FURTHER STUDY OF THE $\pi\pi$ S-WAVE ISOSCALAR AMPLITUDE BELOW THE $K\overline{K}$ THRESHOLD

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We continue the analysis of S-wave production amplitudes for the reaction $\pi^- p \to \pi^+\pi^- n$ involving the data obtained by the CERN–Cracow–Munich collaboration on a transversely polarized target at 17.2 GeV/c π^- momentum. This study deals with the region below the $K\overline{K}$ threshold. In particular, we study the "up-steep" solution containing a narrow S-wave resonance under the $\rho(770)$. This solution exhibits a considerable inelasticity η which does not have any physical interpretation. Assuming that this inelasticity behaviour represents an unlikely fluctuation we impose $\eta \equiv 1$ for all data points. This leads to non-physical results in one third of the $\pi^+\pi^-$ effective mass bins and in the remaining mass bins some parameters behave in a queer way. The situation is even worse for the "down-steep" solution. We conclude that the 17.2 GeV data cannot be described by a relatively narrow $f_0(750)$. The "down-flat" and "up-flat" solutions which easily pass the $\eta \equiv 1$ constraint exhibit a slow increase of phase shifts in the $\rho(770)$ mass range.

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1. Introduction

Scalar mesons are one of the main puzzles of light quark spectroscopy. Even in the lowest mass region (below the $K\overline{K}$ threshold) the situation is far from being clear. In addition to a broad $f_0(500)$ interpreted either as a $q-\overline{q}$ object (see *e.g.* Ref. [1]) or as a glueball by Ochs [2], the relatively narrow $f_0(750)$ has been persistently claimed by Svec [3–5]. Arguments against the narrow $f_0(750)$ has been given *e.g.* by Morgan in Ref. [6]. The main source of information in this mass region is the $\pi\pi$ Partial Wave Analysis (PWA) yielding the S-wave. It should be stressed that a study of S-wave

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objects *does require* the partial wave analysis to "subtract" the dominant contribution of leading $\rho(770)$ meson.

Virtually all PWA's in last decades were based on the old CERN–Munich experiment [7], which supplied 3×10^5 events of the reaction

$$\pi^- p \to \pi^+\pi^- n \tag{1}$$

at 17.2 GeV/c. The number of observables provided by such experiment is much smaller than the number of real parameters needed to describe the partial waves. Consequently, the dominance of pseudoscalar exchange, equivalent to the absence of pseudovector exchange and several other physical assumptions have been made in previous studies [7-8]. These results have been generally used *without even mentioning the assumptions* essential for their derivation.

In our previous paper [9] (hereafter called paper I) we have used the results of PWA performed in the effective mass $m_{\pi\pi}$ range from 600 MeV to 1600 MeV at four-momentum transfer squared $|t| = (0.005\text{–}0.200)$ GeV$^2/c^2$ using additionally the results of the polarized target experiment. This experiment, performed 25 years ago by the CCM (CERN–Cracow–Munich) collaboration [10], provided 1.2×10^6 events of the reaction

$$\pi^- p_\uparrow \to \pi^+\pi^- n \tag{2}$$

also at 17.2 GeV/c. Combination of results of both experiments yields a number of observables sufficient for performing a quasi-complete and energy independent PWA. This analysis is only quasi-complete because of an unknown phase between two sets of transversity amplitudes. Nevertheless, *full* (containing both π and $a_1$exchange) intensities of partial waves could be determined in a model-independent way. The original study of the CCM collaboration [11] removed ambiguities appearing in earlier studies, except for the "up-down" ambiguity [12]. The "up" solution contains an S-wave resonance just under the $\rho(770)$ and of similar width, while the "down" S-wave modulus stays high and nearly constant all the way to the $f_0(980)$.

