HOW HETEROGENOUS STRUCTURE OF TISSUE EFFECTS ITS DIELECTRIC CHARACTERISTICS*

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In the presented paper a new mathematical model of dispersion β in tissue dielectric response is introduced. It is proposed that interfacial phenomena and scaling properties of tissue account for power-law form of this region. The response β of tissue is considered with regard to probabilistic nature of its membrane components. The system is represented by an electric circuit of parallel R–C subcircuits with randomly distributed R and C values. It is shown that for the power law behaviour of tissue dielectric susceptibility $\chi(\omega)$ in the β response area the distribution of the variate $(RC)^{-1}$, representing the relaxation rate of a single subcircuit, should have heavy tails. The results indicate that the variations in local environment (local randomness) can provide a basis for self-similar relaxation dynamics without the need for hierarchically constrained fractal models.

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1. Introduction

The frequency domain measurements of various plant and animal tissues reveal [4] four distinct regions in their dielectric response (figure 1). At the lowest frequency range the non-ideal diffusion of ions through biological membranes takes place. Between 1 Hz and 10^6 Hz the highly dispersive

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response α is observed. It results from lateral movement of slowly mobile charge carriers along biological membranes. This process, which involves transport and storage of ions in the system, is an example of yet little understood Low Frequency Dispersion (LFD) phenomenon [10]. As frequency reaches high enough value (about 10^6 Hz) we observe dispersion β. This region, which is of our special interest in this paper, stems from interfacial phenomena. In the very high frequency area (above 10^9 Hz) the response γ is observed. It is hardly distinctive for the examined material, since it originates in dielectric properties of tissue electrolytes and in relaxation of bound water dipoles. (The cited crossover values are characteristic for the jade leaves — Crassula Portulacaceae [4]).

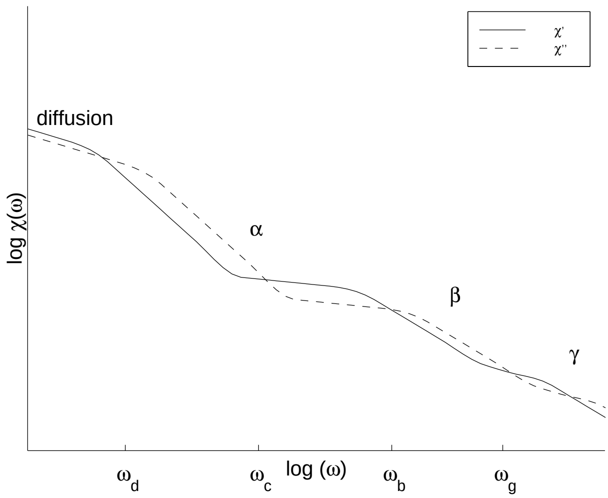

Fig. 1. Typical dielectric characteristics of tissue

The region of dispersion β exhibits power law dielectric characteristics, which has been identified as the Constant Phase Angle (CPA) response [4,9]. It means that the real and imaginary components of dielectric susceptibility $\chi(\omega) = \chi^{'}(\omega) - i\chi^{''}(\omega)$ follow the same function of frequency, which is of power law form:

$$\chi^{'}(\omega) \propto \chi^{''}(\omega) \propto \omega^{s-1}, \qquad 0 < s < 1. \tag{1}$$

The first dielectric model of animal tissue, given by Schwan [12], treated it as a conducting medium (extra-cellular fluid) with suspended spherical cells. The cells are enclosed in almost non-conducting biological membranes, filled with a conducting fluid. Since there are also sub-cellular and supra-cellular structures of the same organization, this model was extended [4] into a deterministic fractal of a self-similar, hierarchical structure. The units are of the same construction but on different size scales. On the basis of this model many attempts have been taken to investigate the phenomena responsible for the typical dielectric response of tissue, among others an electric circuit approach. Below we introduce an electric model of tissue and employ it to explain dielectric properties of tissue in the range of dispersion β. Next, the conditions necessary for the CPA type of this response are mathematically derived and the results of computer simulation are presented.

