\delta^2}{\delta j^\nu_5 \delta j^5_\nu} - 4\frac{\delta^2}{\delta j^\mu \delta j^5_\mu}\frac{\delta^2}{\delta j^\nu \delta j^5_\nu} \right. \\ & + \frac{\delta^2}{\delta j^\mu_5 \delta j^5_\mu}\frac{\delta^2}{\delta j^\nu_5 \delta j^5_\nu} + \frac{1}{16}\,\frac{\delta^4}{\delta |h|^4} + \frac{1}{8\,|h|}\,\frac{\delta^3}{\delta |h|^3} - \frac{1}{16\,|h|^2}\,\frac{\delta^2}{\delta |h|^2} \\ & + \frac{1}{16\,|h|^3}\,\frac{\delta}{\delta |h|} - \frac{1}{2}\,\frac{\delta^2}{\delta |h|^2}\frac{\delta^2}{\delta j^\mu \delta j_\mu} - \frac{1}{2\,|h|}\,\frac{\delta}{\delta |h|}\frac{\delta^2}{\delta j^\mu \delta j_\mu} \\ & \left. + \frac{1}{2}\,\frac{\delta^2}{\delta |h|^2}\frac{\delta^2}{\delta j^\mu_5 \delta j^5_\mu} + \frac{1}{2\,|h|}\,\frac{\delta}{\delta |h|}\frac{\delta^2}{\delta j^\mu_5 \delta j^5_\mu} \right\} \delta(j^\mu)\delta(j^\mu_5)\delta(|h|^2)\,, \end{aligned} \tag{2.10}$$

and

$$\begin{aligned} \mathcal{L}[v_\mu, j^\mu, j^\mu_5, |h|, \alpha] \;=\; & -\frac{1}{4}\left(\partial_{[\mu}\, v_{\nu]}\right)^2 - e\, j^\mu v_\mu + \frac{1}{2}\, j^\mu_5 (\partial_\mu \alpha) \\ & + \frac{1}{8\,|h|^2}\, \varepsilon^{\alpha\beta\mu\gamma}\, j_\alpha j^5_\beta (\partial_\mu j_\gamma) - 2\, m\, |h| \cos\alpha\,. \end{aligned} \tag{2.11}$$

3. Effective bosonized QED

Observe that the integral kernel $K[\,j^\mu,\, j_5^\mu,\, |h|^2\,]$ has the form $K = \mathcal{D}\left\{\delta(j^\mu)\delta(j_5^\mu)\delta(|h|^2)\right\}$, where $\mathcal{D}$ is a differential operator containing functional derivatives with respect to j^μ, j_5^μ and $|h|$, multiplied by singular coefficients. *A priori*, this expression does not make sense. In order to regularize it, we replace it by a Gaussian measure with three free parameters, α_j, α_{j^5} and α_h, which have to be fixed by physical requirements later. This way we get:

$$Z = \mathcal{N} \int \prod_x \{dv_\mu\, dj^\mu\, dj_5^\mu\, d|h|^2\, d\alpha\}\, \mathrm{e}^{\mathrm{i}\int d^4x\, \mathcal{L}[v_\mu, j^\mu, j_5^\mu, |h|, \alpha]}\,, \tag{3.12}$$

where now

$$\begin{aligned}\mathcal{L}[v_\mu, j^\mu, j_5^\mu, |h|, \alpha] = &-\frac{1}{4}(\partial_{[\mu}\, v_{\nu]})^2 - e j^\mu v_\mu + \frac{1}{2} j_5^\mu (\partial_\mu \alpha) + \frac{1}{8|h|^2}\varepsilon^{\alpha\beta\mu\gamma}\, j_\alpha j_\beta^5 (\partial_\mu j_\gamma) \\ &-2m\,|h|\cos\alpha - \frac{1}{2\alpha_j}\, j^\mu j_\mu - \frac{1}{2\alpha_{j^5}}\, j_5^\mu j_\mu^5 - \frac{1}{2\alpha_h}\,|h|^4\,.\end{aligned} \tag{3.13}$$

It is interesting to note that this regularization can be achieved by a technical trick similar to that used in the Faddeev–Popov procedure: One can average the singular kernel (2.10) with a functional depending on three auxiliary fields corresponding to the variables j^μ, j_5^μ and $|h|$. It was shown in [11] that the requirement to obtain the above Gaussian measure after this averaging determines this functional uniquely.

We see that the Lagrangian (3.13) does not contain derivatives of the chiral current j_μ^5, *i.e.* j_μ^5 enters the theory as a non-dynamical field. Thus, we can carry out the simple Gaussian integral over the chiral current, which yields

$$Z = \mathcal{N} \int \prod_x \{dv_\mu\, dj^\mu\, d|h|^2\, d\alpha\}\, \mathrm{e}^{\mathrm{i}\int d^4x\, \mathcal{L}[v_\mu, j^\mu, |h|, \alpha]}\,, \tag{3.14}$$

where the effective Lagrangian has taken the form:

$$\begin{aligned}\mathcal{L}[v_\mu, j^\mu, |h|, \alpha] \;=\; &-\frac{1}{4}(\partial_{[\mu}\, v_{\nu]})^2 - e\, j^\mu v_\mu + \frac{\alpha_{j^5}}{16\,|h|^2}\,\varepsilon^{\alpha\mu\beta\gamma}\,(\partial_\mu\alpha) j_\alpha (\partial_\beta j_\gamma) \\ &+\frac{\alpha_{j^5}}{128\,|h|^4}\,\varepsilon^{\alpha\mu\beta\gamma}\,\varepsilon_{\delta\mu\rho\sigma}\, j_\alpha (\partial_\beta j_\gamma) j^\delta (\partial^\rho j^\sigma) \\ &-\frac{1}{2\alpha_j}\, j^\mu j_\mu + \frac{\alpha_{j^5}}{8}\,(\partial^\mu\alpha)(\partial_\mu\alpha) - \frac{1}{2\alpha_h}\,|h|^4 - 2m\,|h|\cos\alpha\,.\end{aligned} \tag{3.15}$$

