STOCHASTIC RESONANCE IN TWO COUPLED THRESHOLD ELEMENTS WITH PHASE-SHIFTED INPUT SIGNALS*

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Stochastic resonance in a system of two coupled threshold elements (neurons) forming a small neural network is investigated numerically. Periodic signals at inputs of the elements are phase-shifted with respect to each other up to a half of the period, but their frequencies and amplitudes are identical. The signal-to-noise ratio at outputs of the elements has a maximum as a function of the input noise intensity for any phase shift. For proper coupling, dependent on the phase shift, this ratio is enhanced over that of a single uncoupled element. The enhancement is usually observed for positive (excitory) coupling if the phase shift is less than one fourth of the period, and for negative (inhibitory) coupling otherwise, but minor deviations from these rules are possible for high periodic signal frequency. Adiabatic theory of stochastic resonance in coupled threshold elements is also formulated which describes qualitatively the dependence of the signal-to-noise ratio on the coupling for various phase shifts.

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1. Introduction

Stochastic resonance (SR) occurs mainly in nonlinear systems driven by a combination of periodic and stochastic signals, in which the periodicity of the output signal can be maximized by tuning properly the intensity of the input noise [1] (for review see [2]). A simple example of a stochastic resonator is a threshold element with the input periodic signal $s(t) = A \sin \omega_s t$, input noise $\eta(t)$ being white Gaussian noise with variance σ^2, and with the

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threshold $b > A$, whose output $x(t)$ is given by $x(t) = \Theta[s(t) + \eta(t) - b]$, where Θ is the Heaviside step function. Periodicity of the signal $x(t)$ is maximum for $\sigma > 0$ [3, 4]. Recently, SR has been intensively investigated in high-dimensional systems, *e.g.* in systems of globally or locally coupled bistable [5–8] and threshold [9, 10] elements, in model neural networks [11–14], spatially extended systems [15–19], in the Ising model [20, 21] *etc.* In the coupled systems, it has been in general concluded that due to a proper coupling the SR effect observed either in a whole system or in a single element coupled to other similar elements is enhanced over that of a single uncoupled element. The most often studied case is that with individual elements driven by independent noises with identical distribution and intensity, and by a common periodic signal. Other cases, as *e.g.* a neural network in which there is a certain distribution of amplitudes of the periodic signal over neurons, are less frequently analyzed [14].

In this paper we consider threshold elements (formal neurons) with coupling typical of artificial neural networks, each of which is driven by a periodic signal with the same frequency and amplitude, but with arbitrary phase. This can be a typical situation *e.g.* in spatially extended systems in which the signals in two different points can be phase shifted due to finite propagation time of the signal. Here we are only interested in the simplest case of two coupled elements and in the enhancement of SR of a single element due to the coupling. However, a similar problem investigated in a larger system could lead to SR with a spatio-temporal periodic input signal, an effect which, to our knowledge, has not been reported in the literature so far. Similarly, a first step in the investigation of array-enhanced SR in a chain of bistable noisy oscillators [15, 16] is to consider only two coupled oscillators [17].

2. The system and the methods of analysis

The system under study consists of two coupled threshold elements (neurons) denoted as 1, 2, forming a small neural network with discrete-time dynamics and parallel updating. Both elements are driven by a periodic signal with amplitude A, frequency ω_s and initial phase ϕ, and by independent white Gaussian noises $\eta^{(1)}$, $\eta^{(2)}$ with variance σ^2; the periodic signal in the element 2 is shifted in phase by $\Delta\phi$ with respect to that in the element 1. The coupling strength (the synaptic connection weight) w is symmetric. The outputs of the elements, denoted as $x^{(1)}$, $x^{(2)}$, can assume only discrete values 0 (quiescent state) or 1 (firing), if the total input to the given element is below or above the threshold b, respectively; it is assumed that $b > A$. The equations for the time dependence of $x^{(1)}$, $x^{(2)}$ read

$$
\begin{aligned}
x_{n+1}^{(1)} &= \Theta\left[A\sin\left(\omega_s n+\phi\right)+\eta_n^{(1)}+wx_n^{(2)}-b\right] , \\
x_{n+1}^{(2)} &= \Theta\left[A\sin\left(\omega_s n+\phi+\Delta\phi\right)+\eta_n^{(2)}+wx_n^{(1)}-b\right] ,
\end{aligned} \tag{1}
$$

where n denotes the step number.