2. Electric model of dispersion β

For a long time, numerous attempts have been made to design the electric circuit that could reflect all dielectric properties of tissue. In the 1920's Philippson proposed one of the first electric models of the cell, as a simple R–C circuit (figure 2), where R_e is the resistance of the cell interior, C_m the capacitance of the cell membrane and R_m the membrane resistance [11].

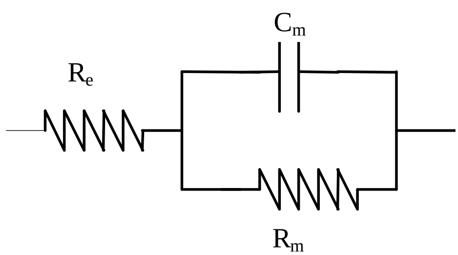

Fig. 2. Electric model of a cell by Philippson

In the following years various electric models of tissue were proposed. However, there has never been any fully successful result obtained. None of the proposed electric circuits could model all dielectric phenomena occurring in tissue, that is: diffusion, interfacial phenomena and dipole relaxation.

Nevertheless, the electric circuit approach seems to be a good idea to model dispersion β since it results from non-equal permittivities and conductivities of membranes within a tissue. Such heterogenous systems exhibit frequency-dependent properties, which are different from either of the constituent phases. The phenomenon of this type is known as the Maxwell-Wagner effect [3, 8] and is usually modelled by resistor-capacitor networks.

Among other electric models of tissue, there was one [4] arising from self-similar structure of tissue, based on fractal electric circuit of percolation type. Although it can model dispersion β quite well, the results are constrained by assumption of deterministic self-similar scaling of the tissue structure and therefore fixed relationship between the impedance of membranes and their interior.

We should, however, stress that the local randomness is ubiquitous for biological systems. Thus, there is a variety of membrane dielectric parameters, which results in the diversity of their characteristics. Hence, we propose another electric recursive model, in which we take into account the random nature of a biological medium. The elementary membrane structure is represented by the irreducible circuit of the shape similar to that proposed by Philippson.

Following the fractal construction (figure 3) we obtain a circuit, which represents dielectric features of tissue, provided small size of electrodes, in the range of dispersion β. It is worth noting that, in contradiction to other fractal circuits, we do not assume equal, deterministic values of each R and C values. Instead, we allow for their random distribution. We also assume independence between electric parameters of each membrane and its interior.

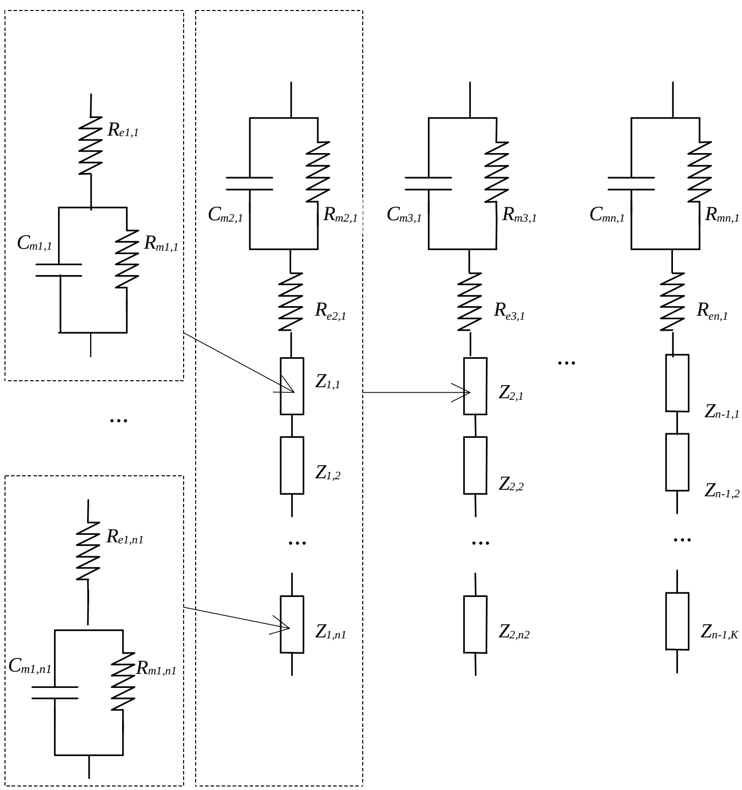

Fig. 3. Successive embedding of the biological membrane system, such as tissue, in electric representation.