Comparing this Lagrangian with (3.13) we observe that j^5_μ has been replaced by the gradient of α. More precisely, we have the remarkable relation

$$\partial_\mu \alpha = \frac{2}{\alpha_{j^5}}\, j^5_\mu \ . \tag{3.16}$$

This is the 4-dimensional analog of the bosonization rule in the 2-dimensional Schwinger model, see [12]. (In [4] we have obtained this rule for the Schwinger model using our approach.)

Observe that the field $|h|^2$ enters (3.15) in a non-dynamical way, too. Thus, in principle, one should integrate it out. This, however, cannot be done explicitly. In a first approximation, $|h|^2$ could be replaced by a constant. We will come to that point later.

Finally, let us write down the generating functional integral of our effective theory (3.14), (3.15):

$$Z[\zeta^\mu, \xi^\mu, \eta^\mu] = \mathcal{N} \int \prod_x \{dv_\mu\, dj^\mu\, d|h|^2\, d\alpha\}\, \mathrm{e}^{\mathrm{i}\int d^4x\, \mathcal{L}[\zeta^\mu, \xi^\mu, \eta^\mu; v_\mu, j^\mu, |h|, \alpha]} \ , \tag{3.17}$$

with

$$\begin{aligned}
&\mathcal{L}[\zeta^\mu, \xi^\mu, \eta^\mu; v_\mu, j^\mu, |h|, \alpha] \\
&= -\frac{1}{4}(\partial_{[\mu} v_{\nu]})^2 - e\, j^\mu v_\mu + \frac{\alpha_{j^5}}{16\,|h|^2}\, \varepsilon^{\alpha\mu\beta\gamma}\, (\partial_\mu \alpha) j_\alpha (\partial_\beta j_\gamma) \\
&\quad + \frac{\alpha_{j^5}}{128\,|h|^4}\, \varepsilon^{\alpha\mu\beta\gamma}\, \varepsilon_{\delta\mu\rho\sigma}\, j_\alpha (\partial_\beta j_\gamma) j^\delta (\partial^\rho j^\sigma) - \frac{1}{2\alpha_j}\, j^\mu j_\mu - \frac{1}{2\alpha_h}\, |h|^4 \\
&\quad + \frac{\alpha_{j^5}}{8}\, (\partial^\mu \alpha)(\partial_\mu \alpha) - 2m\,|h| \cos\alpha + \zeta^\mu (\partial_\mu \alpha) + \xi^\mu j_\mu + \eta^\mu v_\mu \ ,
\end{aligned} \tag{3.18}$$

where ζ^μ, ξ^μ and η^μ denote the source currents for $(\partial^\mu \alpha)$, j^μ and v^μ, respectively.

4. The current–current propagator

In this section we want to calculate the current–current propagator $\langle 0|T j^\mu(y_1) j^\nu(y_2)|0\rangle$ using the effective theory obtained in the last section. We restrict ourselves to the massless case, *i.e.* we put $m = 0$ in the generating functional integral (3.17).

The current–current propagator is given by

$$\begin{aligned}
&\langle 0|T j^\mu(y_1)\, j^\nu(y_2)|0\rangle \\
&= \frac{1}{Z[0,0,0]}\, \frac{1}{\mathrm{i}^2}\, \frac{\delta}{\delta \xi_\mu(y_1)}\, \frac{\delta}{\delta \xi_\nu(y_2)}\, Z[\zeta^\mu, \xi^\mu, \eta^\mu]\Big|_{\zeta^\mu, \xi^\mu, \eta^\mu = 0} \\
&= \frac{1}{Z[0,0,0]}\, \frac{1}{\mathrm{i}^2}\, \frac{\delta}{\delta \xi_\mu(y_1)}\, \frac{\delta}{\delta \xi_\nu(y_2)}\, Z[0, \xi^\mu, \eta^\mu]\Big|_{\xi^\mu, \eta^\mu = 0}.
\end{aligned} \tag{4.19}$$

