APERIODIC STOCHASTIC RESONANCE IN A SYSTEM OF COUPLED CHAOTIC OSCILLATORS*

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Noise-free aperiodic stochastic resonance is investigated numerically in a system of two coupled chaotic Rössler oscillators. The aperiodic input signal is obtained from a different chaotic system and applied either to one of the parameters of one oscillator or added to the coupling term. When the coupling constant is decreased the oscillators lose synchronization via attractor bubbling. The output signal is analyzed which reflects the sequence of synchronized (laminar) phases and non-synchronized bursts in the time evolution of the oscillators. The correlation function between the input and output signals shows maximum as a function of the coupling constant. The dependence of the correlation function on the mean frequency of oscillations of the input signal and on the parameter mismatch between the oscillators is very complex. The correlation increases non-monotonically with decreasing frequency, and the parameter mismatch can cause that the output and input signals are anticorrelated.

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1. Introduction

The idea of aperiodic stochastic resonance (ASR) [1–8] is an extension of the concept of stochastic resonance (SR) [9] to the case of aperiodic input signals (for review of SR and ASR see Ref. [10]). In certain systems driven by noise and an aperiodic input signal the noise intensity can be tuned so that the output signal shows maximum correlation with the input one. Similarly, the idea of noise-free ASR [11] is an extension of that of noise-free SR [12–18]. In this case, the internal dynamics of a chaotic system is changed by varying a control parameter, whose value can be chosen so that to obtain maximum correlation between the aperiodic input and output signals.

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In previous papers [17] noise-free SR in a system of two coupled chaotic oscillators at the edge of synchronization was considered. In this paper noise-free ASR in such a system is studied. The system under study in general can be written as

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{F}\left(p + \delta s\left(t\right), \boldsymbol{x}_1\right), \quad \dot{\boldsymbol{x}}_2 = \boldsymbol{F}\left(p + \Delta p, \boldsymbol{x}_2\right) + k\boldsymbol{G}\left(\boldsymbol{x}_1 + \varepsilon s\left(t\right)\boldsymbol{n} - \boldsymbol{x}_2\right) . \tag{1}$$

Here, $\boldsymbol{x}_{1,2}$ are the state vectors of two coupled chaotic subsystems whose dynamics is given by a vector field $\boldsymbol{F}$, $\boldsymbol{G}$ is the coupling function and k is the coupling constant. The subsystems are assumed to be identical up to a possible small mismatch Δp between a selected parameter p; this imitates experiments in which the parameters of *e.g.* two electric circuits are never identical. The aperiodic signal $s\left(t\right) > 0$ is added either to the parameter p in the system 1 with the amplitude $\delta \ll 1$ or to the variable $\boldsymbol{x}_1$ transmitted from the system 1 to 2 with the amplitude $\varepsilon \ll 1$. In the latter case the signal can be added to the components of $\boldsymbol{x}_1$ with weights given by the components of the vector $\boldsymbol{n}$. This signal can be produced *e.g.* by another chaotic system.

In the case $\Delta p = \delta = \varepsilon = 0$ it is assumed that a critical value k_c of the coupling constant exists such that if $k > k_c$ the two subsystems are synchronized, *i.e.* the absolute value of the argument of the coupling term $\Delta x\left(t\right) = \left|\boldsymbol{x}_1\left(t\right) + \varepsilon s\left(t\right)\boldsymbol{n} - \boldsymbol{x}_2\left(t\right)\right| \longrightarrow 0$ for $t \longrightarrow \infty$ [19, 20] (for review of chaotic synchronization see Ref. [21]). If $k < k_c$ the two subsystems do not synchronize. However, if k is only slightly below k_c there are long time intervals during which $\Delta x\left(t\right) \approx 0$ (laminar phases) interrupted by chaotic bursts during which $\Delta x\left(t\right)$ increases substantially. The mean duration of laminar phases decreases with $k \longrightarrow 0$. It is known that $\Delta x\left(t\right)$ exhibits on-off intermittency (OOI) [22, 23]. If $\Delta p \neq 0$ but still $\delta = \varepsilon = 0$ the synchronization is never perfect and the bursts in $\Delta x\left(t\right)$ are observed even for $k > k_c$; this phenomenon is called attractor bubbling (AB) [24, 25].

