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NEUTRINO MASSES AND MIXINGS*

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We discuss models of neutrino masses that, in the context of the see-saw mechanism, could lead to a large mixing angle for the atmospheric neutrino oscillations without requiring too much fine-tuning between the Dirac and the Majorana sectors. These models are compatible with Abelian flavour symmetries and with the picture of flavour expected in grand unified theories.

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1. Introduction

Recent data from SuperKamiokande [1] have provided a more solid experimental basis for neutrino oscillations as an explanation of the atmospheric neutrino anomaly. In addition, also the solar neutrino deficit, observed by several experiments [2], is probably an indication of a different sort of neutrino oscillations. Neutrino oscillations imply neutrino masses. The extreme smallness of neutrino masses in comparison with quark and charged lepton masses indicates a different nature of the former, presumably linked to lepton-number violation and the Majorana nature of neutrinos. Thus neutrino masses provide a window on the very large energy scale where lepton number is violated and on Grand Unified Theories (GUTs). Experimental facts on neutrino masses and mixings could give an important feedback on the problem of quark and charged lepton masses, as all these masses are possibly related in GUTs. In particular the observation of a nearly maximal mixing angle for $\nu_\mu \to \nu_\tau$ is particularly interesting. Perhaps also solar neutrinos may occur with large mixing angle. At present solar neutrino mixings can be either large or very small, depending on which particular solution will eventually be established by the data. Large mixings in the neutrino sector

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are very interesting because a first guess was in favour of small mixings, in analogy to what is observed for quarks. If confirmed, single or double maximal mixings can provide an important hint on the mechanisms that generate neutrino masses.

The experimental status of neutrino oscillations is still very preliminary. Thus, in order to be able to proceed, the theorist has to make a number of assumptions on how the data will finally look when the experimental situation will be completely clarified.

1.1. Three light neutrinos

Here we assume that only two distinct oscillation frequencies exist, the largest being associated with atmospheric neutrinos and the smallest with solar neutrinos. We assume that the hint of an additional frequency from the LSND experiment [3], not confirmed by the Karmen experiment [4] (but yet far from being completely excluded), will disappear. Thus we avoid the introduction of new sterile neutrino species and can deal with only the three known species of light neutrinos[1].

We interpret the atmospheric neutrino oscillations as nearly maximal $\nu_\mu \to \nu_\tau$ oscillations, in agreement with the Chooz results [6]. The solar-neutrino oscillations correspond to the disappearance of ν_e into nearly equal fractions of ν_μ and ν_τ. A priori we are open minded about which of the three most likely solutions for solar neutrino oscillations is adopted: the two MSW solutions with small (SA) or large (LA) mixing angle, or the vacuum oscillation solution (VO).

1.2. A useful parametrization

Maximal atmospheric neutrino mixing and the requirement that the electron neutrino does not participate in the atmospheric oscillations, as indicated by the SuperKamiokande [1] and Chooz [6] data, lead directly to the following structure of the U_{fi} $(f = e,\mu,\tau,\ i = 1,2,3)$ real orthogonal mixing matrix, apart from sign convention redefinitions (here we are not interested

[1] Can three light neutrinos accommodate solar, atmospheric and LSND oscillations? This would seem a priori possible if the solar Cl experiment were affected by a large unknown systematic error. In this case an energy–independent suppression of the solar neutrino flux by approximately a factor of two could reasonably describe the data, leaving the solar frequency undetermined in a vast range. Thus we might associate the two independent frequencies to LSND and atmospheric oscillations. It has been observed that such an attractive scenario is incompatible with the combined results of the Chooz and SuperKamiokande experiments, when also the atmospheric neutrino asymmetries are considered [5].

in CP violation effects: all matrices are taken real)

$$U_{fi} = \begin{bmatrix} c & -s & 0 \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \end{bmatrix} . \qquad (1)$$