In paper I we have made a further step in the analysis of 17.2 GeV/c data bridging two sets of transversity amplitudes. We required the phases of the leading P, D and F-transversity amplitudes to follow the phases of the Breit–Wigner $\rho(770)$, $f_2(1270)$ and $\rho_3(1690)$ resonant amplitudes in the low, medium and high mass region, respectively. Further, using the measured phase differences between the S-wave and the higher waves we determined the *absolute* phases of the S-wave transversity amplitudes. Once the phases are known, the S-wave amplitudes of different transversity can be combined which allows us to determine explicitly for the first time the pseudoscalar and pseudovector exchange amplitudes in the S-wave. This

has been done using much weaker assumptions[1] than those made in *any* earlier analysis. The price we pay is a fourfold ambiguity in our pseudoscalar exchange S-wave amplitude. In addition to "up-down" ambiguity of the old CCM analysis [11] there are ambiguities resulting from adding or subtracting the phase difference since the PWA yields only the absolute value of the phase difference. Thus we have "down-flat", "down-steep", "up-flat" and "up-steep" solutions. Differences between "flat" and "steep" refer mainly to the behaviour of the S-wave phase shifts below the $f_0(980)$. Above the $f_0(980)$ all the solutions are fairly similar. It is the region below the $f_0(980)$ which is a subject of the present paper. The main difference is that both "steep" solutions contain a relatively narrow[2] resonance under the $\rho(770)$ (like the old "up" solution) while both "flat" solutions indicate a $f_0(500)$ state with a width of about 500 MeV. In particular the "down-flat" solution is very similar to the old solution of the CERN-Munich group [7].

In paper I we have determined inelasticities η for isoscalar π-exchange amplitudes in all solutions (see Fig. 1). It is obvious that both "flat" solutions easily pass the $\eta \leq 1$ test and the "down-steep" solution is not acceptable. Unfortunately, the situation is not as simple as presented in the last edition of Review of Particle Properties [13]. The authors of "Note on scalar mesons" discussing a hypothetical narrow state at 750 MeV write: "Such a solution is also found by (Kaminski 97)... However they show that unitarity is violated for this solution; therefore a narrow light f_0 state below 900 MeV seems to be excluded". The point is that our "up-steep" solution although exhibiting "a puzzling behaviour of inelasticity" cannot be excluded so simply as our "down-steep" solution and it also contains a narrow $f_0(750)$. It should be recalled, that contrary to the Svec analysis which uses only moduli of the unseparated (pseudoscalar and pseudovector exchange) S-wave, we study inelasticity and phase shift of the pure $\pi\pi \to \pi\pi$ isoscalar S-wave. In this paper we discuss the feasibility of an interpretation of 17.2 GeV data in terms of a narrow $f_0(750)$.

The paper is organized as follows. In Sect. 2 we study the inelasticity behaviour in more detail. In Sect. 3 we impose strict $\eta \equiv 1$ condition on all solutions for the effective $\pi\pi$ masses below the $K\overline{K}$ threshold. The results are discussed in Sect. 4 and summarized in Sect. 5.

[1] The main assumption was neglecting of a possible influence of the a_1 exchange in $I = 2$ S-wave as well as in P, D and F waves.

[2] Hereafter "relatively narrow" means "with a width close to $\Gamma_\rho = 150$ MeV".