The effect of setting $\delta \neq 0$ or $\varepsilon \neq 0$ is similar, and AB is then observed even for $\Delta p = 0$. However, since $s\left(t\right)$ is time dependent, the bursts occur more and less frequently for higher and lower values of the aperiodic signal, respectively. In this paper we distinguish between the laminar phases and bursts by considering a discretized signal $X\left(t\right) = \Theta\left[\Delta x\left(t\right) - \theta\right]$, where Θ denotes the Heaviside step function and θ is the threshold for burst. It follows from our numerical simulations and prevoius works [17] that if $X\left(t\right)$ is considered as the output signal of the system (1) noise-free SR or ASR is observed with varying k. In particular, the correlation function

$$C_1 = \frac{\langle s\,(t)\,X\,(t)\rangle - \langle s\,(t)\rangle\langle X\,(t)\rangle}{\sqrt{[\langle s^2\,(t)\rangle - \langle s\,(t)\rangle^2]\,[\langle X^2\,(t)\rangle - \langle X\,(t)\rangle^2]}}\,, \tag{2}$$

where the brackets denote the time average, has a maximum as a function of k.

Systems like (1) are usually considered in connection with the secure communication problem [26-28]. The system 1 is the transmitter, 2 is the receiver, and the transmitted variable is $\boldsymbol{x}_1$. The information signal $s\,(t)$ with small amplitude δ or ε is added to one of the parameters of the transmitter (chaotic communication scheme) or to the transmitted signal (chaotic signal masking scheme), respectively [27]. If $k \gg k_c$ and $\delta, \varepsilon \ll 1$ addition of the signal will not lead to large chaotic bursts typical of AB, since the time necessary for the occurrence of a burst is extremely long [25]. Instead, $\Delta x\,(t)$ follows the signal $s\,(t)$ closely. Hence the information signal can be reproduced almost exactly at the receiver, where $\boldsymbol{x}_2$ is known. This communication scheme is "secure" since precise reproduction of $s\,(t)$ requires the equality of all parameters of the receiver and the transmitter, whose values cannot be extracted from the transmitted signal $\boldsymbol{x}_1$ or $\boldsymbol{x}_1 + \varepsilon s\,(t)$. If this is not the case (*e.g.* $\Delta p \neq 0$) then $\Delta x\,(t)$ is corrupted by chaotic bursts due to AB and does not follow $s\,(t)$.

In this paper the opposite limit $k \approx k_c$ is considered, when $\Delta x\,(t)$ does not reproduce $s\,(t)$ even if $\Delta p = 0$. The problem addressed is if the coupling constant can be tuned in order to reproduce the (generally aperiodic) information signal $s\,(t)$ with a significant degree of accuracy using the idea of noise-free ASR. To this purpose, the variable $\Delta x\,(t)$ is passed through a threshold device to obtain $X\,(t)$. Passing through a threshold nonlinearity is known to produce SR in the case of OOI and AB signals [16-18]. Signals with various amplitudes δ, ε and average frequencies are considered, and the influence of the parameter mismatch $\Delta p \neq 0$ is also analyzed. It turns out that the correlation function C_1 (2) can reach quite big values, in particular for slowly varying signals (the adiabatic limit) with relatively large amplitudes. In many cases non-trivial dependence of C_1 on the average frequency of the input signal and the parameter mismatch is found. Of course, in any case the quality of reproduction of the signal using ASR with $k \approx k_c$ is not better than using directly $\Delta x\,(t)$ in the limit of large k. The amplitudes δ, ε are, however, chosen in purpose so that in the limit $k \gg k_c$ we have $\Delta x\,(t) < \theta$. Thus using noise-free ASR for reproducing the information signal (in the way discussed in this paper) makes sense in two cases. First, if the information signal has such a small amplitude that pure $\Delta x\,(t)$ is not useful at the receiver, where our apparatus can be insensitive to signals below a given threshold θ. Second, if there are any constraints for the maximum allowable k.