This result is obtained by a simple generalization of the analysis of Ref. [7] (also discussed in Ref. [8]) to the case of arbitrary solar mixing angle ($s \equiv \sin\theta_{\rm sun}$, $c \equiv \cos\theta_{\rm sun}$): $c = s = 1/\sqrt{2}$ for maximal solar mixing (*e.g.* for vacuum oscillations $\sin^2 2\theta_{\rm sun} \sim 0.75$) , while $\sin^2 2\theta_{\rm sun} \sim 4s^2 \sim 5.5 \times 10^{-3}$ for the small angle MSW [9] solution. The vanishing of U_{e3} guarantees that ν_e does not participate in the atmospheric oscillations and the relation $|U_{\mu 3}| = |U_{\tau 3}| = 1/\sqrt{2}$ implies maximal mixing for atmospheric neutrinos. Note that we are assuming only two frequencies, given by

$$\Delta_{\rm sun} \propto m_2^2 - m_1^2 , \qquad \Delta_{\rm atm} \propto m_3^2 - m_{1,2}^2 . \qquad (2)$$

The effective light neutrino mass matrix is given by $m_\nu = U m_{\rm diag} U^T$ with $m_{\rm diag} = {\rm Diag}[m_1, m_2, m_3]$. For generic s one finds

$$m_\nu = \begin{bmatrix} 2\varepsilon & \delta & \delta \\ \delta & \frac{m_3}{2} + \varepsilon_2 & -\frac{m_3}{2} + \varepsilon_2 \\ \delta & -\frac{m_3}{2} + \varepsilon_2 & \frac{m_3}{2} + \varepsilon_2 \end{bmatrix} , \qquad (3)$$

with

$$\varepsilon = \frac{m_1 c^2 + m_2 s^2}{2} , \qquad \delta = \frac{(m_1 - m_2) cs}{\sqrt{2}} , \qquad \varepsilon_2 = \frac{m_1 s^2 + m_2 c^2}{2} . \qquad (4)$$

We see that the existence of one maximal mixing and $U_{e3} = 0$ are the most important input that leads to the matrix form in Eqs. (3), (4). The value of the solar neutrino mixing angle can be left free. While the simple parametrization of the matrix U in Eq. (1) is quite useful to guide the search for a realistic pattern of neutrino mass matrices, it should not be taken too literally. In particular the data do not exclude a non-vanishing U_{e3} element. In most of the SuperKamiokande allowed region the bound by Chooz [6] amounts to $|U_{e3}| < 0.2$. In the region not covered by Chooz $|U_{e3}|$ can even be larger [5, 10]. Thus neglecting $|U_{e3}|$ with respect to s in Eq. (1) is not really justified. Also note that in presence of a large hierarchy $|m_3| \gg |m_{1,2}|$ the effect of neglected parameters in Eq. (3) can be enhanced by $m_3/m_{1,2}$ and produce sizeable corrections. A non vanishing U_{e3} term can lead to different $(m_\nu)_{12}$ and $(m_\nu)_{13}$ terms. Similarly, a deviation from maximal mixing $U_{\mu 3} \neq U_{\tau 3}$ distorts the ε_2 terms in the 23 sector of m_ν. Therefore, especially for a large hierarchy, there is more freedom in the small terms in order to construct a model that fits the data than it is apparent from Eq. (3).

1.3. Hierarchical spectrum

Since neutrino oscillations only measure differences of squared masses, the observed differences $(\Delta m^2)_{\rm atm} = |m_3^2 - m_2^2| \gg (\Delta m^2)_{\rm sun} = |m_2^2 - m_1^2|$ could correspond to (A) hierarchical eigenvalues $|m_3| \gg |m_{2,1}|$ or to partial or total near degeneracy: (B) $|m_1| \sim |m_2| \gg |m_3|$ or (C) $|m_1| \sim |m_2| \sim |m_3|$. The configurations (B) and (C) imply a very precise near degeneracy of squared masses. For example, the case (C) is the only one that could in principle accommodate neutrinos as hot dark matter together with solar and atmospheric neutrino oscillations. We think that it is not at all clear at the moment that a hot dark matter component is really needed [11] but this could be a reason in favour of the fully degenerate solution. Then the common mass should be around 1–3 eV. The solar frequency could be given by a small 1–2 splitting, while the atmospheric frequency could be given by a still small but much larger 1,2–3 splitting. A strong constraint arises in this case from the non observation of neutrinoless double beta decay which requires that the *ee* entry of m_ν must obey $|(m_\nu)_{ee}| \leq 0.46$ eV [12]. As observed in Ref. [13], this bound can only be satisfied if bimixing is realized (that is double maximal mixing, with solar neutrinos explained by the VO or MSW-LA solutions). But we would need a relative splitting $|\Delta m/m| \sim \Delta m^2_{\rm atm}/2m^2 \sim 10^{-3} - 10^{-4}$ and a much smaller one for solar neutrinos explained by vacuum oscillations: $|\Delta m/m| \sim 10^{-10} - 10^{-11}$. Such a tiny relative mass splitting, arranged at the large energy scale where lepton number is violated, can be easily upset by the renormalization group evolution down to the electroweak scale [14], unless a suitable flavour symmetry protects it during the running [15].