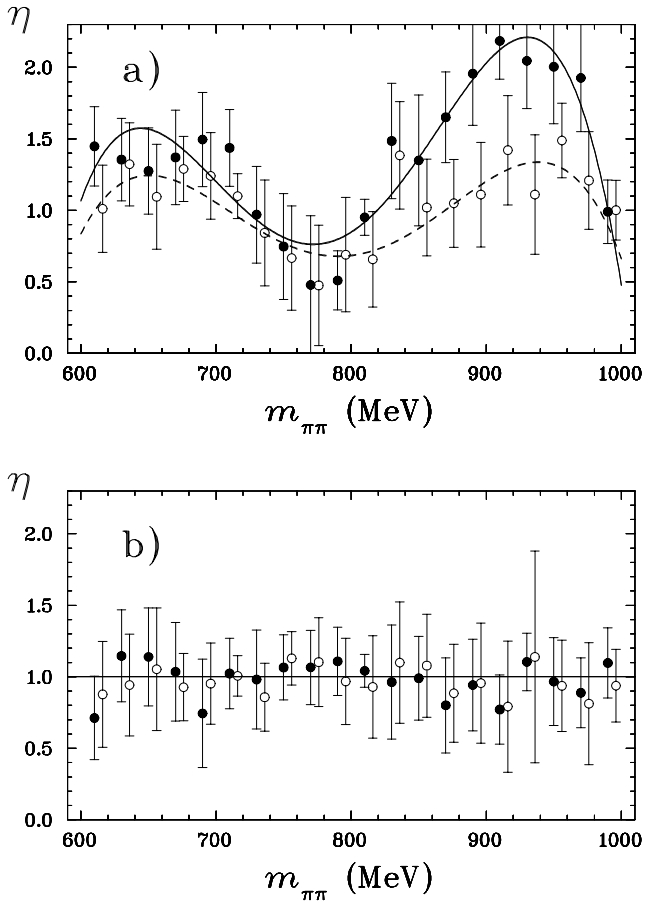

Fig. 1. a) Scalar-isoscalar $\pi\pi$ inelasticity coefficient η versus the effective $\pi\pi$ mass for the "down-steep" (full circles) and "up-steep" (open circles, shifted by 6 MeV) solutions. Solid and dashed lines represent fits of the fourth order polynomial to the coefficient η for the "down-steep" and "up-steep" solutions, respectively. b) Same as in a) but for the "down-flat" (full circles) and "up-flat" (open circles) solutions. Solid line shows the $\eta = 1$ value fitting the experimental data.

2. *S*-wave inelasticity

As seen in Fig. 1a the inelasticity for the "up-steep" solution behaves in a non-trivial way. Below 720 MeV and above 820 MeV $\eta > 1$ while in the intermediate mass region $\eta < 1$. Qualitatively this behaviour of the "up-steep" solution is very similar to the "down-steep" solution. The latter solution was excluded in paper I since the corresponding inelasticity significantly exceeded unity for $m_{\pi\pi} > 820$ MeV. A general trend of η points

for the "up-steep" solution resembles very much a trend of the "down-steep" data in the region below 820 MeV. Above 820 MeV the "up-steep" values of inelasticity lie also above unity. They are, however, closer to unity than the "down-steep" inelasticities. This fact prevented us to reject in paper I the "up-steep" solution solely on a basis of the minimization fit of the sum $\sum_i(\eta_i - 1)^2$ in the whole energy region between 600 MeV and 1000 MeV.

Now we examine in detail the $m_{\pi\pi}$ dependence of the inelasticity η corresponding to the four solutions. Let us define by (a), (b) and (c) three ranges of $m_{\pi\pi}$: $600 \leq m_{\pi\pi} \leq 720$ MeV, $720 \leq m_{\pi\pi} \leq 820$ MeV and $820 \leq m_{\pi\pi} \leq 1000$ MeV, respectively. Then we make a simple fit to inelasticities in the above three ranges by a constant N. We minimize the sum $\sum_i(\eta_i - N)^2/\Delta\eta_i{}^2$, where $\Delta\eta_i$ are the experimental errors of η. The results are shown in Table I. Let us notice that for the "steep" solutions the second value in the intermediate region (b) is definitely lower than unity, this is also different from the values in the ranges (a) and (c) which in turn are higher than unity. In this aspect the "up-steep" solution is very similar to the solution "down-steep" already rejected in paper I. On the other hand both solutions "down-flat" and "up-flat" do not exhibit any dip of η in the range (b); the constants N are very close to unity everywhere. If we determine one common value of N in the whole range between 600 MeV and 1000 MeV then we obtain 1.00 ± 0.06 for the solution "down-flat" and 0.98 ± 0.06 for the solution "up-flat". Both are compatible with unity as seen in Fig. 1b.

TABLE I

Constants N fitted to η in three ranges: $600 < m_{\pi\pi} < 720$ MeV (a), $720 < m_{\pi\pi} < 820$ MeV (b) and $820 < m_{\pi\pi} < 1000$ MeV (c)

Solution	(a)	(b)	(c)
"up-steep"	1.17 ± 0.10	0.67 ± 0.17	1.18 ± 0.10
"down-steep"	1.40 ± 0.12	0.82 ± 0.10	1.67 ± 0.11
"up-flat"	0.97 ± 0.10	1.02 ± 0.11	0.95 ± 0.12
"down-flat"	0.97 ± 0.13	1.05 ± 0.09	0.96 ± 0.09