To handle the non-linear (self-interaction) term

$$\frac{\alpha_{j^5}}{128\,|h|^4}\,\varepsilon^{\alpha\mu\beta\gamma}\,\varepsilon_{\delta\mu\rho\sigma}\,j_\alpha(\partial_\beta j_\gamma)j^\delta(\partial^\rho j^\sigma)$$

occurring in (3.18) we introduce the vector field

$$k^\mu[j_\mu,|h|] := \frac{1}{|h|^2}\,\varepsilon^{\alpha\mu\beta\gamma}\,j_\alpha(\partial_\beta j_\gamma)\;, \tag{4.20}$$

and decompose it into its longitudinal and transversal parts:

$$k^\mu[j_\mu,|h|] = \partial^\mu k[j_\mu,|h|] + k^{\mu\perp}[j_\mu,|h|]\;, \tag{4.21}$$

where $\partial_\mu k^{\mu\perp}[j_\mu,|h|]=0$. This yields

$$\begin{aligned}
&\mathcal{L}[\zeta^\mu,\xi^\mu,\eta^\mu;v_\mu,j^\mu,|h|,\alpha]\\
=\;& -\frac{1}{4}\,(\partial_{[\mu}v_{\nu]})^2 - e\,j^\mu v_\mu + \frac{\alpha_{j^5}}{16}\,(\partial_\mu\alpha)\,(\partial^\mu k[j_\mu,|h|])\\
&+\frac{\alpha_{j^5}}{128}(\partial^\mu k[j_\mu,|h|])(\partial_\mu k[j_\mu,|h|])+\frac{\alpha_{j^5}}{128}k^{\mu\perp}[j_\mu,|h|]\,k^\perp_\mu[j_\mu,|h|]-\frac{1}{2\alpha_j}j^\mu j_\mu\\
&-\frac{1}{2\alpha_h}\,|h|^4+\frac{\alpha_{j^5}}{8}\,(\partial^\mu\alpha)(\partial_\mu\alpha)+\zeta^\mu(\partial_\mu\alpha)+\xi^\mu j_\mu+\eta^\mu v_\mu\,.
\end{aligned} \tag{4.22}$$

The term $\frac{\alpha_{j^5}}{16}\,(\partial_\mu\alpha)\,k^{\mu\perp}[j_\mu,|h|]$ vanishes by partial integration.

Transforming

$$\alpha' = \alpha + \frac{1}{4\,\alpha_{j^5}}\,k[j_\mu,|h|]\,, \tag{4.23}$$

we obtain

$$\begin{aligned}
&\mathcal{L}[0,\xi^\mu,\eta^\mu;v_\mu,j^\mu,|h|,\alpha']\\
=\;& -\frac{1}{4}\,(\partial_{[\mu}v_{\nu]})^2 - e\,j^\mu v_\mu + \frac{\alpha_{j^5}}{128}\,k^{\mu\perp}[j_\mu,|h|]\,k^\perp_\mu[j_\mu,|h|]\\
&-\frac{1}{2\,\alpha_j}\,j^\mu j_\mu - \frac{1}{2\,\alpha_h}|h|^4+\frac{\alpha_{j^5}}{8}\,(\partial^\mu\alpha')(\partial_\mu\alpha')+\xi^\mu j_\mu+\eta^\mu v_\mu.
\end{aligned} \tag{4.24}$$

Observe that the transformation (4.23) leaves the integral measure $d\alpha$ invariant. Thus, α' can be integrated out trivially and we get

$$\begin{aligned}
&\langle 0|Tj^\mu(y_1)\,j^\nu(y_2)|0\rangle\\
=\;& \frac{1}{Z[0,0,0]}\int\prod_x\{dv_\mu\,dj^\mu\,d|h|^2\}\\
&\qquad\times\frac{1}{\mathrm{i}^2}\,\frac{\delta}{\delta\xi_\mu(y_1)}\,\frac{\delta}{\delta\xi_\nu(y_2)}\,\mathrm{e}^{\mathrm{i}\int d^4x\,\mathcal{L}[0,\xi^\mu,\eta^\mu;v_\mu,j^\mu,|h|]}\Big|_{\xi^\mu,\eta^\mu=0}\,,
\end{aligned} \tag{4.25}$$

where

$$\begin{aligned}
&\mathcal{L}[0,\xi^\mu,\eta^\mu;v_\mu,j^\mu,|h|] \\
&\quad = -\frac{1}{4}\,(\partial_{[\mu}v_{\nu]})^2 - e\,j^\mu v_\mu + \frac{\alpha_{j^5}}{128}\,k^{\mu\perp}[j_\mu,|h|]\,k^\perp_\mu[j_\mu,|h|] \\
&\qquad -\frac{1}{2\,\alpha_j}\,j^\mu j_\mu - \frac{1}{2\,\alpha_h}|h|^4 + \xi^\mu j_\mu + \eta^\mu v_\mu\,. \qquad (4.26)
\end{aligned}$$

The further analysis of the term $\frac{\alpha_{j^5}}{128}\,k^{\mu\perp}[j_\mu,|h|]\,k^\perp_\mu[j_\mu,|h|]$ needs some care, due to its non-linear (and non-local) character. Expanding the exponential of this term in a series, we obtain