It could be fairly easy to predict a behaviour of such a system provided a small number of components and their constant values [3]. However, we deal with a large number of randomly distributed R, C elements. In such a case, instead of the standard electric analysis, we can employ the probabilistic approach.

3. The probabilistic origins of the power law form of dispersion β in tissue

For a complex biological system, such as tissue, where the properties of its elements never take exactly the same constant value and can be approximated only by their most frequent values, the probabilistic approach may provide an explanation of global characteristics, without exact parameters of each constituent membrane. Such models give strict constraints on the mathematical form of the relaxation function and in natural way lead from local randomness to the global determinism, characteristic of complex biological system.

The objective of our analysis is to provide information on the stochastic properties of the investigated electric circuit which are responsible for the CPA behavior (1) of its dielectric characteristics.

Due to the relation between impedance $Z(\omega)$ of the system and its dielectric susceptibility $\chi(\omega)$:

$$Z(\omega) \propto (i\omega\chi(\omega))^{-1}$$

property (1) may be expressed in terms of the impedance, $Z(\omega)$, as:

$$Z'(\omega) \propto Z''(\omega) \propto \omega^{-s}, \qquad 0 < s < 1, \tag{2}$$

for the β response area of frequency. On the other hand impedance $Z(\omega)$ of the analyzed electric circuit equals:

$$Z(\omega) = \sum_{k=1}^{N}(\frac{1}{R_k} + i\omega C_k)^{-1} = \sum_{k=1}^{N} \frac{R_k}{1 + i\omega/b_k}, \tag{3}$$

where N represents the number of subcircuits, R_k and $b_k = (R_k C_k)^{-1}$ denote resistance and relaxation rate of the k-th subcircuit, respectively.

Considering single elements parameters as random variables whose values reflect the real physical situation one can see that variates R_k, b_k, $k = 1, \ldots, N$, form a sample taken from a joint distribution of random variables R, B, representing the resistance and the relaxation rate of a single subcircuit, respectively. Then, in a system consisting of a very large number N of subcircuits, formula (3) can be replaced by its approximation

$$Z(\omega) = N \cdot \left\langle \frac{R}{1+i\omega/B} \right\rangle$$

according to the Law of Large Numbers. Assuming stochastic independence of random variables R and B one obtains that

$$Z(\omega) = N \cdot \langle R \rangle \cdot \left\langle \frac{1}{1+i\omega/B} \right\rangle . \tag{4}$$

(Let us notice that such an assumption does not contradict the physical intuition of the phenomenon since B depends only on dielectric properties of the material, while R is greatly influenced by its geometry.)

Since the expected value $\langle R \rangle$ is independent of the frequency ω the impedance $Z(\omega)$ of the form (4) responds to the CPA type of behavior (1), expressed in terms of the impedance $Z(\omega)$ in relation (2), only provided that:

$$K(\omega) = \left\langle \frac{1}{1+i\omega/B} \right\rangle \propto \omega^{-s}, \qquad 0 < s < 1, \tag{5}$$

for ω taken from the region of dispersion β.

Now, the distribution of relaxation rate B that results in the dependence (5) should be specified. We have

$$K(\omega) = \int_0^\infty \frac{1}{1+i\omega/b} f(b) db,$$

where $f(b)$ is a density function of the random variable B. Since

$$\frac{1}{1+i\omega/b} = \int_0^\infty \mathrm{e}^{-i\omega t} b \mathrm{e}^{-bt} dt,$$

we receive that:

$$K(\omega) = \int_0^\infty k(t) \mathrm{e}^{-i\omega t} dt \tag{6}$$

for

$$k(t) = \int_0^\infty f(b) b \mathrm{e}^{-bt} db. \tag{7}$$

According to the Fourier transform quality [1] we have for $0 < \eta < 1$ and non negative function $h(t)$

$$\int_0^\infty h(t)\mathrm{e}^{-i\omega t}dt \propto \omega^{-\eta} \;\text{ for }\; \omega \to \infty$$