2. The system under study

As a particular example of a system (1) a set of two coupled chaotic Rössler oscillators is considered

$$
\begin{aligned}
\dot{x}_1 &= -\left(y_1 + z_1\right) , \\
\dot{y}_1 &= x_1 + \left[a + \delta s\left(t\right)\right] y_1 , \\
\dot{z}_1 &= b + z_1 \left(x_1 - c\right) , \\
\dot{x}_2 &= -\left(y_2 + z_2\right) , \\
\dot{y}_2 &= x_2 + \left(a + \Delta a\right) y_2 + k\left[y_1 + \varepsilon s\left(t\right) - y_2\right] , \\
z_2 &= b + z_2 \left(x_2 - c\right) .
\end{aligned}
\tag{3}
$$

The parameters are $a = 0.2$, $b = 0.2$, $c = 10$, and Δa is the parameter mismatch. The coupling is via the scalar y variable. For this coupling the two oscillators synchronize for $k > k_c \approx 0.24$. The aperiodic input signal $s\left(t\right)$ is obtained from the well-known Lorenz system $\dot{u} = -\mu\sigma\left(u - v\right)$, $\dot{v} = \mu\left(-uw + \rho u - v\right)$, $\dot{w} = \mu\left(uv - \beta w\right)$ with $\sigma = 10$, $\rho = 28$ and $\beta = 8/3$ as $s\left(t\right) = \left|w\left(t\right) - 8\right|$. This results in a signal varying between 0 and c.a. 37. The parameter μ sets the characteristic time scale of oscillations (frequency) of the Lorenz system: the larger μ the faster the signal $s\left(t\right)$ varies in time. Putting aside the exact definition of frequency in chaotic oscillators [29] it can be estimated *e.g.* as a mean number of maxima of $x_1\left(t\right)$ or $w\left(t\right)$ per unit time for the Rössler and Lorenz systems, respectively. For the parameters above and $\mu = 1$ this frequency is $\omega \approx 1$Hz for the Rössler system and is c.a. ten times higher for the Lorenz one. Thus for $\mu \approx 0.1$ both systems oscillate with comparable frequencies. The variable $\Delta x\left(t\right)$ is henceforth defined as $\Delta x\left(t\right) = \left|y_1\left(t\right) + \varepsilon s\left(t\right) - y_2\left(t\right)\right|$ and the output signal is defined as in Sec. 1 with $\theta = 0.1$.

Numerical solutions of Eq. (3) were obtained using a fourth-and-fifth order Runge-Kutta algorithm with permanent step size and error control. The integration step size was varied with μ and ranged from 0.001 to 0.1. As a measure of ASR the correlation function C_1 (2) was used. Care was taken to calculate the time averages in Eq. (2) correctly, taking into account the above-mentioned changes in the integration step size, and it was checked that the results did not depend on this size.

3. Numerical results and discussion

3.1. The chaotic signal masking scheme

The results for the case $\delta = 0$, $\varepsilon \neq 0$ are summarized in Figs. 1,2 where C_1 *vs* k for various μ and Δa is shown. It can be seen that for $\Delta a = 0$ the curves show a single maximum typical of ASR. This maximum in general

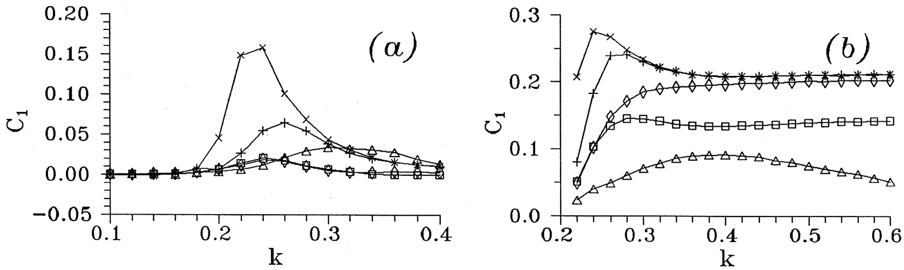

Fig. 1. C_1 *vs* k for the system (3) with $\Delta a = 0$, $\delta = 0$ and (a) — $\varepsilon = 1.5 \times 10^{-3}$, (b) — $\varepsilon = 2.5 \times 10^{-3}$; $\mu = 1$ (□), 10^{-1} (△), 10^{-2} (◇), 10^{-3} (+), 10^{-4} (×).