1.4. See–saw mechanism

For reasons of simplicity, we consider the simplest version of the see-saw mechanism with one Dirac, $m_{\rm D}$, and one Majorana, M, mass matrix, related to the neutrino mass matrix m_ν, in the basis where the charged lepton mass matrix is diagonal, by

$$m_\nu = m_{\rm D}^T M^{-1} m_{\rm D}\,. \tag{5}$$

As well known this is not the most general see-saw mechanism because we are not including the left–left Majorana mass block. It is implausible that starting from hierarchical Dirac matrices we end up via the see-saw mechanism into a nearly perfect degeneracy of squared masses and the assumption of hierarchical Dirac masses and the see-saw mechanism naturally leads to a pattern of type A with $|m_3| \gg |m_2| \gg |m_1|$. Models with degenerate neutrinos (see, for example, Refs. [16]) could be natural if the dominant contributions directly arise from non renormalizable operators which are a priori unrelated to other fermion masses, but we will not explore this possibility here.

2. Two interesting mechanisms

In general large mass splittings correspond to small mixings because normally only close-by states are strongly mixed. In a 2 by 2 matrix context, the requirement of large splitting and large mixing leads to a condition of vanishing determinant. For example the matrix

$$m \propto \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix} \tag{6}$$

has eigenvalues 0 and $1+x^2$ and for x of $O(1)$ the mixing is large. Thus, in the limit of neglecting small mass terms of order $m_{1,2}$, the demands of large atmospheric neutrino mixing and dominance of m_3 translate into the condition that the subdeterminant 23 of the 3 by 3 mass matrix vanishes. The problem is to show that this vanishing can be arranged in a natural way without fine tuning.

Without loss of generality we can go to a basis where both the charged lepton Dirac mass matrix m_{D}^l and the Majorana matrix M for the right-handed neutrinos are diagonal. In fact, after diagonalization of the charged lepton Dirac mass matrix, we still have the freedom of a change of basis for the right-handed neutrino fields, in that the right-handed charged lepton and neutrino fields, as opposed to left-handed fields, are uncorrelated by the SU(2)×U(1) gauge symmetry. We can use this freedom to make the Majorana matrix diagonal: $M^{-1} = V^T d_M V$ with $d_M = \mathrm{Diag}[1/M_1, 1/M_2, 1/M_3]$. Then if we parametrize the matrix $V m_{\mathrm{D}}$ by z_{ab} we have:

$$(m_\nu)_{ab} \equiv (m_{\mathrm{D}}^T M^{-1} m_{\mathrm{D}})_{ab} = \sum_c \frac{z_{ca} z_{cb}}{M_c}\,. \tag{7}$$

From this expression we see that, while we can always arrange the twelve parameters z_{ab} and M_a to arbitrarily fix the six independent matrix elements of m_ν, the hierarchical case is special in that it can be approximately reproduced in two particularly simple ways, without relying on precise cancellations among different terms:

(i) there are only two large entries in the z matrix, $|z_{c2}| \sim |z_{c3}|$, and the three eigenvalues M_a are of comparable magnitude (or, at least, with a less pronounced hierarchy than for the z matrix elements). Then, the subdeterminant 23 vanishes and one only needs the ratio $|z_{c2}/z_{c3}|$ to be close to 1. This possibility was discussed for instance in [17];

(ii) one of the right-handed neutrinos is particularly light and, in first approximation, it is only coupled to μ and τ. Thus, $M_c \sim \eta$ (small) and $z_{c1} \sim 0$. In this case the 23 subdeterminant vanishes, and again one only needs the ratio $|z_{c2}/z_{c3}|$ to be close to 1. This possibility has been especially emphasized in Refs. [5, 18–20].