if and only if

$$h(t) \propto t^{\eta-1} \;\text{ for }\; t \to 0_+ .$$

Similarly, from the Tauberian theorem [1, 5] the relation

$$\int_0^\infty g(b)\mathrm{e}^{-bt}db \propto t^{\eta} \;\text{ for }\; t \to 0_+$$

for $0 < \eta < 1$ and nonnegative function $g(b)$ is satisfied only provided that

$$g(b) \propto b^{-\eta-1} \;\text{ for }\; b \to \infty .$$

It follows from the above properties of Fourier and Laplace transforms that $K(\omega)$, given by formula (6) with $k(t)$ of the form (7), fulfils relation (5) (and therefore (2)) for large ω if and only if the density function $f(b)$ of the distribution of B satisfies the condition

$$f(b) \propto b^{-s-1} \;\text{ for large }\; b. \tag{8}$$

Condition (8) means [5, 13, 15] that the relaxation rate of a single R–C unit has heavy-tailed distribution from the domain of attraction of a Lévy-stable law with stability index equal to the parameter s.

4. Model and details of computer simulation

The theoretical result of the previous section suggests that the CPA characteristics (1), for the systems represented by the electric circuit of the proposed shape with randomly distributed R and C values, requires heavy–tailed, satisfying condition (8) distribution of relaxation rate $B = (RC)^{-1}$. However, it was formally derived only for $\omega \to \infty$, which is not a physical situation. To show the usefulness of the above considerations for finite frequency values (from β response region) the departure from the theoretical result has been examined by means of computer simulations. We have investigated the dielectric susceptibility characteristics, obtained from the electric model according to the formula

$$\chi(\omega) = \left(i\omega\langle R\rangle \sum_{k=1}^{N} \frac{1}{1+i\omega/b_k} \right)^{-1},$$

where $b_1, \ldots, b_N$ are variates of a heavy–tailed random variable $(RC)^{-1}$.

Computations were conducted for three distributions satisfying condition (8): a stable distribution [13,15] with appropriate index of stability, and two, Pareto and Burr, distributions from its domain of attraction [2,7,14]. Values of random variable S_κ, $0 < \kappa < 1$, distributed according to the Lévy-stable law with stability index κ have been generated by means of the relation [6]:

$$S_\kappa = w_0 A_\kappa \frac{\sin(\kappa(V + \pi/2))}{(\cos(V))^{1/\kappa}} \cdot \left(\frac{\cos(V - \kappa\,(V + \pi/2)}{E}\right)^{(1-\kappa)/\kappa},$$

where:

$$A_\kappa = \left(1 + \tan^2 \frac{\pi\kappa}{2}\right)^{\frac{1}{2\kappa}},$$

V is a random variable uniformly distributed on $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$, E is independent of V exponential random variable with mean value 1, and $w_0 > 0$ is a scale parameter.

Variates of the Pareto random variable P_k with parameter k have been obtained via the transformation [14]

$$P_k = w_0(U^{-1/k} - 1) \tag{9}$$

from a uniform random variable U on $[0,1]$. Similarly, for generating values of Burr random variable $B_{p,k}$ with parameters p, k the transformation

$$B_{p,k} = w_0 \left(U^{-1/k} - 1\right)^{1/p} \tag{10}$$

has been used. In both formulas $w_0 > 0$ is a scale parameter. Additionally, to provide that the distribution of P_k, $B_{p,k}$ belongs to the domain of attraction of stable law S_κ, $0 < \kappa < 1$, we have taken $k = 1/\kappa$ in case of Pareto formula (9), and $k = p/\kappa$ in case of Burr transformation (10).

5. Results

The results of simulation for Pareto distribution for $k = 2$ ($s = 0.5$), $w_0 = 10^{-5}$, for two different numbers of elementary circuits are presented in figures 4 and 5. In accordance with the theoretical results, for $N = 10^5$, we have received the linear dependence of $\chi'(\omega)$ and $\chi''(\omega)$ in the log–log scale with the slopes $a'_{\mathrm{sym}} = -0.49$, $a''_{\mathrm{sym}} = -0.52$, while the theoretical values are $a' = a'' = -0.50$. It has been also observed that an increase in the number of subcircuits results in the expansion of the CPA region to the direction of higher frequencies.