In this paper, we characterize SR in any of the elements by the Signal-to-Noise Ratio (SNR), which is obtained from the power spectrum density $S^{(1)}\left(\omega\right)$ or $S^{(2)}\left(\omega\right)$ of the respective time series $x_n^{(1)}$, $x_n^{(2)}$. We define $\mathrm{SNR}^{(1,2)} = 10\log\left[S_P^{(1,2)}\left(\omega_s\right)/\ S_N^{(1,2)}\left(\omega_s\right)\right]$, where $S_P^{(1,2)}\left(\omega_s\right) = S^{(1,2)}\left(\omega_s\right) - S_N^{(1,2)}\left(\omega_s\right)$ is the height of the peak in the power spectrum density at $\omega = \omega_s$, and $S_N^{(1,2)}\left(\omega_s\right)$ is the noise background in the vicinity of ω_s. In systems with SR, SNR as a function of σ has a maximum [2]. In our numerical simulations the power spectrum density is evaluated from $N = 4096$ points of the time series $x_n^{(1,2)}$ and averaged over 100 consecutive runs. Then, SNR is evaluated and averaged over 10 random initial conditions for $x^{(1)}$, $x^{(2)}$ and ϕ. All results for the SNR are normalized to the frequency bandwidth $\Delta f = 2^{-12}$Hz [22]. The results of numerical simulations are presented in Sec. 4.

3. Theory in the adiabatic approximation

In threshold elements with discrete-time dynamics it is possible to evaluate SNR analytically using the method of Ref. [4]. Here we do this in the adiabatic limit $\omega_s \longrightarrow 0$ [22].

First we obtain the probability that $x_n^{(1,2)}=1$, denoted as $\Pr\left(x_n^{(1,2)}=1\right)$. Since the processes $x_n^{(1,2)} = 1$ are non-stationary, this probability is time-dependent and periodic in time [23]. The probability densities for the random variables $w_n^{(1,2)}=wx_n^{(1,2)}$ are $\rho\left(w_n^{(1,2)}\right)=\Pr\left(x_n^{(1,2)}=1\right)\delta\left(w_n^{(1,2)}-w\right)+\left[1-\Pr\left(x_n^{(1,2)}=1\right)\right]\delta\left(w_n^{(1,2)}\right)$. Since the random variables $\eta_n^{(1,2)}$ and $x_n^{(2,1)}$ are independent, the probability densities for the variables $\xi_n^{(1)} = \eta_n^{(1)}+w_n^{(2)}$, $\xi_n^{(2)} = \eta_n^{(2)}+w_n^{(1)}$ become

$$
\begin{aligned}
\rho\left(\xi_n^{(1)}\right) &= \int\limits_{-\infty}^{+\infty} \rho_{\eta_n^{(1)}}\left(\xi_n^{(1)}-w_n^{(2)}\right)\rho\left(w_n^{(2)}\right)dw_n^{(2)} \\
&= \frac{1}{\sqrt{2\pi}\sigma}\left\{\left[1-\Pr\left(x_n^{(2)}=1\right)\right]\mathrm{e}^{-\left(\xi_n^{(1)}\right)^2/2\sigma^2}\right.
\end{aligned}
$$

$$+ \Pr\left(x_n^{(2)}=1\right) \mathrm{e}^{-\left(\xi_n^{(1)}-w\right)^2/2\sigma^2}\Bigg\},$$