In a 2 by 2 matrix context (in the 23 sector), a typical example of mechanism *(i)* is given by a Dirac matrix $m_{\rm D}$, defined by $\bar{R}m_{\rm D}L$, which takes the approximate form:

$$m_{\rm D} \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & x & 1 \end{bmatrix} . \tag{8}$$

This matrix has the property that for a generic Majorana matrix M one finds:

$$m_\nu = m_{\rm D}^T M^{-1} m_{\rm D} \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{bmatrix} . \tag{9}$$

The only condition on M^{-1} is that the 33 entry is non zero. It is important for the following discussion to observe that $m_{\rm D}$ given by Eq. (8) under a change of basis transforms as $m_{\rm D} \to V^\dagger m_{\rm D} U$ where V and U rotate the right and left fields respectively. It is easy to check that in order to make $m_{\rm D}$ diagonal we need large left mixings. More precisely $m_{\rm D}$ is diagonalized by taking $V = 1$ and U given by

$$U = \begin{bmatrix} c & -s & 0 \\ sc_\gamma & cc_\gamma & -s_\gamma \\ ss_\gamma & cs_\gamma & c_\gamma \end{bmatrix} , \tag{10}$$

with

$$s_\gamma = -\frac{x}{r} , \qquad c_\gamma = \frac{1}{r} , \qquad r = \sqrt{1+x^2} . \tag{11}$$

The matrix U is directly the neutrino mixing matrix. The mixing angle for atmospheric neutrino oscillations is given by:

$$\sin^2 2\theta = 4 s_\gamma^2 c_\gamma^2 = \frac{4x^2}{(1+x^2)^2} . \tag{12}$$

Thus the bound $\sin^2 2\theta > 0.8$ translates into $0.6 < |x| < 1.6$. As is clear, this mechanism is based on asymmetric Dirac matrices, with, in the case of the example, a large left-handed mixing already present in the Dirac matrix.

If, for some reason, one prefers symmetric or nearly so matrices, then one can use mechanism *(ii)*. For example, one could want to preserve left–right symmetry at the GUT scale. Then, the observed smallness of left-handed mixings for quarks would also demand small right-handed mixings. So we now assume that $m_{\rm D}$ is nearly diagonal (always in the basis where $m^l_{\rm D}$ and M are diagonal) with all its off-diagonal terms proportional to some small parameter ε. Working in the subsector 23 and starting from

$$m_{\rm D} \propto \begin{bmatrix} \varepsilon^p & x\varepsilon \\ x\varepsilon & 1 \end{bmatrix}, \qquad M^{-1} \propto \begin{bmatrix} r_2 & 0 \\ 0 & 1 \end{bmatrix}, \tag{13}$$

where x is of $O(1)$ and $r_2 \equiv M_3/M_2$, we obtain:

$$m_\nu \propto \begin{bmatrix} \varepsilon^{2p} r_2 + x^2\varepsilon^2 & x\varepsilon^{p+1} r_2 + x\varepsilon \\ x\varepsilon^{p+1} r_2 + x\varepsilon & x^2\varepsilon^2 r_2 + 1 \end{bmatrix}. \tag{14}$$

For sufficiently small M_2 the terms in r_2 are dominant. For $p = 1, 2$, which we consider as typical cases, it is sufficient that $\varepsilon^2 r_2 \gg 1$. Assuming that this condition is satisfied, consider first the case with $p = 2$. We have

$$m_\nu \propto x^2\varepsilon^2 r_2 \begin{bmatrix} \dfrac{\varepsilon^2}{x^2} & \dfrac{\varepsilon}{x} \\ \dfrac{\varepsilon}{x} & 1 \end{bmatrix}. \tag{15}$$

In this case the determinant is naturally vanishing (to the extent that the terms in r_2 are dominant), so that the mass eigenvalues are widely split. However, the mixing is nominally small: $\sin 2\theta$ is of $O(2\varepsilon/x)$. It could be numerically large enough if $1/x \sim 2$–3 and ε is of the order of the Cabibbo angle $\varepsilon \sim 0.20$–0.25. This is what we call "stretching": the large neutrino mixing is explained in terms of a small parameter; this is not so small and can give a perhaps sufficient amount of mixing if enhanced by a possibly large coefficient. This minimalistic view was endorsed in Refs. [21].