The simulations for Burr distribution (figure 6) for $p = 5$, $k = 10$ ($s = 0.5$), $w_0 = 10^{-5}$, and for the stable distribution (figure 7) with stability

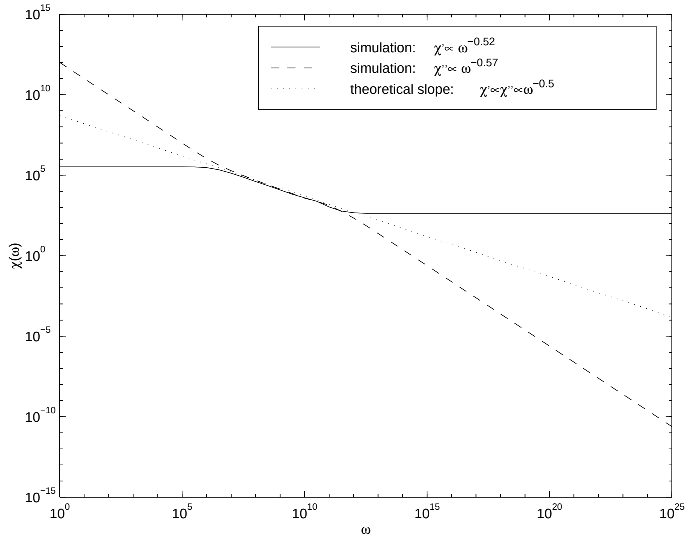

Fig. 4. Dielectric characteristics of the investigated electric circuit for Pareto distribution ($s = 0.5$, $w_0 = 10^{-5}$) of each subcircuit relaxation time and 10^3 subcircuits.

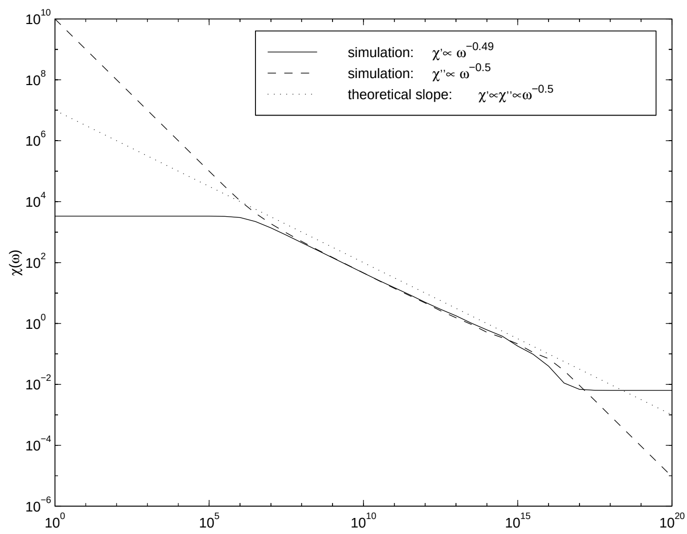

Fig. 5. Dielectric characteristics of the investigated electric circuit for Pareto distribution ($s = 0.5$, $w_0 = 10^{-5}$) of each subcircuit relaxation time and 10^5 subcircuits.

index $\kappa = 0.5$ ($s = 0.5$), $w = 10^{-5}$, have also been close to the theoretical result. The examination for other values of distribution parameters (which correspond to different values of parameter s), for all investigated distributions, did not show any significant departure from the theoretical slopes either. At the end of the simulation we executed reliability tests, in which the computations were repeated one hundred times for each case. The tests proved that the mean linear slopes are very close to their theoretical values and the standard deviations are very minor.