Obviously we cannot obtain constant fits to η of a similar quality for the "steep" solutions as those fits to the "flat" solutions. In order to probe rather strong energy variation of η in the former solutions we have tried to fit it by polynomials of different powers. We treat these fits only as an ad hoc description of data in the particular region between 600 and 1000 MeV. It turned out that in order to obtain good fits we need polynomials of the order as high as four (see Fig. 1a). Once again one can see the similar depen-

dence of inelasticities when one compares a shape of the fitted curves to the "up-steep" and "down-steep" data points. A minimum of η for $m_{\pi\pi}$ around 800 MeV, indicating an inelasticity, will be discussed below. Before passing to a physical discussion of possible consequences of the strong energy dependence of inelasticity for the "up-steep" solution below the $K\bar{K}$ threshold, let us discuss a possibility of the fluctuation of η values around unity. It is true that the deviation is not significant (see paper I: $\chi^2 = 15$ for 17 points below 940 MeV) *if we ignore the shape* of the η distribution. However, the chance of all five $\eta < 1$ points falling accidentally in the effective mass region between 720 and 820 MeV is only 2×10^{-3}. This value was calculated by taking into account a number of all the possibilities to choose five lowest values of η among twenty available points in the effective mass range between 600 and 1000 MeV and then grouping them together in the range (b).

In paper I in addition to inelasticities we have calculated the phase shift values. For both "flat" solutions phase shifts grow slowly with $m_{\pi\pi}$. For the "steep" solutions we have obtained rather fast increase of the S-wave I=0 $\pi\pi$ phase shifts near 770 MeV — the value close to the ρ-meson mass. We have tentatively fitted inelasticities and phase shifts of the solution "up-steep" by a single resonance. This was done for $m_{\pi\pi} < 940$ MeV in order to avoid a possible influence of the $f_0(980)$. We additionally allowed a simultaneous change of all η values by the same factor R_η since in paper I they were fixed only by minimizing the $\sum_i(\eta_i - 1)^2$ value. In fact a combined fit (χ^2/NDF= 26/30) yields the reduction factor $R_\eta = 0.71 \pm 0.10$ and the following resonance parameters: $m = (754 \pm 5)$ MeV, $\Gamma = (162 \pm 9)$ MeV and $x = 0.74^{+0.10}_{-0.08}$, x being an elasticity of the resonance. The mass and the width agree with $m = (753 \pm 19)$ MeV, $\Gamma = (108 \pm 53)$ MeV claimed by Svec [5], on the basis of the same data. However, such a considerable inelasticity is inconsistent with the available experimental data on the 4π system which is the only kinematically possible channel.

The lowest mass of the 4π system is probably available in the central production due to a $1/m^2_{4\pi}$ flux factor. In fact the WA91 [14] and WA102 [15] collaborations have found a tiny peak around 800 MeV in the mass distribution of their 4π system produced at 450 GeV in the reaction

$$pp \to p_{\rm f}\pi^+\pi^+\pi^-\pi^-p_{\rm s}, \tag{3}$$

where $p_{\rm f}$ and $p_{\rm s}$ stand for fast and slow proton, respectively. This is however well explained by the reflection from the $\eta' \to \eta\pi^+\pi^-$ decay with the loss of the slow π^0 from the subsequent $\eta \to \pi^+\pi^-\pi^0$ decay. The IHEP-IISN-LANL-LAPP collaboration [16] studying at low $|t|$ a reaction similar to reaction (1) *i.e.*

$$\pi^-p \to \pi^0\pi^0\pi^0\pi^0n \tag{4}$$

has found that the effective mass distribution of their $4\pi^0$ system starts only around 800 MeV and rises smoothly. The LRL group [17] studying the reaction

$$\pi^+ p \to \pi^+\pi^+\pi^-\pi^-\Delta^{++} \tag{5}$$

found hardly any events with a 4π mass below 1 GeV. The 4π mass spectrum from annihilation $\overline{p}N \to 5\pi$ starts at even higher mass as shown in [18–21] . Thus the non-zero inelasticity in the "up-steep" solution does not have any reasonable physical interpretation. In next section we will assume $\eta \equiv 1$.