$$\begin{aligned}
\rho\left(\xi_n^{(2)}\right) &= \int\limits_{-\infty}^{+\infty} \rho_{\eta_n^{(2)}}\left(\xi_n^{(2)}-w_n^{(1)}\right)\rho\left(w_n^{(1)}\right)dw_n^{(1)} \\
&= \frac{1}{\sqrt{2\pi}\sigma}\Bigg\{\left[1-\Pr\left(x_n^{(1)}=1\right)\right]\mathrm{e}^{-\left(\xi_n^{(2)}\right)^2/2\sigma^2} \\
&\quad + \Pr\left(x_n^{(1)}=1\right) \mathrm{e}^{-\left(\xi_n^{(2)}-w\right)^2/2\sigma^2}\Bigg\}.
\end{aligned} \tag{2}$$

From (1) it follows that *e.g.* $x_{n+1}^{(1)}=1$ if $\xi_n^{(1)} > b - A\sin\left(\omega_s n+\phi\right)$, thus

$$\begin{aligned}
\Pr\left(x_{n+1}^{(1)}=1\right) &= \int\limits_{b-A\sin(\omega_s n+\phi)}^{\infty} \rho\left(\xi_n^{(1)}\right)d\xi_n^{(1)}, \\
\Pr\left(x_{n+1}^{(2)}=1\right) &= \int\limits_{b-A\sin(\omega_s n+\phi+\Delta\phi)}^{\infty} \rho\left(\xi_n^{(2)}\right)d\xi_n^{(2)}.
\end{aligned} \tag{3}$$

In the adiabatic limit $\omega_s \longrightarrow 0$ we assume that $\Pr\left(x_{n+1}^{(1,2)}=1\right)=\Pr\left(x_n^{(1,2)}=1\right)$. Then, after inserting Eq. (2) into Eq. (3) we obtain a system of two linear equations for $\Pr\left(x_n^{(1,2)}=1\right)$ whose solution is

$$\begin{aligned}
\Pr\left(x_n^{(1)}=1\right) &= \frac{\alpha_n^{(1)}+\left(\beta_n^{(1)}-\alpha_n^{(1)}\right)\alpha_n^{(2)}}{1-\left(\beta_n^{(1)}-\alpha_n^{(1)}\right)\left(\beta_n^{(2)}-\alpha_n^{(2)}\right)}, \\
\Pr\left(x_n^{(2)}=1\right) &= \frac{\alpha_n^{(2)}+\left(\beta_n^{(2)}-\alpha_n^{(2)}\right)\alpha_n^{(1)}}{1-\left(\beta_n^{(1)}-\alpha_n^{(1)}\right)\left(\beta_n^{(2)}-\alpha_n^{(2)}\right)},
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
\alpha_n^{(1)} &= \frac{1}{2}+\frac{1}{\sqrt{2\pi}\sigma}\int\limits_{b-A\sin(\omega_s n+\phi)}^{0} \mathrm{e}^{-\xi^2/2\sigma^2}d\xi, \\
\alpha_n^{(2)} &= \frac{1}{2}+\frac{1}{\sqrt{2\pi}\sigma}\int\limits_{b-A\sin(\omega_s n+\phi+\Delta\phi)}^{0} \mathrm{e}^{-\xi^2/2\sigma^2}d\xi,
\end{aligned}$$

$$\beta_n^{(1)} = \frac{1}{2} + \frac{1}{\sqrt{2\pi}\sigma} \int\limits_{b-A\sin(\omega_s n+\phi)-w}^{0} \mathrm{e}^{-\xi^2/2\sigma^2} d\xi \,,$$

$$\beta_n^{(2)} = \frac{1}{2} + \frac{1}{\sqrt{2\pi}\sigma} \int\limits_{b-A\sin(\omega_s n+\phi+\Delta\phi)-w}^{0} \mathrm{e}^{-\xi^2/2\sigma^2} d\xi \,. \tag{5}$$