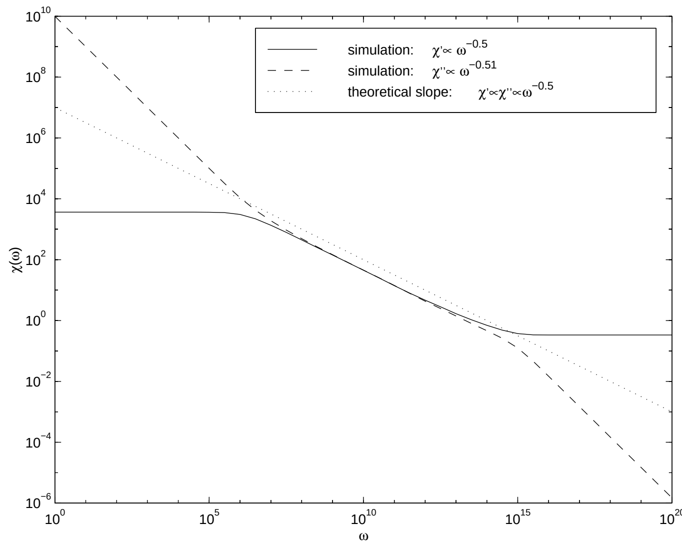

Fig. 6. Dielectric characteristics of the investigated electric circuit for Burr distribution ($p = 5$, $k = 10$, $w_0 = 10^{-5}$) of each subcircuit relaxation time and 10^5 subcircuits.

The results confirmed that the theoretical model can be applied for physical phenomenon modelling, also in the finite range β of frequency values.

It should be noted, however, that the simulation revealed two limits of the CPA region. While the low frequency boundary was theoretically predicted since the reasoning is valid only for high enough frequencies, the upper limit origin should be clarified. The explanation of this phenomenon is obvious if we take into regard the difference between electrical and probabilistic approaches. The probabilistic model assumes infinite number of subcircuits in order to introduce the averaging procedure (4), which is then used to obtain condition (8) for the distribution of the relaxation rate. The simulation based on more realistic electrical model, has been conducted for finite number of subcircuits, without the averaging. Therefore, for larger number of subcircuits the upper boundary is shifted to higher frequencies (compare figures 4 and 5). Nevertheless, this limitation does not stand in the way to apply the model. In the experimental characteristics the CPA regions are also limited from both sides.

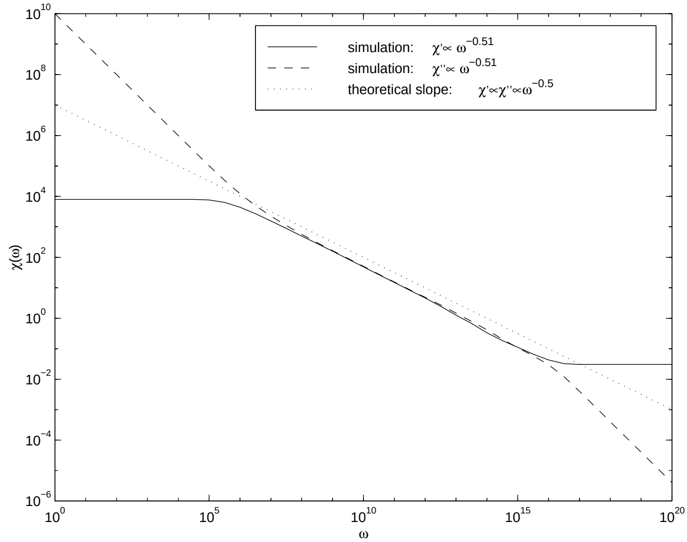

Fig. 7. Dielectric characteristics of the investigated electric circuit for stable distribution ($s = 0.5$, $w_0 = 10^{-5}$) of each subcircuit relaxation time and 10^5 subcircuits.

6. Summary

In this paper we have proposed an alternative to deterministic fractal approach in modelling dielectric phenomena by electric circuits. The analysis was presented for a circuit constructed as a series of parallel subcircuits, which is often used to model Maxwell-Wagner phenomenon. We have shown that CPA characteristics of power-law form can be obtained not only for the circuit of the deterministic nature. Namely, in the framework of a probabilistic approach which assumes randomly distributed R and C elements, we have found the relation between the CPA characteristics and heavy tails of the distribution of a single subcircuit's relaxation rate $b = (RC)^{-1}$. Theoretical asymptotical results have been illustrated by computer simulations for finite frequency values.

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