3. Another approach to the $\pi\pi$ scalar-isoscalar phase shifts

In Sect. 2 we have shown that the 4π channel is very weak below 1000 MeV. Therefore now we assume that the $\pi\pi$ S-wave inelasticity is exactly equal to unity up to 990 MeV and we shall make a new analysis of the $\pi\pi$ isoscalar-scalar phase shifts obtained from the $\pi^- p \to \pi^+\pi^- n$ data at 17.2 GeV/c. Let us recall that in paper I a separation of the S-wave pseudoscalar A_0 and pseudovector B_0 exchange amplitudes for the production process was performed and that we have calculated the S-wave $\pi^+\pi^- \to \pi^+\pi^-$ amplitude a_S from the following formula:

$$a_S = K f A_0. \tag{6}$$

In (6) K is the proportionality factor

$$K = -\frac{8\pi p_\pi \sqrt{s q_\pi}}{m_{\pi\pi}\sqrt{2 \cdot \frac{g^2}{4\pi}}} \frac{1}{2M}, \tag{7}$$

where p_π is the incoming π^- momentum in the $\pi^- p$ centre of mass frame, s is the square of the total energy in the same frame, q_π is the final pion momentum in the $\pi\pi$ rest frame, $g^2/4\pi$ is the pion-nucleon coupling constant (taken as 14.6 in paper I) and M is the proton mass. The coefficient f is the complex correction factor which (when averaged over the four momentum transfer squared) represents the t-dependence of the pion-nucleon vertex function, the off-shellness of the exchanged pion and a possible phase of the pion-exchange propagator — a case where at high energy the exchanged pion is treated as a Regge particle. This coefficient was calculated from the requirement that the sum $\sum_i (\eta_i - 1)^2$ was minimal for the inelasticities of the scalar-isoscalar $\pi\pi$ amplitude a_0 for a set of points depending on the $\pi\pi$ effective mass up to the $K\overline{K}$ threshold mass. The amplitude a_0 is connected to the amplitude a_S and the isospin 2 S-wave amplitude a_2 in the following way:

$$a_0 = 3a_S - \frac{1}{2}a_2. \tag{8}$$

The a_0 is also related to the isospin 0 S-wave phase shift δ_0 and inelasticity η:

$$a_0 = \frac{\eta e^{2i\delta_0} - 1}{2i}. \tag{9}$$

Motivated by the results of Sect. 2 we impose now the condition $\eta \equiv 1$ in the whole $m_{\pi\pi}$ range below 1 GeV. Then, from (9)

$$a_0 = \sin\delta_0 e^{i\delta_0}, \tag{10}$$

so the modulus of a_0 is uniquely related to the phase shift value δ_0. Since in (6) A_0 is the complex amplitude calculated with some errors coming from experimental uncertainties, then (8) and (10) are not necessarily satisfied if we require $\eta \equiv 1$. In order to keep this assumption valid we have to include at least one additional real factor n in equation (6) at each $m_{\pi\pi}$, so now

$$a_S^{\text{new}} = n a_S. \tag{11}$$

We shall also assume that the isospin 2 amplitude a_2 is fully elastic and can be described by the corresponding phase shift δ_2

$$a_2 = \sin\delta_2 e^{i\delta_2}. \tag{12}$$

Then, following (9) and inserting (11) into (8) one has to satisfy equations

$$\eta^2 = |1 + 2ia_0|^2 = \left|1 + 2i(3Kfn A_0 - \frac{1}{2}a_2)\right|^2 \equiv 1. \tag{13}$$

We treat (13) as a quadratic equation for n. Its roots are

$$n = \frac{1}{6\,|a_S|}(b \pm \sqrt{b^2 - 3\sin^2\delta_2}), \tag{14}$$

where

$$b = (1 + \sin^2\delta_2)\sin\alpha + \frac{1}{2}\sin^2\delta_2\cos\alpha \tag{15}$$

and α denotes the phase of a_S:

$$a_S = |a_S|\, e^{i\alpha}. \tag{16}$$

Let us remark that in the limit of δ_2 going to 0 (vanishing a_2) we should consider only the upper sign (+) in (14). In general, both solutions for n are possible; we have, however, used only the root with the upper sign since its value was closer to unity. The isotensor phase shift δ_2 is calculated according to the parametrization given in paper I which fits well the data of Ref. [22] obtained by method B.