Next the discrete Fourier transform of the probabilities in Eq. (4) is performed

$$P_k^{(1,2)} = \frac{1}{T_s} \sum_{n=0}^{T_s-1} \Pr\left(x_n^{(1,2)} = 1\right) \exp\left(-2\pi i \frac{nk}{T_s}\right) \,, \tag{6}$$

where $T_s = 2\pi/\omega_s$ is the period. According to Ref. [4] $\mathrm{SNR}^{(1,2)}$ normalized to a given bandwidth $\Delta f = 1/N$ (here, $N = 4096$) can be evaluated from Eqs. (4),(6) as

$$\mathrm{SNR}^{(1,2)} = \frac{N\left|P_1^{(1,2)}\right|^2}{\left\langle \Pr\left(x_n^{(1,2)} = 1\right)\right\rangle - \left\langle \Pr^2\left(x_n^{(1,2)} = 1\right)\right\rangle} \,, \tag{7}$$

where the brackets denote the time average over T_s. It should be pointed out, however, that Eq. (7) is exact only in the case of a threshold element driven by a sum of a deterministic periodic signal and *white* noise [4], while in Eq. (1) the total random noises $\xi_n^{(1,2)}$ are non-white and the variables $x_n^{(1)}$, $x_n^{(2)}$ are correlated. Thus Eq. (7) is exact only for $w = 0$, *i.e.* for uncoupled elements, and the difference between the numerical and analytic values of SNR increases with $|w|$. As it will be shown in Sec. 5, only certain qualitative predictions concerning the dependence of SNR on w can be made using Eq. (7).

4. Numerical results and their discussion

In this section, the numerical results for $\mathrm{SNR}^{(1,2)}$ *vs.* σ for the system (1) with parameters $A = 0.5$, $b = 0.6$ and various w and ω_s are presented.

We start with a slowly varying signal and take $T_s = 128$. For $\Delta\phi = 0$ there is no phase shift between the periodic signals in both elements and the results are rather typical of coupled systems. In particular, due to the symmetry $\mathrm{SNR}^{(1)} = \mathrm{SNR}^{(2)}$. If $w < 0$ SNR decreases in comparison with the case $w = 0$, and the location of the maximum of SNR does not change (Fig. 1(a)). If $w > 0$, there exists an optimum value of the coupling $w_{\rm opt} \approx$

1.0 for which the maximum of SNR assumes the largest value, which exceeds by several dB that for the $w = 0$ curve (Fig. 1(b)). For $0 < w < w_{\text{opt}}$ SNR for a given σ generally increases with w and the maximum shifts towards smaller σ; for $w > w_{\text{opt}}$ SNR generally decreases with w and the maximum shifts towards larger σ (Fig. 1(b)). Different results are obtained for $\Delta\phi = \pi$, where again due to the symmetry $\text{SNR}^{(1)} = \text{SNR}^{(2)}$. Then for $w \longrightarrow -\infty$ SNR increases slightly, but without any visible shift of both the value and location of the maximum (Fig. 2(a)), while for $w \longrightarrow +\infty$ SNR decreases and the maximum shifts towards larger σ (Fig. 2(b)).

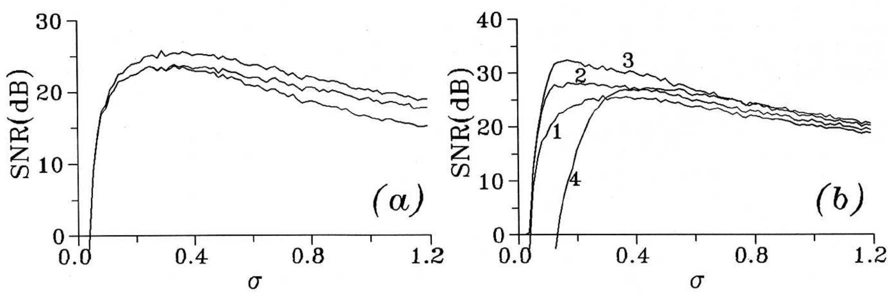

Fig. 1. $\text{SNR}^{(1,2)}$ *vs.* σ for the system (1) with $T_s = 128$, $\Delta\phi = 0$ and other parameters as in the text. (a) from bottom to top: $w = -2.0$, $w = -0.5$, $w = 0$; (b) curve 1 for $w = 0$, 2 for $w = 0.4$, 3 for $w = 1.0$, 4 for $w = 1.5$.