A more peculiar case is obtained for $p = 1$, which gives:

$$m_\nu \propto \varepsilon^2 r_2 \begin{bmatrix} 1 & x \\ x & x^2 \end{bmatrix}. \tag{16}$$

In this case the small parameter ε is completely factored out and for $x \sim 1$ the mixing is nearly maximal. The see-saw mechanism has created *large mixing from almost nothing* [22]: all relevant matrices entering the see-saw mechanism are nearly diagonal. Clearly, the crucial factorization of the small parameter ε^2 only arises for $p = 1$, that is the light Majorana eigenvalue is coupled to ν_μ and ν_τ with comparable strength. It is straightforward to extend the previous model to the 3 by 3 case [22]. In that case it is possible to reproduce both the SA and the LA MSW solutions. The required hierarchy among the matrix elements can be supported by a suitable Abelian flavour symmetry, which can be realized also at the level of an SU(5) grand unified theory. Moreover, this hierarchy is not spoiled by the renormalization group evolution from the unification scale down to low energy. In a similar class of models all Dirac mixings are small, but large mixing are introduced via M [23].

3. An explicit model

We have seen that, in order to explain in a natural way widely split light neutrino masses together with large mixings, we need an automatic vanishing of the 23 subdeterminant. This in turn is most simply realized within mechanism *i*, by allowing some large left-handed mixing terms in the Dirac neutrino matrix. By left-handed mixing we mean non diagonal matrix elements that can only be eliminated by a large rotation of the left-handed fields. Thus the question is how to reconcile large left-handed mixings in the leptonic sector with the observed near diagonal form of $V_{\rm CKM}$, the quark mixing matrix. Strictly speaking, since $V_{\rm CKM} = U_u^\dagger U_d$, the individual matrices U_u and U_d need not be near diagonal, but $V_{\rm CKM}$ does, while the analogue for leptons apparently cannot be near diagonal. However nothing forbids for quarks that, in the basis where m_u is diagonal, the d quark matrix has large non diagonal terms that can be rotated away by a pure right-handed rotation. We suggest that this is so and that in some way right-handed mixings for quarks correspond to left-handed mixings for leptons.

In the context of (Susy) SU(5) [24] there is a very attractive hint of how the present mechanism can be realized [17,25]. In the $\bar{5}$ of SU(5) the d^c singlet appears together with the lepton doublet (ν, e). The (u, d) doublet and e^c belong to the 10 and ν^c to the 1 and similarly for the other families. As a consequence, in the simplest model with mass terms arising from only Higgs pentaplets, the Dirac matrix of down quarks is the transpose of the charged lepton matrix: $m_{\rm D}^d = (m_{\rm D}^l)^T$. Thus, indeed, a large mixing for right-handed down quarks corresponds to a large left-handed mixing for charged leptons. In the same simplest approximation with 5 or $\bar{5}$ Higgs, the up quark mass matrix is symmetric, so that left and right mixing matrices are equal in this case[2]. Then small mixings for up quarks and small left-handed mixings for down quarks are sufficient to guarantee small $V_{\rm CKM}$ mixing angles even for large d quark right-handed mixings. When the charged lepton matrix is diagonalized the large left-handed mixing of the charged leptons is transferred to the neutrinos. Note that in SU(5) we can diagonalize the u mass matrix by a rotation of the fields in the 10, the Majorana matrix M by a rotation of the 1 and the effective light neutrino matrix m_ν by a rotation of the $\bar{5}$.

[2] Up to a diagonal matrix of phases.

In this basis the d quark mass matrix fixes V_{CKM} and the charged lepton mass matrix fixes neutrino mixings. It is well known that a model where the down and the charged lepton mass matrices are exactly the transpose of one another cannot be exactly true because of the e/d and μ/s mass ratios [24]. It is also known that one remedy to this problem is to add some Higgs component in the 45 representation of SU(5) [26]. A different solution [27] will be described later. But the symmetry under transposition can still be a good guideline if we are only interested in the order of magnitude of the matrix entries and not in their exact values. Similarly, the Dirac neutrino mass matrix m_{D} is the same as the up quark mass matrix in the very crude model where the Higgs pentaplets come from a pure 10 representation of SO(10): $m_{\mathrm{D}} = m_{\mathrm{D}}^u$. For m_{D} the dominance of the third family eigenvalue as well as a near diagonal form could be an order of magnitude remnant of this broken symmetry. Thus, neglecting small terms, the neutrino Dirac matrix in the basis where charged leptons are diagonal could be directly obtained in the form of Eq. (8).

