GAMOW–TELLER BETA-DECAY STRENGTHS OF NEUTRON-DEFICIENT TIN ISOTOPES: COMPARISON OF FFST AND *pn*BCS+QRPA RESULTS

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Microscopic analysis of recent data on beta-decay of even neutron-deficient nuclides between ^{100}Sn and ^{108}Sn is performed within the self-consistent finite Fermi-system theory and BCS plus Quasiparticle Random Phase Approximation with G-matrix interaction and proton–neutron pairing. Strength functions of Gamow–Teller β-decay are calculated. The mechanisms of reduction of the GT strength are discussed.

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1. Introduction

Evolution of shell structure of exotic nuclei near new double-shell closures and its isospin dependence is an attractive problem. The interest in this field is mainly related with the progress in radioactive beams experiments and the important role of the exotic nuclei in explosive astrophysical processes. Study of the β-decay modes is often the only way to extend our knowledge of nuclear properties to region far from stability. For the proton rich nuclei the β-decay is dominated by the Gamow–Teller (GT) transitions. This is the fastest channel of the β-decay in the region of nuclei where super-allowed Fermi decay is suppressed by isospin selection rules. In the region of doubly-magic very neutron deficient ^{100}Sn the Gamow–Teller β^+ transition is built on $(1\pi g_{9/2}, 1\nu g_{7/2})$ shell-model configuration. The simplicity of the decay mode and the selectivity of the GT-transitions with $\Delta L = 0$, $\Delta S = 1$,

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and no parity change is a good reference point for checking various microscopic approaches to exotic nuclei. The maint points of interest are studying of GT-strength distribution and determining its magnitude. The accurate consideration of the problem requires the self-consistent study of the ground and excited states properties and pairing in the nuclei with $N \approx Z$ and the description of the splitting of the GT-strength over several final states in daughter nuclei.

2. Finite Fermi-system theory

The self-consistent finite Fermi-system (FFS) theory [1] has much in common with the HFB method with effective forces. The main problem of its practical application is the choice of the form and parametrization of the appropriate density functional. In the present work we have used the density functional by [2] which was fitted to the ground state properties of spin-zero nuclei. As a consequence, the scheme is not fully self-consistent in the case of spin–isospin excitations, and one has some freedom in choosing the effective spin–isospin NN-interaction.

In general, the method includes the description of the ground states of superfluid nuclei [2] and excited states of even and odd-A nuclei with pairing correlations [3, 4]. It consists of two steps:

1) calculating the self-consistent potential and the quasiparticle basis, taking into account the effects of pairing correlations. We use the density functional method and the quasiparticle Hamiltonian with a free kinetic energy operator[1]. Quasiparticle spectrum and wave functions are calculated in a self-consistent mean field which is the first functional derivative of the interaction energy with respect to normal density, the pairing potential being obtained as the functional derivative of the pairing energy with respect to the anomalous density (see *e.g.* [1]);

2) solving the QRPA[2]-type FFS theory equations and calculating the strength functions of nuclear charge-exchange excitations in sperfluid nuclei [3, 4].

The strength function determining the response of a superfluid nucleus to the charge-exchange external field $V^0 \propto \tau_\pm$ is given in the FFS theory by [5]

$$S(\omega) = \left[\int \hat{e}_q V_0(\vec{r}) \rho_{\rm tr}(\vec{r};\omega) \mathrm{d}\vec{r} \right]^2 , \qquad (1)$$

[1] The quasiparticle effective mass m^* being equal to the bare nucleon mass m.
[2] Quasiparticle Random Phase Approximation.

where $\hat{e}_q$ is the quasiparticle local charge with respect to the external field V_0 and the transition density $\rho_{\rm tr}$ of a nuclear excitation with a frequency ω_s is calculated in the FFS theory as

$$\rho_{\rm tr}(\vec{r};\omega_s) = C\,{\rm Im}\int {\rm d}\vec{r}\,' \Big[L(\vec{r},\vec{r}\,';\omega_s)V_{pn}(\vec{r}\,';\omega_s) + M(\vec{r},\vec{r}\,';\omega_s)V^{\rm h}_{\rm pn}(\vec{r}\,';\omega_s) + N^1(\vec{r},\vec{r}\,';\omega_s)d^1(\vec{r}\,';\omega_s) + N^2(\vec{r},\vec{r}\,';\omega_s)d^2(\vec{r}\,';\omega_s)\Big]\,, \qquad (2)$$

where the normalization constant C is calculated through matrix element of the nuclear transition from the ground state to the excited one:

$$M^2_{0\to s} = \left[\int {\rm d}\vec{r} e_q V_0(\vec{r})\rho_{\rm tr}(\vec{r},\omega_s)\right]^2 = \int\limits_{\Delta\omega} S(\omega){\rm d}\omega\,. \qquad (3)$$

Here $\Delta\omega$ is an energy interval in which the contribution of some specific maximum (or resonance) in $S(\omega)$ at $\omega=\omega_s$ can be extracted.