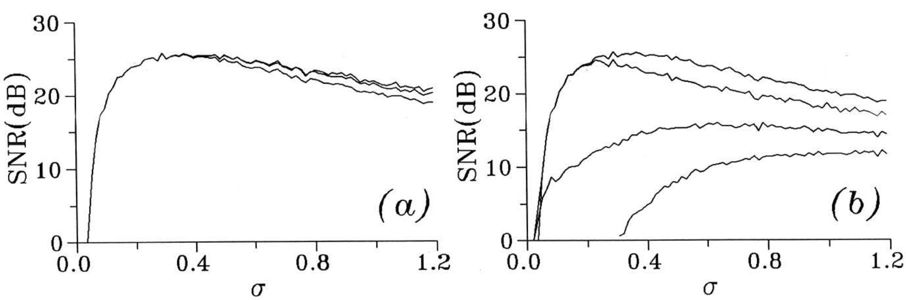

Fig. 2. $\text{SNR}^{(1,2)}$ *vs.* σ for the system (1) with $T_s = 128$, $\Delta\phi = \pi$ and other parameters as in the text. (a) from bottom to top: $w = 0$, $w = -0.5$, $w = -2.0$; (b) from bottom to top: $w = 1.5$, $w = 1.0$, $w = 0.5$, $w = 0.0$.

These results can be interpreted using arguments similar to these applied to the case of coupled bistable noisy oscillators in Refs. [15,16]. The positive coupling $w > 0$ increases the probability of simultaneous firing of both elements, since it increases the effective noise $\xi_n^{(1,2)}$ acting on a given element when the other element fires. In turn, $w < 0$ decreases this probability. If

$\Delta\phi = 0$ both probabilities $\Pr\left(x_n^{(1,2)} = 1\right)$ reach their maximum values at the same time, when the common periodic signal is maximum. Thus if $w > 0$ and one element fires then probably the other one will also fire, and both of them will fire in phase with the common periodic signal. This leads to the increase of periodicity in the time series $x_n^{(1,2)}$. If $\Delta\phi = \pi$ then the moments in which the probabilities $\Pr\left(x_n^{(1)} = 1\right)$ and $\Pr\left(x_n^{(2)} = 1\right)$ reach their maximum values are shifted by $T_s/2$. Thus $w > 0$ increases the probability that a given element will fire not only when the periodic signal at its input is maximum, but also when the signal is minimum. Hence the periodicity of $x_n^{(1,2)}$ is decreased. Quite the reverse, $w < 0$ decreases the above-mentioned probability, thus increasing the periodicity of $x_n^{(1,2)}$.

If $0 < \Delta\phi < \pi$ the system symmetry is broken and the SNRs of the two elements need not be equal. But for $T_s = 128$ we did not observe significant differences between $\mathrm{SNR}^{(1)}$ and $\mathrm{SNR}^{(2)}$. For $0 < \Delta\phi < \pi/2$ the observed dependence of SNR on w resembles that for $\Delta\phi = 0$, and for $\pi/2 < \Delta\phi < \pi$ — that for $\Delta\phi = \pi$. Arguments similar to the ones given above explain this dependence, since for $0 < \Delta\phi < \pi/2$ periodic signals at the inputs of both elements have the same sign during most of the period, as in the $\Delta\phi = 0$ case, and for $\pi/2 < \Delta\phi < \pi$ they have opposite signs, as in the $\Delta\phi = \pi$ case. For $\Delta\phi = \pi/2$ the value $w = w_{\mathrm{opt}} \approx 1.0$ still exists for which the maximum of the SNR *vs.* σ curve assumes the largest value, but the decrease of SNR for $w < 0$ is very small. The maximum of SNR for $w = w_{\mathrm{opt}}$ decreases slightly with increasing $\Delta\phi$ for $0 < \Delta\phi < \pi/2$.