We give here an explicit example of the mechanism under discussion in the framework of a unified Susy SU(5) theory with an additional $\mathrm{U}(1)_F$ flavour symmetry [28]. This model is to be taken as merely indicative, in that some important problems, like, for example, the cancellation of chiral anomalies are not tackled here. But we find it impressive that the general pattern of all what we know on fermion masses and mixings is correctly reproduced at the level of orders of magnitude. We regard the present model as a low-energy effective theory valid at energies close to $M_{\mathrm{GUT}} \ll M_{\mathrm{Pl}}$. We can think to obtain it by integrating out the heavy modes from an unknown underlying fundamental theory defined at an energy scale close to M_{Pl}. From this point of view the gauge anomalies generated by the light supermultiplets listed below can be compensated by another set of supermultiplets with masses above M_{GUT}, already eliminated from the low-energy theory. In particular, we assume that these additional supermultiplets are vector-like with respect to SU(5) and chiral with respect to $\mathrm{U}(1)_F$. Their masses are then naturally expected to be of the order of the $\mathrm{U}(1)_F$ breaking scale, which, in the following discussion, turns out to be near M_{Pl}. We have explicitly checked the possibility of cancelling the gauge anomalies in this way but, due to our ignorance about the fundamental theory, we do not find particularly instructive to illustrate the details here. In this model the known generations of quarks and leptons are contained in triplets Ψ_{10}^a and $\Psi_{\bar{5}}^a$, $(a = 1, 2, 3)$ transforming as 10 and $\bar{5}$ of SU(5), respectively. Three more SU(5) singlets Ψ_1^a describe the right-handed neutrinos. We assign to these fields the following F-charges:

$$\Psi_{10} \sim (3,2,0)\,, \quad (17)$$
$$\Psi_{\bar{5}} \sim (3,0,0)\,, \quad (18)$$
$$\Psi_{1} \sim (1,-1,0)\,. \quad (19)$$

We start by discussing the Yukawa coupling allowed by $\mathrm{U}(1)_F$-neutral Higgs multiplets φ_5 and $\varphi_{\bar{5}}$ in the 5 and $\bar{5}$ SU(5) representations and by a pair θ and $\bar{\theta}$ of SU(5) singlets with $F=1$ and $F=-1$, respectively.

In the quark sector we obtain[3]:

$$m_{\mathrm{D}}^u = (m_{\mathrm{D}}^u)^T = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_u\,, \qquad m_{\mathrm{D}}^d = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_d\,, \quad (20)$$

from which we get the order-of-magnitude relations:

$$\begin{aligned} m_u : m_c : m_t &= \lambda^6 : \lambda^4 : 1\,, \\ m_d : m_s : m_b &= \lambda^6 : \lambda^2 : 1\,, \end{aligned} \quad (21)$$

and

$$V_{us} \sim \lambda\,, \qquad V_{ub} \sim \lambda^3\,, \qquad V_{cb} \sim \lambda^2\,. \quad (22)$$

Here $v_u \equiv \langle\varphi_5\rangle$, $v_d \equiv \langle\varphi_{\bar{5}}\rangle$ and λ denotes the ratio between the vacuum expectation value of $\bar{\theta}$ and an ultraviolet cut-off identified with the Planck mass M_{Pl}: $\lambda \equiv \langle\bar{\theta}\rangle/M_{\mathrm{Pl}}$. To correctly reproduce the observed quark mixing angles, we take λ of the order of the Cabibbo angle. For non-negative F-charges, the elements of the quark mixing matrix V_{CKM} depend only on the charge differences of the left-handed quark doublet [28]. Up to a constant shift, this defines the choice in Eq. (17). Equal F-charges for $\Psi_{\bar{5}}^{2,3}$ (see Eq. (18)) are then required to fit m_b and m_s. We will comment on the lightest quark masses later on.