For each value of $m_{\pi\pi}$ we have to check whether the roots exist. With $\eta = 1$ the amplitude a_0 must satisfy the elastic unitarity condition

$$\text{Im } a_0 = |a_0|^2 \,. \tag{17}$$

Therefore the following inequality must be fulfilled

$$|b| \geq \sqrt{3}\,|\sin \delta_2| \,. \tag{18}$$

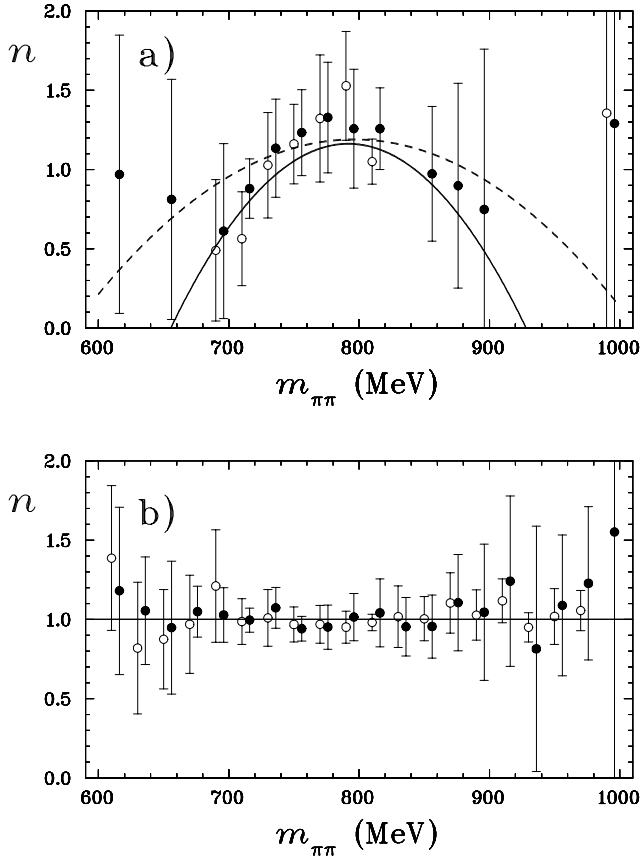

Fig. 2. a) Effective mass dependence of the parameter n for the "down-steep" (full circles) and "up-steep" (open circles, shifted by 6 MeV) solutions. Solid and dashed lines represent fits of the second order polynomial to n for the "down-steep" and "up-steep" solutions, respectively. Note that in many bins no physical solution could be found. b) Same as in a) but for the "down-flat" (full circles) and "up-flat" (open circles) solutions. Solid line represents values of the constant parameters fitted to n for the "down-flat" and "up-flat" solutions.