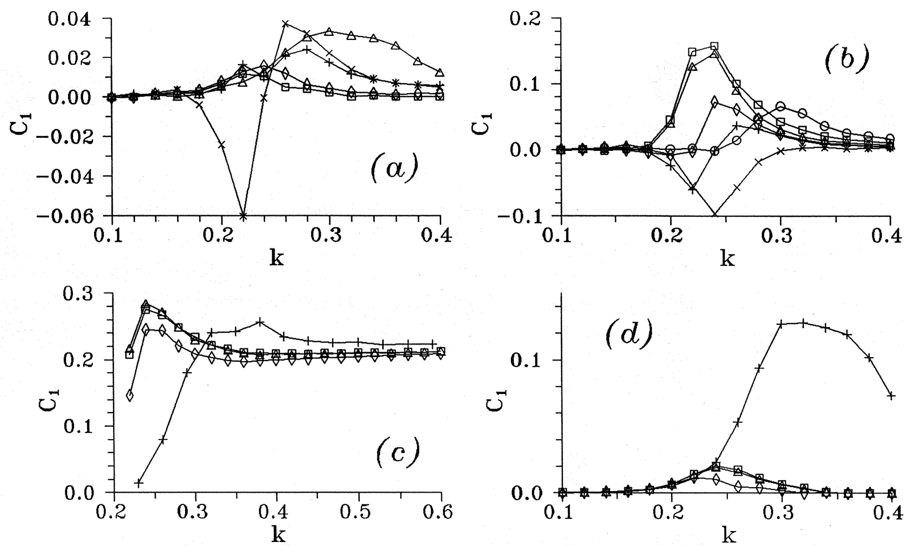

Fig. 2. C_1 *vs* k for the system (3) with $\delta = 0$ and (a) — $\varepsilon = 1.5 \times 10^{-3}$, $\Delta a = 10^{-4}$ and $\mu = 1$ (□), 10^{-1} (△), 10^{-2} (◇), 10^{-3} (+), 10^{-4} (×); (b) — $\varepsilon = 1.5 \times 10^{-3}$, $\mu = 10^{-4}$ and $\Delta a = 0$ (□), 5×10^{-5} (△), 8×10^{-5} (◇), 10^{-4} (+), 2×10^{-4} (×), 10^{-3} (○); (c) — $\varepsilon = 2.5 \times 10^{-3}$, $\mu = 10^{-4}$ and $\Delta a = 0$ (□), 10^{-5} (△), 10^{-4} (◇), 10^{-3} (+); (d) — $\varepsilon = 1.5 \times 10^{-3}$, $\mu = 1$ and $\Delta a = 0$ (□), 10^{-5} (△), 10^{-4} (◇), 10^{-3} (+).

increases with $\mu \longrightarrow 0$ and its position is shifted towards $k \approx k_c$ (Fig. 1). Such dependence of C_1 on frequency is common in noise-free ASR, and the effect of ASR is usually most noticeable in the adiabatic limit of slowly varying input signals [11]. However, for $\mu \approx 0.1$ deviations from a monotonic increase of C_1 are observed. Then, in the case of a weak input signal with $\varepsilon = 1.5 \times 10^{-3}$ and $\varepsilon s\,(t) \ll \theta$ the maximum of the C_1 *vs* k curve increases and shifts towards larger k (Fig. 1(a)), while in the case of a strong input signal with $\varepsilon = 2.5 \times 10^{-3}$ and $\varepsilon s(t) \approx \theta$ it decreases and shifts towards larger k (Fig. 1(b)). In the latter case the increase of C_1 not only for $\mu \longrightarrow 0$ but also for $\mu > 0.1$ is observed.

If $\Delta a \neq 0$ differences between the cases of weak and strong input signals also can be seen (Fig. 2). In Figs. 2(a), (b) results for the weak signal with $\varepsilon = 1.5 \times 10^{-3}$ are shown: in Fig. 2(a) for given Δa and decreasing μ, and in Fig. 2(b) for given μ, yielding a slowly varying signal $s(t)$, and increasing Δa. If the input signal has small frequency a substantial decrease of C_1 with the increase of the parameter mismatch is observed. Moreover, there are such intervals of Δa, μ and k for which the output and input signals are anticorrelated, *i.e.* $C_1 < 0$. This means that $\Delta x(t)$ is above or below the threshold θ most often when the signal is close to its minimum or maximum, respectively, just the opposite of what can be expected. In the case of the strong input signal with $\varepsilon = 2.5 \times 10^{-3}$ the anticorrelation is not observed within the range of parameters investigated, and for slowly varying $s(t)$ the maximum of the C_1 *vs* k curve just decreases and shifts towards larger k with increasing parameter mismatch (Fig. 2(c)). For both weak and strong input signal, if its frequency is very high, first slight decrease and then substantial increase of C_1 with the rise of Δa is observed, of course within reasonable limits of $\Delta a \ll \theta$ (Fig. 2(d)).