For fast varying periodic signals the results are more difficult to analyze, since $\mathrm{SNR}^{(1)} = \mathrm{SNR}^{(2)}$ only for $\Delta\phi = 0$ and $\Delta\phi = \pi$. For other values of the phase shift the differences between the SNRs increase with $|w|$ and decrease with T_s. The general tendencies in the dependence of SNR on w and $\Delta\phi$ are similar to these for $T_s = 128$. Usually, for $0 < \Delta\phi < \pi/2$ and for $w < 0$ SNR decreases, whereas for $w > 0$ the value $w_{\mathrm{opt}} \approx 1.0$ exists for which the maximum of SNR assumes the largest value. For $\pi/2 < \Delta\phi < \pi$ and $w < 0$ SNR increases or at least does not change, whereas for $w > 0$ it monotonically decreases. However, for very small T_s deviations from these tendencies can appear. *E.g.* for $T_s = 8$, $\Delta\phi = 0.75\pi$ and $w > 0$ $\mathrm{SNR}^{(1)}$ first increases, reaches its maximum values for $w = w_{\mathrm{opt}} \approx 0.8$ and only then decreases with w, while $\mathrm{SNR}^{(2)}$ decreases monotonically, as usually (Fig. 3). Such an anomalous behaviour of $\mathrm{SNR}^{(1)}$ changes into the usual decrease with $w \longrightarrow +\infty$ already for $T_s = 16$.

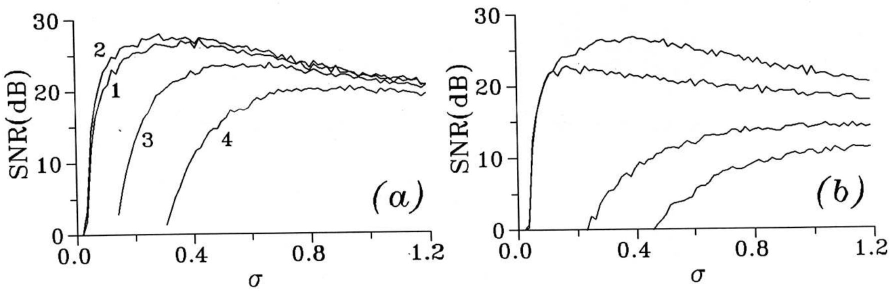

Fig. 3. (a) $\mathrm{SNR}^{(1)}$ *vs.* σ for the system (1) with $T_s = 8$, $\Delta\phi = 0.75\pi$ and other parameters as in the text: curve 1 for $w = 0$, 2 for $w = 0.8$, 3 for $w = 1.5$, 4 for $w = 2.0$; (b) the same for $\mathrm{SNR}^{(2)}$, from bottom to top: $w = 2.0$, $w = 1.5$, $w = 0.8$, $w = 0.0$.

5. Comparison between the numerical and theoretical results

In this section we briefly compare the numerically obtained $\mathrm{SNR}^{(1,2)}$ *vs.* σ curves with the predictions of the adiabatic theory. The theoretical values of $\mathrm{SNR}^{(1,2)}$ were obtained from Eq. (7) with $\phi = 0$, which does not change the generality of the results. In all cases examined we observed that the theoretical values of $\mathrm{SNR}^{(1)}$ and $\mathrm{SNR}^{(2)}$ were equal independently of w and $\Delta\phi$. This is in agreement with the numerical results of Sec. 3 which show that for large T_s the values of both SNRs are equal even if the system symmetry is broken, *i.e.* for $\Delta\phi \neq 0$ or $\Delta\phi \neq \pi$, and significant differences appear only in the non-adiabatic limit of small T_s. The theory predicts also qualitatively the dependence of SNR on w and $\Delta\phi$ in the limit of $\omega_s \longrightarrow 0$. However, the quantitative agreement is much worse, mainly due to the violation of certain basic assumptions made during the derivation of Eq. (7) in Ref. [4], as mentioned in Sec. 3. As an example the case $T_s = 128$ and $\Delta\phi = \pi$ is considered. For $w = 0$ the theory is exact and the numerical and theoretical values of SNR coincide. For $w < 0$ the theory predicts slight increase of SNR for $w \longrightarrow \infty$, in accordance with numerical results in Fig. 2(a). For $w > 0$ the theory predicts the decrease of SNR with w, but this decrease is much slower than the one obtained from numerical experiments (Fig. 4). The difference between the numerical and theoretical values of SNR increases with w and reaches *ca.* 10 dB already for $w = 1.0$ and moderate σ.