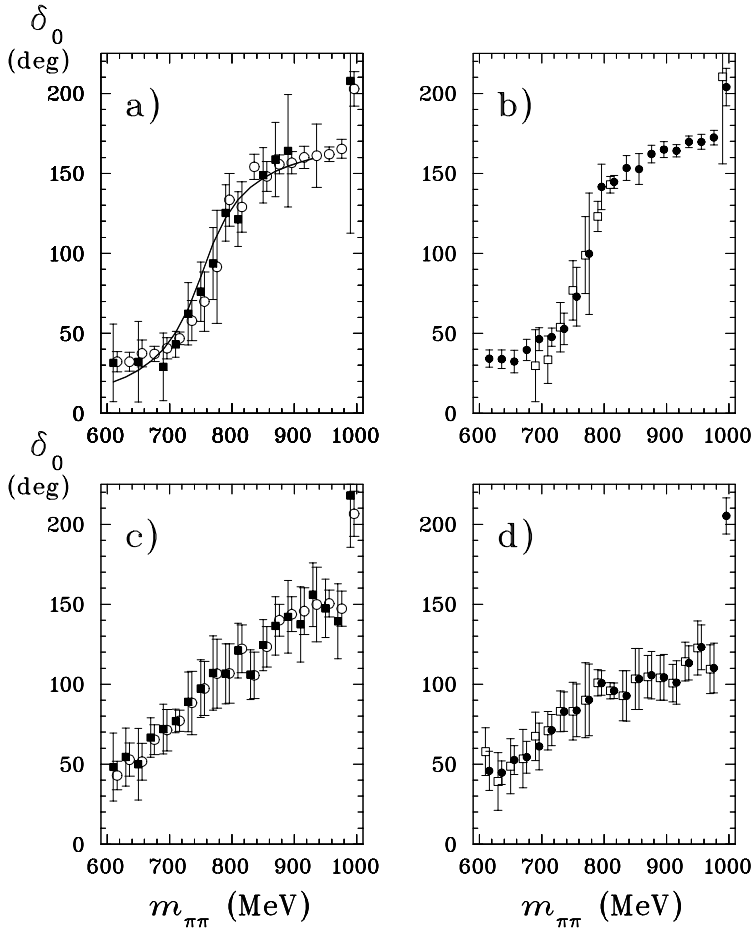

Fig. 3. Comparison of the scalar-isoscalar $\pi\pi$ phase shifts obtained in paper I (circles, shifted in $m_{\pi\pi}$ by 6 MeV) with the new phase shifts (squares) calculated under the assumption $\eta \equiv 1$. a) For the "up-steep" solution. Solid line shows the Breit–Wigner fit to the data marked by open circles as described in Sect. 2. b) For the "down-steep" solution. c) For the "up-flat" solution. d) For the "down-flat" solution.

We have obtained the following numerical results for the solutions discussed in I: the inequality (18) was satisfied for all twenty $m_{\pi\pi}$ points of the solution "up-flat" and for 19 points (except of the extreme point at 990 MeV) corresponding to the solution "down-flat". However for 7 points of the solution "up-steep" and for 12 points of the solution "down-steep" the condition (18) was violated. This fact casts a serious doubt on a validity of both

"steep" solutions. The resulting values of n calculated in cases when (18) was satisfied are shown in Fig. 2. The errors of n are due to experimental errors of the modulus $|a_S|$ and the phase α extracted from experiment. No errors of δ_2 were taken into account since δ_2 values were calculated using the smooth theoretical parametrization. We see in Fig. 2b that both "flat" solutions are well fitted by constants very close to unity (0.994 ± 0.03 for the "down-flat" and 0.997 ± 0.04 for the "up-flat" solution). On the other hand a variation of n with $m_{\pi\pi}$ for two "steep" solutions is better described by a parabola than by a constant (see Fig. 2a). Such strong dependence on $m_{\pi\pi}$ of the coefficient n corresponding to both "steep" solutions can be used as a fairly strong argument against the acceptance of these solutions as good physical solutions.

For completeness we present in Fig. 3 new $\pi\pi$ phase shifts calculated from (10) using $a_S^{\rm new}$ given by (11). Obviously we show only these points for which the corresponding values of n do exist. The new phase shifts, calculated under the assumption that $\eta \equiv 1$, agree very well with those presented in paper I for the "flat" solutions. For the "steep" solutions this agreement is not so good and the new errors of δ_0 are larger than those shown in paper I.

4. Discussion

In Sections 2 and 3 we have presented arguments that both "steep" solutions have unphysical behaviour. On the other hand both "flat" solutions satisfy well our tests and none of them can be eliminated using the methods described in this paper. Therefore let us discuss common features of these solutions and major differences between them. In Fig. 4 the "flat" solutions are plotted in a wide effective mass range up to 1600 MeV. Their shape is quite similar. One sees an initial steady grow of phase shifts with $m_{\pi\pi}$ above 600 MeV, then at about 1000 MeV, corresponding to the $K\overline{K}$ threshold, there is a jump as high as 140° and further on a fairly steep increase above 1300 MeV. An interpretation of this behaviour of phases in terms of three scalar resonances $f_0(500)$, $f_0(980)$ and $f_0(1500)$, started in paper I, has been continued in more detail in Refs. [23,24]. We do not repeat it here but we underline major differences between the "up-flat" and "down-flat" phase shifts since they lead to different values of the $f_0(500)$ resonance parameters. The $f_0(500)$ mass for the "up-flat" solution is by about 50 MeV higher than the corresponding mass for the "down-flat" solution. The reversed relation for the $f_0(500)$ width leading to a difference between 45 and 50 MeV is also observed. We do not see important differences between the $f_0(980)$ parameters for the above solutions. As seen in Fig. 4 the most important differences, reaching about 45°, exist between 800 and 1000 MeV. They are

related to a difference between the *moduli* of the S-wave pseudoscalar amplitude (see paper I). This difference of moduli is then directly transformed into a difference between the phase shifts because inelasticity for both "flat" solutions below the $K\overline{K}$ threshold is very close to unity. This fact in turn guarantees a fulfillment of the elastic unitarity condition (17). Closer inspection into Fig. 4 allows one to see a continuity of the phase differences above the $K\overline{K}$ threshold, namely the "up-flat" points lie systematically above the "down-flat" points up to about 1300 MeV. Above this value of $m_{\pi\pi}$ we do not observe any systematic difference.