At this level, the mass matrix for the charged leptons is the transpose of m_{D}^d:

$$m_{\mathrm{D}}^l = (m_{\mathrm{D}}^d)^T \quad (23)$$

and we find:

$$m_e : m_\mu : m_\tau = \lambda^6 : \lambda^2 : 1\,. \quad (24)$$

The $O(1)$ off-diagonal entry of m_{D}^l gives rise to a large left-handed mixing in the 23 block which corresponds to a large right-handed mixing in the d

[3] In Eq. (20) the entries denoted by 1 in m_{D}^u and m_{D}^d are not necessarily equal. As usual, such a notation allows for $O(1)$ deviations.

mass matrix. In the neutrino sector, the Dirac and Majorana mass matrices are given by:

$$m_{\rm D} = \begin{bmatrix} \lambda^4 & \lambda & \lambda \\ \lambda^2 & \lambda' & \lambda' \\ \lambda^3 & 1 & 1 \end{bmatrix} v_u \,, \qquad M = \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix} \bar{M} \,, \tag{25}$$

where $\lambda' \equiv \langle\theta\rangle/M_{\rm Pl}$ and $\bar{M}$ denotes the large mass scale associated to the right-handed neutrinos: $\bar{M} \gg v_{u,d}$.

After diagonalization of the charged lepton sector and after integrating out the heavy right-handed neutrinos we obtain the following neutrino mass matrix in the low-energy effective theory:

$$m_\nu = \begin{bmatrix} \lambda^6 & \lambda^3 & \lambda^3 \\ \lambda^3 & 1 & 1 \\ \lambda^3 & 1 & 1 \end{bmatrix} \frac{v_u^2}{\bar{M}} \,, \tag{26}$$

where we have taken $\lambda \sim \lambda'$. The $O(1)$ elements in the 23 block are produced by combining the large left-handed mixing induced by the charged lepton sector and the large left-handed mixing in $m_{\rm D}$. A crucial property of m_ν is that, as a result of the sea-saw mechanism and of the specific $\mathrm{U}(1)_F$ charge assignment, the determinant of the 23 block is <u>*automatically*</u> of $O(\lambda^2)$ (for this the presence of negative charge values, leading to the presence of both λ and λ' is essential [17]).

It is easy to verify that the eigenvalues of m_ν satisfy the relations:

$$m_1 : m_2 : m_3 = \lambda^4 : \lambda^2 : 1 \,. \tag{27}$$

The atmospheric neutrino oscillations require $m_3^2 \sim 10^{-3}\ \mathrm{eV}^2$. From Eq. (26), taking $v_u \sim 250$ GeV, the mass scale $\bar{M}$ of the heavy Majorana neutrinos turns out to be close to the unification scale, $\bar{M} \sim 10^{15}$ GeV. The squared mass difference between the lightest states is of $O(\lambda^4)\ m_3^2$, appropriate to the MSW solution to the solar neutrino problem. Finally, beyond the large mixing in the 23 sector, corresponding to $s_\gamma \sim c_\gamma$ in Eq. (10), m_ν provides a mixing angle $s \sim (\lambda/2)$ in the 12 sector, close to the range preferred by the small angle MSW solution. In general U_{e3} is non-vanishing, of $O(\lambda^3)$.

In general, the charge assignment under $\mathrm{U}(1)_F$ allows for non-canonical kinetic terms that represent an additional source of mixing. Such terms are allowed by the underlying flavour symmetry and it would be unnatural to tune them to the canonical form. We have checked that all the results quoted up to now remain unchanged after including the effects related to the most general kinetic terms, via appropriate rotations and rescaling in the flavour space (see also Ref. [29]).

Obviously, the order of magnitude description offered by this model is not intended to account for all the details of fermion masses. Even neglecting the parameters associated with the CP violating observables, some of the relevant observables are somewhat marginally reproduced. For instance we obtain $m_u/m_t \sim \lambda^6$ which is perhaps too large. However we find it remarkable that in such a simple scheme most of the 12 independent fermion masses and the 6 mixing angles turn out to have the correct order of magnitude. Notice also that our model prefers large values of $\tan\beta \equiv v_u/v_d$. This is a consequence of the equality $F(\Psi^3_{10}) = F(\Psi^3_{\bar{5}})$ (see Eqs. (17) and (18)). In this case the Yukawa couplings of top and bottom quarks are expected to be of the same order of magnitude, while the large m_t/m_b ratio is attributed to $v_u \gg v_d$ (there may be factors $O(1)$ modifying these considerations, of course). We recall here that in supersymmetric grand unified models large values of $\tan\beta$ are one possible solution to the problem of reconciling the boundary condition $m_b = m_\tau$ at the GUT scale with the low-energy data [30]. Alternatively, to keep $\tan\beta$ small, one could suppress m_b/m_t by adopting different F-charges for the $\Psi^3_{\bar{5}}$ and Ψ^3_{10}.