In Eq. (2), $V_{\rm pn}$ and $V^{\rm h}_{\rm pn}$ are the effective fields for particles and holes, respectively, arising due to the action of an external charge-exchange field $V^0 \propto \sigma\tau$ $(V^{0h}_{\rm pn}=0)$ and d^1 and d^2 are the effective changes of the corresponding pairing fields. These effective fields have to be found by solving the system of the FFS theory QRPA-like equations [5] for charge-changing excitations in non-magic nuclei. In (1)–(3) $\omega=\tilde{\omega}-\delta\mu$ with $\delta\mu=\mu^p-\mu^n$ being difference between the proton and neutron chemical potentials, and $\tilde{\omega}$ the excitation energy in daughter nucleus. The propagators L, M, N^1, N^2 are obtained by integrating various products of the normal and abnormal Green's functions G and F (see [5]).

In practice one often uses an ordinary expansion over single-particle wave functions φ_λ[3]. It is impossible to solve the effective field equation in the λ-representation without basis truncation because of single-particle continuum. To overcome this difficulty we are using a method of coordinate representation of the single-particle Green function allowing to include the whole *ph*-continuum in the FFS theory equations (see *e.g.* [6] and Refs. therein). For the charge-exchange excitations of superfluid nuclei a scheme for inclusion the ph-continuum has been developed [3,4] which is similar to the method of mixed (r,λ)-representation used in [7] for the neutral-channel excitations. Note that for nuclei near the drip lines the correct description of pairing should allow for the interaction between the bound nucleon states and continuum [4,8].

[3] The so-called λ-representation with $\lambda^{n(p)} = nljm\tau$ is the standard set of single-particle quantum numbers of the neutron ($\tau=\nu$) or proton ($\tau=\pi$) level with energy ε^τ_λ.

For an accurate description of the single-particle characteristics it is very important to choose an appropriate form and parametrization of the density functional. Here we have used the density functional as suggested in [2], where a dependence on ρ is simulated by simple fractional-linear functions and the surface contribution is related to the finite-range forces which are also density-dependent [9]. The total interaction energy of superfluid nucleus, $E_{\rm int}[\rho,\nu] = \int d\vec{r}\,\varepsilon_{\rm int}(\vec{r})$ is a functional of two densities, the normal, $\rho(\vec{r})$, and the anomalous, $\nu(\vec{r})$. Self-consistent calculation with such a functional looks like the standard variational HFB procedure in which the single-particle Hamiltonian takes the form

$$\mathcal{H} = \begin{pmatrix} h-\mu & -\Delta \\ -\Delta & \mu-h \end{pmatrix}, \tag{4}$$

where

$$h = \frac{p^2}{2m} + \frac{\delta E_{\rm int}[\rho,\nu]}{\delta\rho}, \quad \Delta = -\frac{\delta E_{\rm int}[\rho,\nu]}{\delta\nu}\,. \tag{5}$$

These equations were solved iteratively. The interaction energy density is represented as

$$\varepsilon_{\rm int} = \varepsilon_{\rm main} + \varepsilon_{\rm coul} + \varepsilon_{\rm sl} + \varepsilon_{\rm pair}\,, \tag{6}$$

where $\varepsilon_{\rm main}$ contains the volume isoscalar and isovector parts and these correspond to the surface isoscalar and isovector potential energies generated by the density-dependent finite-range forces [2]. The energy density of the Coulomb interaction $\varepsilon_{\rm coul}$ takes the usual form and includes the exchange part in the Slater approximation, while the spin-orbit term $\varepsilon_{\rm sl}$ in Eq. (6) comes from spin-orbit $\propto (\kappa+\kappa'\vec{\tau}_1\cdot\vec{\tau}_2)[\vec{\nabla}_1\delta(\vec{r}_1-\vec{r}_2)\times(\vec{p}_1-\vec{p}_2)]\cdot(\vec{\sigma}_1+\vec{\sigma}_2)$ and velocity spin-dependent $\propto (g_1+g_1'\vec{\tau}_1\cdot\vec{\tau}_2)(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{p}_1\cdot\vec{p}_2)$ interactions (for details see [2]). The last term in (6) is the pairing energy density $\varepsilon_{\rm pair} = \frac{1}{2}\nu F^{\xi}\nu$, where ν is the anomalous nucleon density and F^{ξ} plays the role of the effective force in the particle–particle channel and has been chosen for simplicity as:

$$F^{\xi}(\vec{r},\vec{r}\,') = -C_0 f^{\xi}\delta(\vec{r}-\vec{r}\,')\,. \tag{7}$$

Here C_0 is the inverse density of states at the Fermi surface in equilibrium nuclear matter (305 MeV$\times$fm^3) and f^{ξ} is a dimensionless interaction constant of the FFS theory [5]. The superscript ξ refers to the energy cut-off parameter which defines the number of single-particle levels taken into account when evaluating the anomalous Green's functions and, correspondingly, when solving equations for pairing fields $\Delta^{\tau}(\vec{r})$ and chemical potentials μ^{τ} as well as the dynamical FFS equations for excited states. In the present calculations pairing was treated in a diagonal approximation on the basis

of all bound single-particle levels and quasi-bound levels within the cut off energy interval of 15 MeV with $f^{\xi}_{nn} = f^{\xi}_{pp} = 0.28$, as we have used in [10] .

Parameters of the density functional (6) were chosen in [9] by fitting binding energies, charge distributions and single-particle spectra for magic nuclei ^{40}Ca, ^{48}Ca, ^{208}Pb and for non-magic ones both with weak superfluidity (^{90}Zr, ^{146}Gd) and developed pairing (even–even stable Sn and Pb isotopes). As for the description of the pairing properties with the interaction (7), the pairing potential Δ is found to be a smooth function of $\vec{r}$ and its non-diagonal matrix elements $\Delta_{\lambda\lambda'}$ are small, *i.e.* $\Delta_{\lambda\lambda'} \approx \Delta_{\lambda}\delta_{\lambda\lambda'}$. For even–even nuclei the level-dependent matrix elements Δ_{λ} are obtained from the standard gap equation, which is solved toghether with the condition on the chemical potential μ^{τ}. This procedure gives the particle number in a nucleus and thus defines the quasiparticle energy $E_{\lambda} = \sqrt{(\varepsilon_{\lambda} - \mu)^2 + \Delta^2_{\lambda}}$.

It is worth to note, that for specific case of the spin–isospin excitations the corresponding effective NN-interaction has not been obtained as the second functional derivative of the density functional with respect to normal spin–isospin density but was defined independently. The effective interaction $F^{\omega}_{\sigma\tau}$ in the particle–hole spin–isospin channel is chosen as in [11]. It contains local δ-part with Landau–Migdal parameter g' and renormalized one-pion exchange amplitude. In the momentum representation it reads

$$F^{\omega}_{\sigma\tau} = 2C_0 \left[g'\vec{\sigma}_1\vec{\sigma}_2 - g^{\pi}(1 - 2\zeta^{\pi}_s)^2 \frac{(\vec{\sigma}_1\vec{k})(\vec{\sigma}_2\vec{k})}{k^2 + m^2_{\pi} + P_{\Delta}(k^2)} \right] , \qquad (8)$$

where $g_{\pi} = -4\pi\ (f^2_{\pi}/m^2_{\pi})/C_0 = -1.45$ and $P_{\Delta}(k^2)$ is the pion irreducible polarization operator in nuclear medium with allowance for virtual production of the Δ isobar. The constant g'=1.0–1.1 has been extracted from positions of the GT and M1 resonances [11]. The parameter ζ^{π}_s characterizes the suppression of spin–isospin vertices in nuclei due to the quasiparticle local charge $e_q[\sigma\tau] = 1 - 2\zeta_s$ [5]. The value of $\zeta_s = 0.1$ was determined by a fit to the observed GT and M1 strength distributions [11].