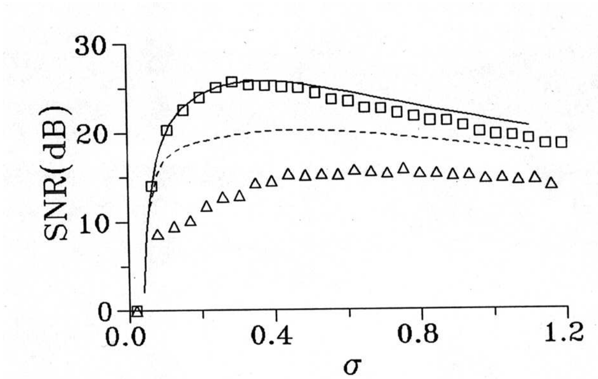

Fig. 4. Comparison between the numerical and theoretical results in the adiabatic approximation for the system (1) with $T_s = 128$, $\Delta\phi = \pi$ and other parameters as in the text: $w = 0.1$ — squares (numerical results) and solid curve (theory); $w = 1.0$ — triangles (numerical results) and dashed curve (theory).

6. Summary and conclusions

In this paper we investigated SR in a system of two coupled threshold elements forming a small neural network (1). Each element was driven by white Gaussian noise, non-correlated with the noise in the other element, and by the periodic signal with the amplitude and frequency identical in both elements; however, there was the phase shift $\Delta\phi$ of the signal between the elements. We investigated SNR in individual elements as a function of the noise intensity σ for various phase shifts, coupling and signal frequencies. We were interested in the enhancement of SNR due to the coupling. The theory of SNR, based on the expression for SNR in a threshold element known from the literature [4], was formulated and its predictions were compared with the numerical results.

We found that even in the case $\Delta\phi \neq 0$ proper coupling enhances SR in a threshold element, similarly as in the widely investigated case of a common signal acting on all coupled elements [5–17]. The SNR varies in the most systematic manner with the coupling and phase shift if the periodic signal frequency is small. Then, positive (excitory) coupling between elements (neurons) $w > 0$ enhances SNR over that of the uncoupled element if the periodic signals at the input of both elements have equal signs during most of the period, *i.e.* $0 < \Delta\phi < \pi/2$. Negative (inhibitory) coupling $w < 0$ enhances SNR if the signals have opposite signs during most of the period, *i.e.* $\pi/2 < \phi < \pi$. In the latter case the enhancement is much smaller than in the former one. In particular, for given w and varying $\Delta\phi$ the enhancement, if any, is always most visible for $\Delta\phi = 0$, and for other $\Delta\phi$

the SNR does not exceed the value for the case of in-phase signal in all elements. If SNR is increased for $w > 0$ then usually such $w = w_{\text{opt}}$ exists for which this enhancement is most pronounced. If SNR is increased for $w < 0$ it usually increases monotonically with $w \longrightarrow -\infty$. Small deviations from these general rules can appear for high-frequency input signals.