$$\begin{aligned}
&\langle 0|T j^\mu(y_1)\,j^\nu(y_2)|0\rangle \\
&\quad = \frac{1}{Z[0,0,0]}\int\prod_x\Big\{dv_\mu\,dj^\mu\,d|h|^2 \\
&\qquad \times\sum_{n=0}^{\infty}\frac{1}{n!}\,\Big(\frac{\alpha_{j^5}}{128}\Big)^n\,\Big(k^{\mu\perp}[j_\mu,|h|]\,k^\perp_\mu[j_\mu,|h|]\Big)^n\Big\} \\
&\qquad \times\frac{1}{\mathrm{i}^2}\,\frac{\delta}{\delta\xi_\mu(y_1)}\,\frac{\delta}{\delta\xi_\nu(y_2)}\,\mathrm{e}^{\mathrm{i}\int d^4x\,\mathcal{L}_0[0,\xi^\mu,\eta^\mu;v_\mu,j^\mu,|h|]}\Big|_{\xi^\mu,\eta^\mu=0} \\
&\quad = \frac{1}{Z[0,0,0]}\int\prod_x\Big\{dv_\mu\,dj^\mu\,d|h|^2 \\
&\qquad \times\sum_{n=0}^{\infty}\frac{1}{n!}\,\Big(\frac{\alpha_{j^5}}{128}\Big)^n\,\Big(k^{\mu\perp}[\frac{\delta}{\mathrm{i}\,\delta\xi^\mu},|h|]\,k^\perp_\mu[\frac{\delta}{\mathrm{i}\,\delta\xi^\mu},|h|]\Big)^n\Big\} \\
&\qquad \times\frac{1}{\mathrm{i}^2}\,\frac{\delta}{\delta\xi_\mu(y_1)}\,\frac{\delta}{\delta\xi_\nu(y_2)}\,\mathrm{e}^{\mathrm{i}\int d^4x\,\mathcal{L}_0[0,\xi^\mu,\eta^\mu;v_\mu,j^\mu,|h|]}\Big|_{\xi^\mu,\eta^\mu=0},
\end{aligned}$$

where

$$\begin{aligned}
Z[0,0,0] =&\int\prod_x\Big\{dv_\mu dj^\mu d|h|^2\sum_{n=0}^{\infty}\frac{1}{n!}\,\Big(\frac{\alpha_{j^5}}{128}\Big)^n\Big(k^{\mu\perp}\,\Big[\frac{\delta}{\mathrm{i}\delta\xi^\mu},|h|\Big]\,k^\perp_\mu\,\Big[\frac{\delta}{\mathrm{i}\delta\xi^\mu},|h|\Big]\Big)^n\Big\} \\
&\times\mathrm{e}^{\mathrm{i}\int d^4x\,\mathcal{L}_0[0,\xi^\mu,\eta^\mu;v_\mu,j^\mu,|h|]}\Big|_{\xi^\mu,\eta^\mu=0}
\end{aligned}$$

and

$$\begin{aligned}
&\mathcal{L}_0[0,\xi^\mu,\eta^\mu;v_\mu,j^\mu,|h|] \\
&\quad = -\frac{1}{4}\,(\partial_{[\mu}v_{\nu]})^2 - e\,j^\mu v_\mu - \frac{1}{2\,\alpha_j}\,j^\mu j_\mu - \frac{1}{2\,\alpha_h}|h|^4 + \xi^\mu j_\mu + \eta^\mu v_\mu\,.
\end{aligned}$$

In the last step we used the fact that the integral kernel consists of a polynomial function in j^μ. Thus, j^μ can be replaced by the corresponding functional derivative with respect to the source current ξ^μ. Now we are left with a Gaussian integral with respect to j^μ, which yields

$$\begin{aligned}&\langle 0|T\, j^\mu(y_1)\, j^\nu(y_2)|0\rangle \\ &= \frac{1}{Z[0,0,0]} \int \prod_x \left\{ dv_\mu d|h|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \Big(\frac{\alpha_{j^5}}{128}\Big)^n \left(k^{\mu\perp}\left[\frac{\delta}{\mathrm{i}\delta\xi^\mu}, |h|\right] k^\perp_\mu \left[\frac{\delta}{\mathrm{i}\delta\xi^\mu}, |h|\right]\right)^n \right\} \\ &\qquad \times \frac{1}{\mathrm{i}^2} \frac{\delta}{\delta\xi_\mu(y_1)} \frac{\delta}{\delta\xi_\nu(y_2)} \, \mathrm{e}^{\mathrm{i}\int d^4x\, \mathcal{L}_0[0,\xi^\mu,\eta^\mu;v_\mu,|h|]}\Big|_{\xi^\mu,\eta^\mu=0} \end{aligned} \tag{4.27}$$

with

$$\begin{aligned} Z[0,0,0] = &\int \prod_x \left\{ dv_\mu d|h|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \Big(\frac{\alpha_{j^5}}{128}\Big)^n \left(k^{\mu\perp}\left[\frac{\delta}{\mathrm{i}\delta\xi^\mu}, |h|\right] k^\perp_\mu \left[\frac{\delta}{\mathrm{i}\delta\xi^\mu}, |h|\right]\right)^n \right\} \\ &\times \mathrm{e}^{\mathrm{i}\int d^4x\, \mathcal{L}_0[0,\xi^\mu,\eta^\mu;v_\mu,|h|]}\Big|_{\xi^\mu,\eta^\mu=0} \end{aligned}$$

and

$$\begin{aligned}&\mathcal{L}_0[0,\xi^\mu,\eta^\mu;v_\mu,|h|] \\ &= -\frac{1}{4}(\partial_{[\mu}v_{\nu]})^2 + \frac{e^2\alpha_j}{2} v^\mu v_\mu + \frac{\alpha_j}{2}\xi^\mu\xi_\mu - \frac{1}{2\alpha_h}|h|^4 - \alpha_j e v^\mu \xi_\mu + \eta^\mu v_\mu\,. \end{aligned} \tag{4.28}$$