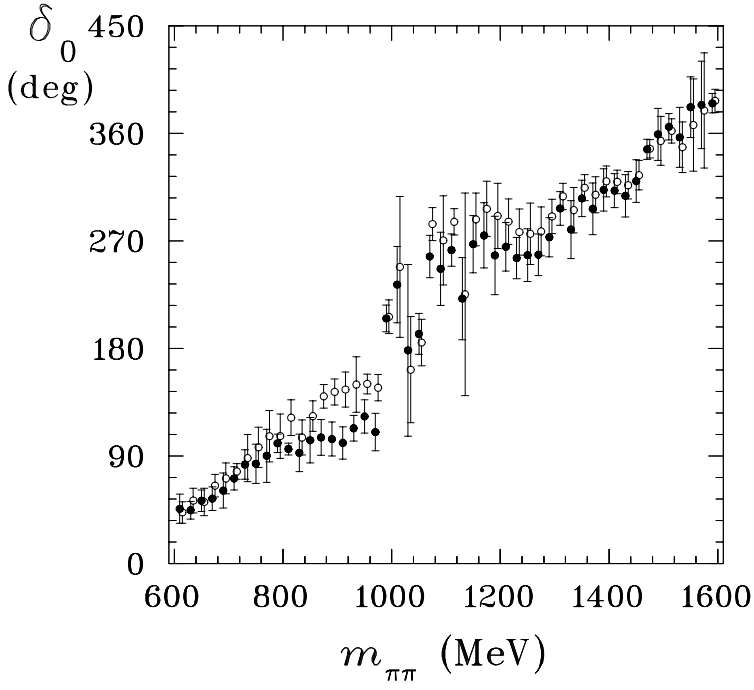

Fig. 4. Comparison of the scalar-isoscalar $\pi\pi$ phase shifts obtained in paper I for the "down-flat" (full circles) and "up-flat" (open circles) solutions.

Since the data points of the "up-flat" solution lie between the points of the (already excluded) "up-steep" solution and the "down-flat" one, we have checked whether the $f_0(750)$ survives in this solution. This was done by fitting elasticities and phase shifts of the "up-flat" solution by a single Breit–Wigner term like it was done in Sect. 2 for the "up-steep" solution. No overall change of η values was needed ($R_\eta = 1.01^{+0.07}_{-0.15}$) and the resonance parameters are $m = (732 \pm 8)$ MeV, $\Gamma = (246^{+37}_{-25})$ MeV and $x = 1.00^{+0.08}_{-0.16}$. The large width is inconsistent with a narrow $f_0(750)$.

5. Summary

In conclusion, we have studied in detail the $\pi\pi$ effective mass dependence of the S-wave isoscalar phase shifts corresponding to four solutions "up-steep", "down-steep", "up-flat" and "down-flat" found in paper I. Both "steep" solutions exhibit an inelasticity behaviour which has no physical interpretation. We do not find any data on the 4π systems which could explain a strong $m_{\pi\pi}$ dependence of the inelasticity corresponding to the "steep" solutions below the $K\overline{K}$ threshold. The "down-steep" solution was already rejected in paper I since its inelasticity substantially exceeded unity for $m_{\pi\pi}$ above 820 MeV. Assuming that the "up-steep" inelasticity is an unusual fluctuation we impose $\eta \equiv 1$ for all points and, for completeness, in all solutions. This leads to non-physical results in 7/20 mass bins for the "up-steep" solution and in 12/20 bins for the "down-steep" solution. In the remaining mass bins the parameters behave in a queer way (compare Figs. 2a and 2b). We conclude that the "up-steep" solution cannot be treated as a good physical set of phase shifts. It can be eliminated together with the "down-steep" solution. However, the "up-flat" and "down-flat" solutions easily pass our tests. We would like to stress that both the $f_0(500)$ and $f_0(980)$ resonances are present in the "flat" solutions. This is not true for a relatively narrow $f_0(750)$.

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