3.2. The chaotic communication scheme

Analogous results in the case $\delta \neq 0$, $\varepsilon = 0$ are summarized in Figs. 3,4. Again, for $\Delta a = 0$ the curves C_1 *vs* k show a single maximum whose value increases and position shifts towards $k \approx k_c$ if $\mu \longrightarrow 0$ (Fig. 3). In the opposite limit of large μ, C_1 decreases to zero and no significant correlation between the input and output signals is observed. This is true for both weak (with $\delta = 10^{-4}$) and strong (with $\delta = 4 \times 10^{-4}$) input signals, although in the latter case C_1 differs noticeably from zero for μ an order of magnitude bigger than in the former case (cf. the values of μ in Fig. 3(a) and Fig. 3(b)). In particular, no increase of the correlation function for the strong input signal and $\mu > 0.1$ is observed. For moderate μ, just when C_1 starts deviating from zero, anticorrelation between the input and output signals is observed, which results in $C_1 < 0$.

In the case $\Delta a \neq 0$ the results again depend on the strength of the input signal (Fig. 4). In Figs. 4(a), (b) results for the weak signal with $\delta = 10^{-4}$ are shown: in Fig. 4(a) for given Δa and decreasing μ, and in Fig. 4(b) for given μ, yielding a slowly varying signal $s(t)$, and increasing Δa. If the input signal has small frequency, then, like in the chaotic signal masking scheme, anticorrelation between the input and output signals, characterized

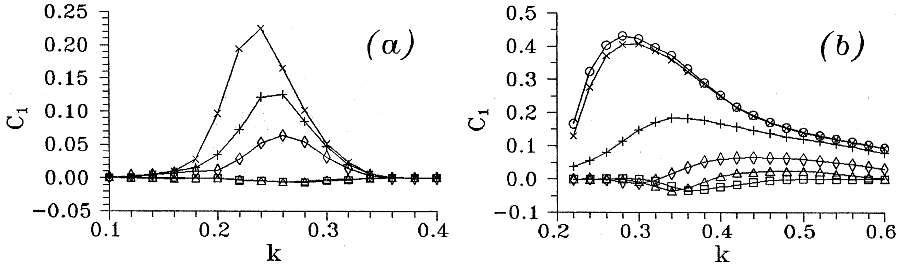

Fig. 3. C_1 *vs* k for the system (3) with $\Delta a = 0$, $\varepsilon = 0$ and (a) — $\delta = 10^{-4}$ and $\mu = 10^{-2}$ (□), 5×10^{-3} (△), 2×10^{-3} (◇), 10^{-3} (+), 10^{-4} (×); (b) — $\delta = 4 \times 10^{-4}$ and $\mu = 10^{-1}$ (□), 5×10^{-2} (△), 3×10^{-2} (◇), 10^{-2} (+), 10^{-3} (×), (○) 10^{-4}.