We remark, that the covector field v_μ has acquired a mass $m_v^2 = \alpha_j e^2$. Thus, in our effective field theory, the original gauge potential A_μ has been replaced by a massive spin-1 field v_μ. Now, observe that due to (4.28) the functional derivative with respect to ξ^μ produces the term $-\alpha_j e v_\mu$. Therefore, the non-linear (and non-local) term $\left(k^{\mu\perp}[\frac{\delta}{\mathrm{i}\delta\xi^\mu}, |h|]\, k^\perp_\mu[\frac{\delta}{\mathrm{i}\delta\xi^\mu}, |h|]\right)$ is effectively of the order e^4, and we can treat it as a perturbation. Performing the functional derivatives in (4.27) with respect to ξ^μ yields a complicated non-local measure, which in full generality cannot be handled analytically. Limiting ourselves to lowest order we get:

$$\begin{aligned}&\langle 0|T\, j^\mu(y_1)\, j^\nu(y_2)|0\rangle_0 = \frac{1}{Z[0]} \int \prod_x \left\{ dv_\mu\, d|h|^2 \right\} \\ &\times \left(\alpha_j\, \eta^{\mu\nu}\, \delta^4(y_1 - y_2) + m_v^2 \alpha_j\, v^\mu(y_1) v^\nu(y_2)\right) \mathrm{e}^{\mathrm{i}\int d^4x\, \mathcal{L}_0[\eta^\mu;v_\mu,|h|]}\big|_{\eta^\mu=0}. \end{aligned} \tag{4.29}$$

Here we have denoted

$$Z[0] = \int \prod_x \left\{ dv_\mu\, d|h|^2 \right\} \, \mathrm{e}^{\mathrm{i}\int d^4x\, \mathcal{L}_0[0;v_\mu,|h|]}$$

and

$$\mathcal{L}_0[\eta^\mu; v_\mu, |h|] = -\frac{1}{4}\,(\partial_{[\mu}v_{\nu]})^2 + \frac{m_v^2}{2}\,v^\mu v_\mu - \frac{1}{2\,\alpha_h}|h|^4 + \eta^\mu v_\mu\,. \qquad (4.30)$$

Next, observe that the integration over $|h|$ decouples, giving

$$\begin{aligned}
&\langle 0|T j^\mu(y_1)\, j^\nu(y_2)|0\rangle_0 \\
&= \frac{1}{Z[0]}\int\prod_x \{dv_\mu\}\big(\alpha_j\eta^{\mu\nu}\delta^4(y_1-y_2)+m_v^2\alpha_j v^\mu(y_1)v^\nu(y_2)\big)\,\mathrm{e}^{\mathrm{i}\int d^4x\mathcal{L}_0[\eta^\mu;v_\mu]}\Big|_{\eta^\mu=0} \\
&\equiv \langle\alpha_j\,\eta^{\mu\nu}\,\delta^4(y_1-y_2) + m_v^2\alpha_j\,v^\mu(y_1)v^\nu(y_2)\rangle\,,
\end{aligned}$$

where now

$$Z[0] = \int\prod_x \{dv_\mu\}\,\mathrm{e}^{\mathrm{i}\int d^4x\,\mathcal{L}_0[0;v_\mu]}$$

and

$$\mathcal{L}_0[\eta^\mu; v_\mu] = -\frac{1}{4}\,(\partial_{[\mu}v_{\nu]})^2 + \frac{m_v^2}{2}\,v^\mu v_\mu + \eta^\mu v_\mu\,.$$

This way we obtain:

Proposition 1 *In lowest order, the current–current propagator*

$$\langle 0|T j^\mu(x) j^\nu(y)|0\rangle$$

of massless QED is given by the vacuum expectation value

$$\langle 0|T j^\mu(y_1)\, j^\nu(y_2)|0\rangle_0 = \langle\alpha_j\,\eta^{\mu\nu}\,\delta^4(y_1-y_2) + m_v^2\alpha_j\,v^\mu(y_1)v^\nu(y_2)\rangle \qquad (4.31)$$

with respect to the functional measure $\prod_x\{dv_\mu\}\mathrm{e}^{\mathrm{i}\int d^4x\mathcal{L}[v_\mu]}$ *and the Lagrangian*

$$\mathcal{L}[v_\mu] = -\frac{1}{4}\,(\partial_{[\mu}v_{\nu]})^2 + \frac{m_v^2}{2}\,v^\mu v_\mu\,. \qquad (4.32)$$

Thus, in the lowest order, we are left with a simple Gaussian integration. We have

$$\mathcal{L}_0[\eta^\mu; v_\mu] = -\frac{1}{4}\,(\partial_{[\mu}v_{\nu]})^2 + \frac{m_v^2}{2}\,v^\mu v_\mu + \eta^\mu v_\mu = -\frac{1}{2}\,v_\mu D^\mu{}_\nu v^\nu + \eta^\mu v_\mu\,,$$

with $D^\nu{}_\mu := \partial^\nu\partial_\mu - \delta^\nu_\mu\partial^\sigma\partial_\sigma - m_v^2\,\delta^\nu_\mu$. We introduce the new field

$$v^{\mu'}(x) := v^\mu(x) + \int d^4y\eta^\nu(y)(D^{-1})^\mu{}_\nu(x-y)\,,$$

where the propagator $(D^{-1})^{\mu}{}_{\nu}(x-y)$ of the massive free field v_μ is defined by

$$D^{\mu}{}_{\alpha}\,(D^{-1})^{\alpha}{}_{\nu}(x-y) = -\,\delta^4(x-y)\,\delta^{\mu}_{\nu}\ .$$