The effective spin–isospin interaction in the particle–particle channel entering FFS theory equations for the charge-exchange excitations was chosen in the simplest form:

$$F^{\xi}_{\sigma\tau}(\vec{r}_1, \vec{r}_2) = -C_0 g'_{\xi}\delta(\vec{r}_1 - \vec{r}_2)\ , (J^{\pi} = 0^-, 1^+, \ldots) . \qquad (9)$$

The value of g'_{ξ} for the spin–isospin strength constant in the pp-channel as deduced from the β-decay data and $(p,n)/(n,p)$ reactions spectra in [3] was found to be 0.2–0.5 (in the present calculations the value of 0.2 has been used).

3. Quasiparticle Random Phase Approximation and excited states

To calculate the matrix elements of beta transitions one needs to know not only the structure of the ground state but of the excited states of the final nucleus as well. Also their energy has to be calculated if one is interested in the discrimination of the individual transition. Here the QRPA formalism proves to be useful. It accounts not only for the particle–hole diagrams but for particle—particle ones as well [18]. The excited states are constructed as one-phonon excitations generated from the ground state of the initial nucleus:

$$|m, J^\pi M\rangle = \sum_{a\sigma\leq b\tau} \frac{1}{\sqrt{1+\delta_{ab}\delta_{\sigma\tau}}} \Big[X^{a\sigma,b\tau}_{m,J^\pi} A^\dagger(a\sigma, b\tau; JM) - Y^{a\sigma,b\tau}_{m,J^\pi} \tilde{A}(a\sigma, b\tau; JM) \Big] |\mathrm{QRPA}\rangle\,, \tag{10}$$

where $A^\dagger(a\sigma, b\tau; JM)$ are the angular-momentum coupled two-quasiparticle creation operators. The usual proton–neutron QRPA formalism has been extended to a full form, taking into account the possibility of mixing between protons and neutrons [19]. Treating X and Y amplitudes as independent variational parameters one gets a matrix equation for them and for the excitation energy of the m^{th} excited state $\hbar\omega_{m,J^\pi} \equiv E_{m,J^\pi} - E_0$:

$$\begin{pmatrix} \mathcal{A}_{J^\pi} & \mathcal{B}_{J^\pi} \\ -\mathcal{B}_{J^\pi} & -\mathcal{A}_{J^\pi} \end{pmatrix} \begin{pmatrix} X_{J^\pi} \\ Y_{J^\pi} \end{pmatrix}_m = \hbar\omega_{m,J^\pi} \begin{pmatrix} X_{J^\pi} \\ Y_{J^\pi} \end{pmatrix}_m . \tag{11}$$

Matrices $\mathcal{A}$ i $\mathcal{B}$ are defined in the following way:

$$\begin{aligned} \mathcal{A}^{a'\sigma',b'\tau',a\sigma,b\tau}_{J^\pi} &\equiv \frac{\langle pn\mathrm{BCS}|A(a'\sigma', b'\tau'; JM)[\hat{H}, A^\dagger(a\sigma, b\tau; JM)]|pn\mathrm{BCS}\rangle}{\sqrt{(1+\delta_{ab}\delta_{\sigma\tau})(1+\delta_{a'b'}\delta_{\sigma'\tau'})}}, \\ \mathcal{B}^{a'\sigma',b'\tau',a\sigma,b\tau}_{J^\pi} &\equiv \frac{\langle pn\mathrm{BCS}|A(a'\sigma', b'\tau'; JM)\tilde{A}(a\sigma, b\tau; JM)\hat{H}|pn\mathrm{BCS}\rangle}{\sqrt{(1+\delta_{ab}\delta_{\sigma\tau})(1+\delta_{a'b'}\delta_{\sigma'\tau'})}}. \end{aligned} \tag{12}$$

The explicit forms of $\mathcal{A}$ i $\mathcal{B}$ matrices are quite lengthy and will not be quoted here — they can be found *e.g.* in [19].

In the QRPA formalism the Gamow–Teller transition matrix elements are expressed as follows:

$$\begin{aligned} B(\mathrm{GT}) &\equiv \Big|\langle m, 1^+||\hat{\beta}^+||\mathrm{QRPA}\rangle\Big|^2 \\ &= \Bigg|\sum_{a\sigma\leq b\tau} \frac{\langle a||\boldsymbol{\sigma}||b\rangle}{\sqrt{1+\delta_{ab}\delta_{\sigma\tau}}} [X^{a\sigma,b\tau}_{m',1^+}(v_{a\sigma p}u_{b\tau n} - u_{a\sigma n}v_{b\tau p}) \end{aligned}$$

$$+Y^{a\sigma,b\tau}_{m',1^+}(u_{a\sigma p}v_{b\tau n} - v_{a\sigma n}u_{b\tau p})]\Big|^2 . \qquad (13)$$

The u and v coefficients are the 2×2 matrices, being the solutions of the generalized BCS problem with proton–neutron pairing taken into account [19].