It is interesting to note that, in our model, in certain cases proper negative (inhibitory) coupling can improve SNR, at least slightly, in particular for $\sigma \longrightarrow \infty$ (Fig. 2(a)). If there is one periodic signal in a whole system only positive coupling enhances SR, *e.g.* diffusive coupling in a chain of bistable noisy oscillators [15, 16] or ferromagnetic coupling in the Ising model [20]. Putting aside the fact that the above-mentioned systems are bistable, the enhancement of SR occurs since positive coupling increases the probability of two neighbouring elements being in the same state. Hence in spatially extended systems the enhancement of SR is connected with maximum spatio-temporal synchronization of the system [15,16]. The spatio-temporal synchronization means also maximum synchronization with the input signal. As explained in Sec. 4, in our system, if the phase shift is big, *e.g.* $\Delta\phi \approx \pi$, the simultaneous firing of both elements decreases the periodicity of their outputs, and $w < 0$ is necessary to enhance this periodicity and thus SNR. Hence a question arises if in a spatially extended noisy system driven by a signal periodic in time and space the possible enhancement of SR is also connected with maximum synchronization of the system to the input signal. In a different formulation this is a problem of the most effective processing of spatio-temporal signals corrupted by noise, using the idea of SR. To our knowledge, this is a new possible application of SR which requires further investigation. It also seems that such spatio-temporal effects can be studied not only in continuous-time systems of coupled oscillators, but also *e.g.* in chains of coupled threshold elements forming spatially extended neural networks. The present paper provides a starting point for such investigations which should be done in much larger systems.

REFERENCES

[1] R. Benzi, A. Sutera, A. Vulpiani, *J. Phys.* **A14**, L453 (1981).

[2] L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni, *Rev. Mod .Phys.* **70**, 223 (1998).

[3] Z. Gingl, L.B. Kiss, F. Moss, *Europhys. Lett.* **29**, 191 (1995).

[4] F. Chapeau-Blondeau, *Phys. Rev.* **E53**, 5469 (1996).

[5] P. Jung, *Phys. Rev.* **A46**, R1709 (1992).

[6] M. Morillo, J. Gómez-Ordoñez, J.M. Casado, *Phys. Rev.* **E52**, 316 (1995).

[7] M.E. Inchiosa, A.R. Bulsara, *Phys. Rev.* **E52**, 327 (1995).

[8] J. Xiao, G. Hu, H. Liu, Y. Zhang, *Eur. Phys. J.* **B5**, 133 (1998).

[9] P. Jung, G. Mayer-Kress, *Phys. Rev. Lett.* **74**, 2130 (1995).

[10] P.C. Gailey, A. Neiman, J.J. Collins, F. Moss, *Phys. Rev. Lett.* **79**, 4701 (1997).

[11] J.J. Collins, C.C. Chow, T.T. Imhoff, *Nature* **376**, 236 (1995).

[12] D.R. Chialvo, A. Longtin, J. Müller-Gerking, *Phys. Rev.* **E55**, 1798 (1997).

[13] G. Mato, *Phys. Rev.* **E59**, 3339 (1999).

[14] F. Liu, J. Wang, W. Wang, *Phys. Rev.* **E59**, 3453 (1999).

[15] J.F. Lindner, B.K. Meadows, W.L. Ditto, M.E. Inchiosa, A.R. Bulsara, *Phys. Rev. Lett.* **75**, 3 (1995).

[16] J.F. Lindner, B.K. Meadows, W.L. Ditto, M.E. Inchiosa, A.R. Bulsara, *Phys. Rev.* **E53**, 2081 (1996).

[17] A. Neiman, L. Schimansky-Geier, *Phys. Lett.* **A197**, 379 (1995).

[18] F. Castelpoggi, H.S. Wio, *Phys. Rev.* **E57**, 5112 (1998).

[19] J.A. González, B.A. Mello, L.I. Reyes, L.E. Guerrero, *Phys. Rev. Lett.* **80**, 1361 (1998).

[20] U. Siewert, L. Schimansky-Geier, *Phys. Rev.* **E58**, 2843 (1998).

[21] K. Leung, Z. Néda, *Phys. Rev.* **E59**, 2730 (1999).

[22] B. McNamara, K. Wiesenfeld, *Phys. Rev.* **A39**, 4854 (1989).

[23] P. Jung, P. Hänggi, *Phys. Rev.* **A44**, 8032 (1991).