Thus,

$$(D^{-1})^{\mu}{}_{\nu}(x-y) = \frac{\partial^{\mu}\partial_{\nu} + m_v^2\,\delta^{\mu}_{\nu}}{m_v^2\,(m_v^2+\Box)}\,\delta^4(x-y)\ . \tag{4.33}$$

Performing the above transformation, the resulting integration over v'_μ decouples and we obtain

$$\begin{aligned}
&\langle 0|T\,j^{\mu}(y_1)\,j^{\nu}(y_2)|0\rangle_0 \\
&= \left(\alpha_j\eta^{\mu\nu}\,\delta^4(y_1-y_2) + m_v^2\alpha_j\,\frac{1}{\mathrm{i}^2}\,\frac{\delta}{\delta\eta_{\mu}(y_1)}\,\frac{\delta}{\delta\eta_{\nu}(y_2)}\right) \\
&\times\ \mathrm{e}^{\mathrm{i}\int d^4x d^4y\,\left\{-\eta_{\nu}(x)\,(D^{-1})^{\nu}{}_{\mu}(x-y)\,\eta^{\mu}(y)\right\}}\Big|_{\eta^{\mu}=0}\ .
\end{aligned} \tag{4.34}$$

Finally, performing the remaining functional derivatives, we get

$$\begin{aligned}
\langle 0|T\,j^{\mu}(y_1)j^{\nu}(y_2)|0\rangle_0 &= \alpha_j\eta^{\mu\nu}\delta^4(y_1-y_2) - \alpha_j\frac{\partial^{\mu}\partial^{\nu}+m_v^2\eta^{\mu\nu}}{m_v^2+\Box}\delta^4(y_1-y_2) \\
&= \alpha_j\frac{\eta^{\mu\nu}\Box - \partial^{\mu}\partial^{\nu}}{m_v^2+\Box}\delta^4(y_1-y_2)\,.
\end{aligned} \tag{4.35}$$

Fourier transforming to momentum space leads to the following expression in lowest order:

$$\Pi_0^{\mu\nu}(p) = \mathcal{F} < 0|T\,j^{\mu}(y_1)\,j^{\nu}(y_2)|0 >_0 = (p^{\mu}p^{\nu} - \eta^{\mu\nu}p^2)\,T(p^2) \tag{4.36}$$

with $T(p^2) := \frac{\alpha_j}{m_v^2-p^2}$. This result has the expected Lorentz structure. Moreover, we obtain the identity $p_{\mu}\,\Pi_0^{\mu\nu}(p) = 0$, which is nothing but the vector Ward identity. Thus, in lowest order of the above defined perturbation series, our result obeys the vector Ward identity.

However, a direct comparison with the well known perturbation series of QED is not possible. This is due to the fact that the mass $m_v^2 = \alpha_j e^2$ of the spin-1 field v_μ occurring in (4.35) contains the bare coupling constant e. Thus, expanding this result around $m_v^2 = 0$, which corresponds to a power expansion in e^2, we obtain non-vanishing contributions to all orders in e^2. Therefore, formula (4.35) can be interpreted as a resumation of particular quantum corrections, which results in an effective ("dynamical") mass for the field v_μ. Unfortunately, higher order contributions (in the sense of our expansion) cannot be calculated analytically.

5. The chiral anomaly

In this section we show that within our approach the correct Adler–Bardeen anomaly [14] is obtained in the lowest order approximation discussed in the previous section. Again we restrict ourselves to the massless case.

Due to the bosonization rule (3.16) the chiral anomaly $\langle \partial_\mu j_5^\mu \rangle$ is given by the vacuum expectation value

$$\begin{aligned}\langle \partial_\mu j_5^\mu \rangle &\equiv \left\langle \frac{\alpha_{j^5}}{2} \left(\partial_\mu \partial^\mu \alpha(y)\right)\right\rangle \\ &= \frac{1}{Z[0,0,0]} \frac{\alpha_{j^5}}{2\mathrm{i}} \left(\partial_\mu \frac{\delta}{\delta \zeta_\mu(y)}\right) Z[\zeta^\mu, \xi^\mu, \eta^\mu]\Big|_{\zeta^\mu, \xi^\mu, \eta^\mu = 0}, \quad (5.37)\end{aligned}$$

where $Z[\zeta^\mu, \xi^\mu, \eta^\mu]$ is given by (3.17) and $\mathcal{L}[\zeta^\mu, \xi^\mu, \eta^\mu; v_\mu, j^\mu, |h|, \alpha]$ by (3.18), or in terms of $k^\mu[j_\mu, |h|]$, by (4.22). Choosing a longitudinal source current $\zeta^\mu := (\partial^\mu \zeta)$ and transforming

$$\alpha'' = \alpha + \frac{4}{\alpha_{j^5}} \zeta + \frac{1}{4\,\alpha_{j^5}} k[j_\mu, |h|] \quad (5.38)$$

yields the Lagrangian in the following form:

$$\begin{aligned}&\mathcal{L}[\zeta^\mu, \xi^\mu, \eta^\mu; v_\mu, j^\mu, |h|, \alpha''] \\ &= -\frac{1}{4}\left(\partial_{[\mu} v_{\nu]}\right)^2 - e\, j^\mu v_\mu - \frac{1}{4\,\alpha_{j^5}} \zeta_\mu k^{\mu\|}[j_\mu, |h|] + \frac{\alpha_{j^5}}{128} k^{\mu\perp}[j_\mu, |h|] k_\mu^\perp[j_\mu, |h|] \\ &\quad - \frac{1}{2\,\alpha_j} j^\mu j_\mu - \frac{1}{2\,\alpha_h}|h|^4 - \frac{2}{\alpha_{j^5}} \zeta_\mu \zeta^\mu + \frac{\alpha_{j^5}}{8} (\partial^\mu \alpha'')(\partial_\mu \alpha'') + \xi^\mu j_\mu + \eta^\mu v_\mu.\end{aligned}$$