4. Details of the calculations and results

4.1. FFST approach

The results of the calculations within the FFST are given in Fig. 1 and Table I, where the GT-strength functions and the total GT-strength within the $Q_{\rm EC}$ window for the β^+-decay of even $^{100-108}$Sn are presented. For all available cases the experimental distributions are also given (for review see [20]). The qualitative features of the β^+-strength distributions in FFST scheme are easily seen. There are two main 1^+ levels fed in the energy region of interest. One of them carring the most of the strength has the ph nature, the other one is attributed to the pairing effects. The energy position and the strength of *ph* level depends mainly on the strength parameters of the effective NN-interaction (g', g_π) and the local charge value, which remains unchanged in our calculations. The energy position of the *ph* level for tin isotopes with $A \leq 110$ is typically below the $Q_{\rm EC}$-value. The characteristics of the second level, the so-called *pp*-level, is sensitive to *pp*-interaction strength. In some cases this level is shifted above the $Q_{\rm EC}$-value. It is important that for β^+-decay the main part of the strength is contained in the *ph*-level which carries the average particle numbers of daughter nuclei. This allows to determine its excitation energy from the calculated transition energy. In the present work the $Q_{\rm EC}$-values are determined in self-consistent way as the atomic mass differences of parent and daughter nuclei in their ground states

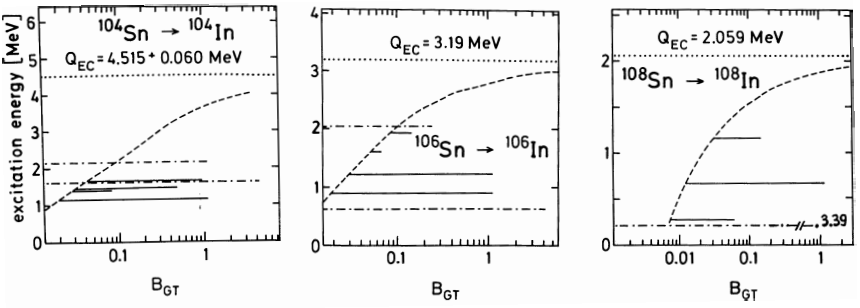

Fig. 1. Experimental GT-strength distribution (solid lines) and calculated within FFST approach (dash-dotted lines). The experimental threshold is marked with the dashed line.

TABLE I

Total B(GT) strength, calculated within the FFST (a), QRPA without (b) and with (c) p–n pairing, together with experimental values (d) [20]. The adopted strength of the spin–isospin interaction in the pp-channel is 60 MeV×fm^3.

Sn → In	$\sum B$(GT)			
A	(a)	(b)	(c)	(d)
100	7.63	9.27	8.95	—
102	6.08	8.02	7.69	—
104	5.30	6.75	6.49	$2.63^{+0.38}_{-0.33}$
106	4.37	5.43	5.24	$2.44^{+0.30}_{-0.27}$
108	3.53	3.99	3.85	$1.37^{+0.10}_{-0.09}$

and were not fitted to experimental ones, as well as single-particle energies. Further calculations of the total GT-strength are influenced by the accuracy of the obtained $Q_{\rm EC}$-values.

It is worth to compare not the details of calculated and experimental spectra, but the energy positions of the centroids of GT-strength distributions. It is seen from Fig. 1 that the main part of the calculated strength for $^{104-108}$ Sn is within the $Q_{\rm EC}$-window. The positions of the centroids of GT-distribution in even–even tin isotopes calculated within the FFST are in resonable agreement with experiment.

The total GT-strength observed below decay energy threshold is substantially lower than calculated in FFST and BCS+pnQRPA approaches. The mechanisms of the strength renormalisation has been intensively studied recently (see *e.g.* [21]). The main source of the GT-strength reduction in is the "core-polarization" mechanism due to the effective interactions in ph and pp channels and the continuum effect. The relative importance of the mentioned factors can be understood on the example of the FFST calculation of the 1^+ state in ^{104}Sn. The quenching factor cased by spin–isospin ph interaction with truncated basis is 2.1, the coupling to continuum drives the reduction factor to 2.5, the inclusion of pp interaction with the strength parameter $g\prime_\xi$=0.2 gives overall reduction of 2.8 (all factors are given relative to the total strength, calculated without any interaction except pairing one). Note that the latest experimental data on the (n,p) reactions [12] favor the higher value of the $g\prime_\xi$=0.4–0.5 which would lead to additional 30–40% reduction in the total B(GT$_+$).