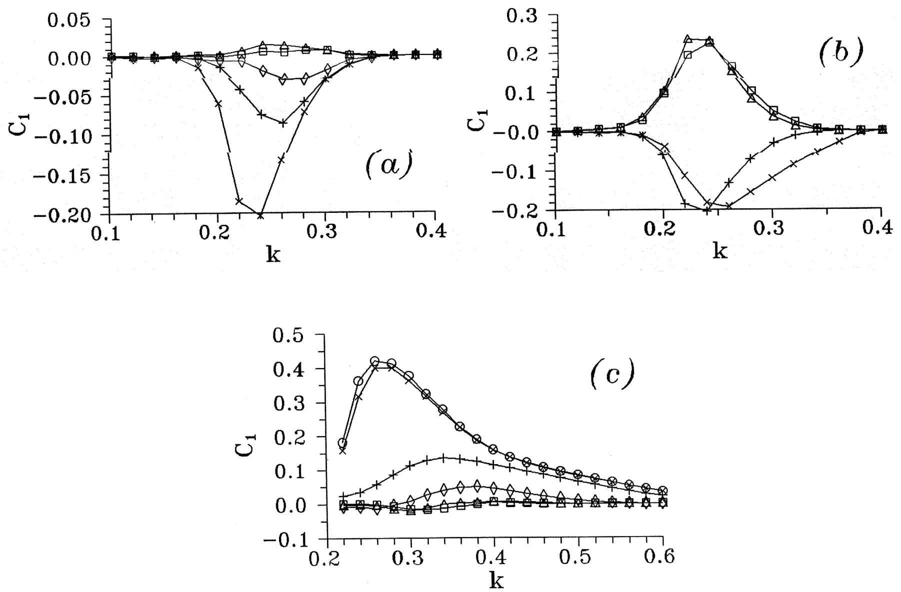

Fig. 4. C_1 *vs* k for the system (3) with $\varepsilon = 0$ and (a) — $\delta = 10^{-4}$, $\Delta a = 5 \times 10^{-4}$ and $\mu = 10^{-2}$ (□), 5×10^{-3} (△), 2×10^{-3} (◇), 10^{-3} (+), 10^{-4} (×); (b) — $\delta = 10^{-4}$, $\mu = 10^{-4}$ and $\Delta a = 0$ (□), 10^{-4} (△), 5×10^{-4} (+), 10^{-3} (×); (c) — $\delta = 4 \times 10^{-4}$, $\Delta a = 5 \times 10^{-4}$ and $\mu = 10^{-1}$ (□), 5×10^{-2} (△), 3×10^{-2} (◇), 10^{-2} (+), 10^{-3} (×), 10^{-4} (○).

by $C_1 < 0$, occurs for non-zero parameter mismatch. This effect appears for $\mu \longrightarrow 0$ (Fig. 4(a)) or for increasing Δa (Fig. 4(b)). The maximum value of C_1 again decreases with increasing parameter mismatch. In the case of the slowly varying strong input signal with $\varepsilon = 4 \times 10^{-4}$ the anticorrelation is not observed within the range of parameters investigated and the values of

C_1 decrease with increasing Δa (*cf.* Fig. 4(c) and Fig. 3(b)). Only for strong input signals with moderate μ we have $C_1 < 0$, but this is true also if there is no any parameter mismatch between the oscillators.

3.3. Discussion

The results for ASR obtained in this paper are comparable with the ones known from our earlier work on noise-free SR with periodic input signals in two coupled chaotic oscillators [17]. In that case a different measure of SR is used, the signal-to-noise ratio (SNR) which yields the strength of the peak at the input signal frequency divided by the strength of the noise background, where both quantities are obtained from the power spectrum density of the output signal. In order to perform a detailed comparison between the SR and ASR cases the results for SR from Ref. [17] have to be extended to a wider range of frequencies and amplitudes of the periodic signal. However, even now certain similarities between these two cases can be found.

In the case of chaotic signal masking scheme, if $\Delta a = 0$ both C_1 and SNR show complex dependence on the input signal frequency. Of particular interest is the case when the input signal frequency is close to the frequency of oscillations of the system (3), here for $\mu = 1$. For such frequencies C_1 and SNR can decrease substantially (Fig. 1(b)), but the opposite effect of the increase of SNR in the case of a weak input signal was not reported, contrary to C_1 (Fig. 1(a)). Also the increase of SNR for both very fast and very slow input signals was found, in analogy with the results for C_1 shown in Fig. 1(b). The increase of SNR for high-frequency input signals is even much more spectacular than that of C_1, by orders of magnitude. The discussion in Ref. [17] shows that in these two limiting cases two different mechanisms are responsible for SR. For fast input signals the SR effect is non-dynamical and high values of SNR are caused by a separation of the fast and slow time scales, connected with the fast and slow oscillations of the input signal and the chaotic system, respectively. For slow input signals (adiabatic limit) SR is a dynamical effect connected with AB, as suggested in Sec. 1. This is also true in the ASR case.