Now α'' can be integrated out trivially. Performing the functional derivative with respect to ζ^μ we obtain

$$\begin{aligned}&\left\langle \frac{\alpha_{j^5}}{2} \left(\partial_\mu \partial^\mu \alpha(y)\right)\right\rangle \\ &= \frac{1}{Z[0,0,0]} \int \prod_x \left\{ dv_\mu\, dj^\mu\, d|h|^2 \right\} \left(-\frac{1}{8} \partial_\mu (\partial^\mu k[j_\mu(y), |h(y)|] + 4\,\zeta^\mu(y)) \right) \\ &\qquad \times \mathrm{e}^{\mathrm{i} \int d^4x\, \mathcal{L}[\zeta^\mu, \xi^\mu, 0; v_\mu, j^\mu, |h|]}\Big|_{\zeta^\mu, \xi^\mu = 0} \\ &= \frac{1}{Z[0,0,0]} \int \prod_x \left\{ dv_\mu\, dj^\mu\, d|h|^2 \right\} \left(-\frac{1}{8} \partial_\mu k^\mu[j_\mu(y), |h(y)|] \right) \\ &\qquad \times \mathrm{e}^{\mathrm{i} \int d^4x\, \mathcal{L}[0, \xi^\mu, 0; v_\mu, j^\mu, |h|]}\Big|_{\xi^\mu = 0},\end{aligned}$$

where

$$\begin{aligned}&\mathcal{L}[0,\xi^\mu,0;v_\mu,j^\mu,|h|]\\ &= -\frac{1}{4}(\partial_{[\mu}v_{\nu]})^2 - ej^\mu v_\mu + \frac{\alpha_{j^5}}{128}k^{\mu\perp}[j_\mu,|h|]k^\perp_\mu[j_\mu,|h|] - \frac{1}{2\alpha_j}j^\mu j_\mu - \frac{1}{2\alpha_h}|h|^4 + \xi^\mu j_\mu\,.\end{aligned}$$

Due to the explicit dependence of $k^\nu[j_\mu(y),|h(y)|]$ on $|h|$, a further exact analytical treatment of this formula is impossible. But, as outlined in Section 3, $|h|$ enters our effective theory in a non-dynamical way and, therefore, in principle it can be "averaged" out. In the simplest approximation we replace $|h|$ by its mean value χ_0. This way we are led to

$$\begin{aligned}&\left\langle \frac{\alpha_{j^5}}{2}\,(\partial_\mu\partial^\mu\alpha(y))\right\rangle\\ &= \frac{1}{Z[0,0,0]}\int\prod_x\{dv_\mu\,dj^\mu\}\,\left(-\frac{1}{8}\,\partial_\mu k^\mu[j_\mu(y),\chi_0]\right)\\ &\qquad\times \mathrm{e}^{\mathrm{i}\int d^4x\,\mathcal{L}[0,\xi^\mu,0;v_\mu,j^\mu]}\Big|_{\xi^\mu=0}\\ &= \frac{1}{Z[0,0,0]}\int\prod_x\{dv_\mu\,dj^\mu\}\left(-\frac{1}{8\,\chi_0^2}\,\varepsilon^{\delta\mu\sigma\xi}(\partial_\mu j_\delta(y))(\partial_\sigma j_\xi(y))\right)\\ &\qquad\times\,\mathrm{e}^{\mathrm{i}\int d^4x\,\mathcal{L}[0,\xi^\mu,0;v_\mu,j^\mu]}\Big|_{\xi^\mu=0},\end{aligned}\tag{5.39}$$

with

$$\begin{aligned}&\mathcal{L}[0,\xi^\mu,0;v_\mu,j^\mu]\\ &= -\frac{1}{4}\,(\partial_{[\mu}v_{\nu]})^2 - e\,j^\mu v_\mu + \frac{\alpha_{j^5}}{128}\,k^{\mu\perp}[j_\mu,\chi_0]k^\perp_\mu[j_\mu,\chi_0] - \frac{1}{2\,\alpha_j}\,j^\mu j_\mu + \xi^\mu j_\mu.\end{aligned}$$