4.2. BCS+QRPA approach

In these calculations we have taken the realistic Hamiltonian with interaction part based on the G-matrix calculated using the Bethe–Goldstone equation with one-boson exchange nucleon–nucleon potential of Bonn type [22], and Woods–Saxon potential with Bertsch parametrization as the single-particle part. For comparison with FFST results, the strength of the spin–isospin effective interactionin the pp-channel has been chosen the same, namely 60 MeV$\times$fm^3. The single-particle basis consisted of 10 orbitals with ^{40}Ca as an inert core. We studied an influence of the proton–neutron pairing on the B(GT) values by switching off the p–n interaction, what corresponds to taking non-diagonal elements of the u and v matrices equal zero and restricting the isospin indices in Eqs. (10) and (13) to values $\sigma = p$ and $\tau = n$. The results are summarised in Table I and in Fig. 2. One can notice slight reduction of B(GT) values when p–n pairing is included. This is due to the fact that the strength is redistributed because of the increased number of possible excitations. But still they overestimate both FFST results and experimental data.

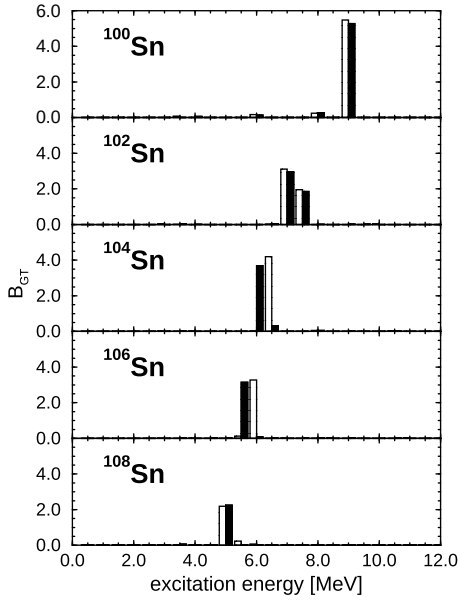

Fig. 2. GT-strength distribution calculated within BCS+QRPA approach with (black bars) and without (white bars) p–n pairing

Important feature of the observed GT-distributions is substantial splitting of the strength between several states. This may cause a significant part of reduction of total GT-strength due to limited experimental sensitiv-

ity, which is energy dependent. The transitions with the excitation energy close to the Q_{EC}-value may be missed even when its strength is rather large.

In order to explain the fragmentation of the GT-strength in terms of QRPA-like schemes one should include the effect of coupling of single-particle and phonon modes. The consistent approach of this type in nuclei with pairing is still missing. The attempts to extend the QRPA model in this direction are known, but quantitative analysis has been not performed yet. Also recently discussed problem of the violation of Pauli principle in the QRPA approach [23] and inclusion of the so-called scattering terms [24] may influence the results. These and other questions remain therefore still open.

5. Summary and conclusions

The analysis of recent experiments on the β-decay of neutron- deficient tin isotopes near ^{100}Sn were performed in order to understand the details of the GT-decays in even–even nuclei in this region. The self-consistent FFST with continuum and the QRPA approach with the G-matrix interaction and proton–neutron pairing was used for the study. The results are consistent with the overall picture of GT-decay governed by the $g_{9/2} \to g_{7/2}$ transition. The effect of proton–neutron pairing on low-spin excitations is found to be small. Although the change in the total strength caused by the *pn*-pairing is in the right direction, it is clearly not enough to solve the problem of the universal quenching of GT-strength. The latter seems to be also an experimental issue.

To explain the rest of difference in total GT-strength for even–even tin isotopes one should include in reliable way the quasiparticle–phonon correlations within the RQRPA-like schemes [23, 24]. This can also provide the link with the shell-model approaches, which forms a basis for understanding of the role of strength splitting in the observed quenching effect. On the other hand, further experimental developments, like the application of well calibrated total gamma absorption spectrometers will help to improve our knowledges on the β-decay properties of exotic nuclei in the region of doubly-magic ^{100}Sn.

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