Another interesting problem is the complicated dependence of the ASR effect on the parameter mismatch Δa. In Ref. [17] this problem was not studied systematically, but in certain cases the increase of SNR with Δa was observed, in analogy with Fig. 2(d) here obtained for the fast input signal. In Fig. 2(a)–(c) it can be seen that for slow input signals C_1 in general decreases with Δa and anticorrelation between the input and output signals can occur. Similar effects have not been reported for SR.

The case of chaotic communication scheme was not discussed for periodic input signals in Ref. [17]. The results of Sec. 3.2 show that for the whole range of μ investigated ASR is a dynamical effect. This is confirmed by the systematic increase of C_1 with decreasing μ and the disappearance of correlation between the output and fast input signal. Namely, if the input signal oscillates too fast the system has no time to react to individual maxima of $s(t)$ and AB is triggered by averaged influence of the signal, so the desynchronization bursts are not correlated with the signal maxima. For moderate μ and $\Delta a = 0$ we have $C_1 < 0$ (Fig. 3). This is probably connected with a kind of a phase shift between the input signal and the desynchronization bursts: the occurrence of maximum in $s(t)$ initiates the burst, but the input signal varies so fast that a maximum of $\Delta x(t)$ appears only when the input signal already reaches its minimum. For $\Delta a \neq 0$ anticorrelation is observed for a wide range of μ when $\mu \longrightarrow 0$.

It turns out that the properties of the ASR effect in (3) depend strongly on the definition of the output signal $X(t)$ and on the input signal. *e.g.* if in the case $\delta = 0$, $\varepsilon \neq 0$ the presence of a desynchronization burst is defined as exceeding the threshold θ by a local maximum of $\Delta x(t)$, the fast oscillations of the output signal are cut off and the increase of C_1 with μ seen in Fig. 1(b) for $\mu < 0.1$ is not observed. If instead of the input signal from the Lorenz system $s(t)$ just randomly switches between 0 and 1, the frequency for such a signal is poorly defined and C_1 just increases monotonically with decreasing mean time between switches. Besides, it should be pointed out that maximum of C_1 *vs* k is observed only if the signal $\Delta x(t)$ is passed through a threshold, both in the chaotic signal masking and chaotic communication schemes. If $\Delta x(t)$ itself is used as the output signal $C_1 \longrightarrow 1$ for $k \longrightarrow \infty$. This exceeds much the maxima of C_1 *vs* k curves in Figs. 1–4 thus proving that ASR cannot be used to increase the quality of retrieving the information signal in secure communication. This is in contrast with the amplification of small signals using AB, recently discussed in Refs. [18, 30, 31]. Small signals can be amplified since the amplitude of desynchronization bursts exceeds by many orders of magnitude that of the input signal, and it seems that the amplification factor as a function of k can exhibit maximum [18]. Nevertheless, the output signals are contaminated with chaotic fluctuations, thus the correlation function and SNR do not reach their highest possible values.

4. Conclusions

In this paper noise-free ASR in a system of coupled chaotic Rössler oscillators at the edge of chaotic synchronization was investigated. The system under study is typical in the secure communication problem. The cases of aperiodic chaotic input signal applied both in the chaotic signal masking and chaotic communication schemes were analyzed. The output signal reflected the sequence of synchronization phases and bursts in AB triggered by the input signal, and maximum of the correlation function between these signals was observed on varying the coupling constant. The correlation function showed strong dependence on the mean frequency of the input signal and the parameter mismatch between the oscillators. The novel effects are the non-monotonic increase of C_1 with decreasing frequency of the input signal, with violation of monotonicity when the frequencies of the input signal and of chaotic oscillations of the system (3) coincide, the possible increase of C_1 for fast input signals and the occurrence of anticorrelation between the input and output signals for non-zero parameter mismatch. The rather non-trivial dependence of SR and ASR in coupled chaotic oscillators on the input signal frequency requires further systematic study, since it may be connected with fundamental differences between the chaotic and stochastic systems. It was also concluded that even though the ASR effect does not lead to the improvement in the quality of recovering information signals in secure communication, it can be used to maximize correlation between the retrieved and information signal *e.g.* if the latter is too weak to be useful at the output.

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