Now the non-trivial coupling term $\frac{\alpha_{j^5}}{128}\,k^{\mu\perp}[j_\mu,\chi_0]\,k^\perp_\mu[j_\mu,\chi_0]$ will be treated similarly as in the previous section. In the lowest order we get

$$\begin{aligned}&\left\langle\frac{\alpha_{j^5}}{2}(\partial_\mu\partial^\mu\alpha(y))\right\rangle_0\\ &= \frac{1}{Z[0,0,0]}\int\prod_x\left\{dv_\mu dj^\mu\right\}\left(-\frac{1}{8\chi_0^2}\varepsilon^{\delta\mu\sigma\xi}\,(\partial_\mu j_\delta(y))\,(\partial_\sigma j_\xi(y))\right)\\ &\qquad\times\mathrm{e}^{\mathrm{i}\int d^4x\mathcal{L}_0[0,\xi^\mu,0;v_\mu,j^\mu]}\Big|_{\xi^\mu=0}\\ &= \frac{1}{Z[0,0,0]}\int\prod_x\left\{dv_\mu\,dj^\mu\right\}\left(-\frac{1}{8\mathrm{i}^2\,\chi_0^2}\varepsilon^{\delta\mu\sigma\xi}\left(\partial_\mu\frac{\delta}{\delta\xi^\delta(y)}\right)\left(\partial_\sigma\frac{\delta}{\delta\xi^\xi(y)}\right)\right)\\ &\qquad\times\mathrm{e}^{\mathrm{i}\int d^4x\mathcal{L}_0[0,\xi^\mu,0;v_\mu,j^\mu]}\Big|_{\xi^\mu=0},\end{aligned}$$

where

$$Z[0,0,0] = \int \prod_x \left\{ dv_\mu \, dj^\mu \right\} \mathrm{e}^{\mathrm{i} \int d^4x \, \mathcal{L}_0[0,0,0;v_\mu,j^\mu]}$$

and

$$\mathcal{L}_0[0,\xi^\mu,0;v_\mu,j^\mu] = -\frac{1}{4}\,(\partial_{[\mu}v_{\nu]})^2 - e\,j^\mu v_\mu - \frac{1}{2\,\alpha_j}\,j^\mu j_\mu + \xi^\mu j_\mu .$$

Performing the Gaussian integration over j^μ and the functional derivatives with respect to ξ^μ, we finally obtain

$$\begin{aligned} & \left\langle \frac{\alpha_{j^5}}{2}\,(\partial_\mu \partial^\mu \alpha(y)) \right\rangle_0 \\ = \; & \frac{e^2\alpha_j^2}{8\chi_0^2}\,\varepsilon^{\mu\nu\alpha\beta}\,\frac{1}{Z[0]} \int \prod_x \{dv_\mu\}\,(\partial_{[\mu}v_{\nu]}(y))(\partial_{[\alpha}v_{\beta]}(y))\,\mathrm{e}^{\mathrm{i}\int d^4x\,\mathcal{L}[v^\mu]} \\ = \; & \frac{e^2\alpha_j^2}{8\chi_0^2}\,\varepsilon^{\mu\nu\alpha\beta}\,\langle (\partial_{[\mu}v_{\nu]}(y))(\partial_{[\alpha}v_{\beta]}(y))\rangle \end{aligned} \tag{5.40}$$

with $Z[0] = \int \prod_x \{dv_\mu\}\,\mathrm{e}^{\mathrm{i}\int d^4x\,\mathcal{L}[v^\mu]}$ and

$$\mathcal{L}[v_\mu] = -\frac{1}{4}\,\left(\partial_{[\mu}v_{\nu]}\right)^2 + \frac{m_v^2}{2}\,v^\mu v_\mu \,. \tag{5.41}$$

Thus, bearing in mind that $F_{\mu\nu} = \partial_{[\mu}v_{\nu]}$ we can formulate the following result:

Proposition 2 *In lowest order, the chiral anomaly of massless QED in* $(3+1)$ *dimensions is given by the vacuum expectation value*

$$\left\langle \frac{\alpha_{j^5}}{2}\,(\partial_\mu \partial^\mu \alpha(y)) \right\rangle_0 = \frac{e^2\alpha_j^2}{8\chi_0^2}\,\varepsilon^{\mu\nu\alpha\beta}\,\langle F_{\mu\nu}\,F_{\alpha\beta}\rangle \tag{5.42}$$

with respect to the functional measure $\prod_x \{dv_\mu\}\,\mathrm{e}^{\mathrm{i}\int d^4x\,\mathcal{L}[v_\mu]}$ *, where*

$$\mathcal{L}[v_\mu] = -\frac{1}{4}\,\left(\partial_{[\mu}v_{\nu]}\right)^2 + \frac{m_v^2}{2}\,v^\mu v_\mu \;. \tag{5.43}$$

If we start with an external electromagnetic field, v_μ becomes external, too. In that case we get

$$\left\langle \frac{\alpha_{j^5}}{2}\,(\partial_\mu \partial^\mu \alpha(y)) \right\rangle_v = \frac{e^2\alpha_j^2}{8\chi_0^2}\,\varepsilon^{\mu\nu\alpha\beta}\,F_{\mu\nu}\,F_{\alpha\beta}\,, \tag{5.44}$$

where $F_{\mu\nu} = \partial_{[\mu}v_{\nu]}$ denotes the external electromagnetic field strength.

Observe that we get the correct coefficient, see [13, 14], for the chiral anomaly if we require the following relation between our parameters:

$$\frac{\alpha_j^2}{\chi_0^2} = \frac{1}{2\,\pi^2}\,. \tag{5.45}$$

This yields

$$\langle \partial^\mu j_\mu^5(y))\rangle_v \equiv \left\langle \frac{\alpha_{j^5}}{2}\,(\partial_\mu \partial^\mu \alpha(y))\right\rangle_v = \frac{e^2}{16\,\pi^2}\,\varepsilon^{\mu\nu\alpha\beta}\,F_{\mu\nu}\,F_{\alpha\beta}\,. \tag{5.46}$$

Thus, the anomaly can be used to fix one of the parameters of our effective theory as discussed in the Introduction.

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