Additional contributions to flavour changing processes and to CP violating observables are generally expected in a supersymmetric grand unified theory. However, a reliable estimate of the corresponding effects would require a much more detailed definition of the theory than attempted here. Crucial ingredients such as the mechanism of supersymmetry breaking and its transmission to the observable sector have been ignored in the present note. We are implicitly assuming that the omission of this aspect of the flavour problem does not substantially alter our discussion.

A common problem of all SU(5) unified theories based on a minimal Higgs structure is represented by the relation $m^l_{\rm D} = (m^d_{\rm D})^T$ that, while leading to the successful $m_b = m_\tau$ boundary condition at the GUT scale, provides the wrong prediction $m_d/m_s = m_e/m_\mu$ (which, however, is an acceptable order of magnitude equality). We can easily overcome this problem and improve the picture [27] by introducing an additional supermultiplet $\bar{\theta}_{24}$ transforming in the adjoint representation of SU(5) and possessing a negative $\mathrm{U}(1)_F$ charge, $-n$ $(n > 0)$. Under these conditions, a positive F-charge f carried by the matrix elements $\Psi^a_{10}\Psi^b_{\bar{5}}$ can be compensated in several different ways by monomials of the kind $(\bar{\theta})^p(\bar{\theta}_{24})^q$, with $p + nq = f$. Each of these possibilities represents an independent contribution to the down quark and charged lepton mass matrices, occurring with an unknown coefficient of $O(1)$. Moreover the product $(\bar{\theta}_{24})^q\varphi_{\bar{5}}$ contains both the $\bar{5}$ and the $\overline{45}$ SU(5) representations, allowing for a differentiation between the down quarks and the charged leptons. The only, welcome, exceptions are given by the $O(1)$ entries that do not require any compensation and, at the leading order, remain the same for charged leptons and down quarks. This preserves the good $m_b = m_\tau$ prediction. Since a perturbation of $O(1)$ in the subleading matrix elements is sufficient to cure the bad $m_d/m_s = m_e/m_\mu$ relation, we can safely assume that $\langle\bar{\theta}_{24}\rangle/M_{\rm Pl} \sim \lambda^n$, to preserve the correct order-of-magnitude predictions in the remaining sectors.

We have not dealt here with the problem of recovering the correct vacuum structure by minimizing the effective potential of the theory. It may be noticed that the presence of two multiplets θ and $\bar{\theta}$ with opposite F charges could hardly be reconciled, without adding extra structure to the model, with a large common VEV for these fields, due to possible analytic terms of the kind $(\theta\bar{\theta})^n$ in the superpotential. We find therefore instructive to explore the consequences of allowing only the negatively charged $\bar{\theta}$ field in the theory.

It can be immediately recognized that, while the quark mass matrices of Eqs. (20) are unchanged, in the neutrino sector the Dirac and Majorana matrices get modified into:

$$m_{\rm D} = \begin{bmatrix} \lambda^4 & \lambda & \lambda \\ \lambda^2 & 0 & 0 \\ \lambda^3 & 1 & 1 \end{bmatrix} v_u \,, \qquad M = \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & 0 & 0 \\ \lambda & 0 & 1 \end{bmatrix} \bar{M} \,. \tag{28}$$

The zeros are due to the analytic property of the superpotential that makes impossible to form the corresponding F invariant by using $\bar{\theta}$ alone. These zeros should not be taken literally, as they will be eventually filled by small terms coming, for instance, from the diagonalization of the charged lepton mass matrix and from the transformation that put the kinetic terms into canonical form. It is however interesting to work out, in first approximation, the case of exactly zero entries in $m_{\rm D}$ and M, when forbidden by F.

The neutrino mass matrix obtained via see-saw from $m_{\rm D}$ and M has the same pattern as the one displayed in Eq. (26). A closer inspection reveals that the determinant of the 23 block is identically zero, independently from λ. This leads to the following pattern of masses:

$$m_1 : m_2 : m_3 = \lambda^3 : \lambda^3 : 1 \,, \qquad m_1^2 - m_2^2 = {\rm O}(\lambda^9)\; m_3^2 \,. \tag{29}$$

Moreover the mixing in the 12 sector is almost maximal:

$$\frac{s}{c} = \frac{\pi}{4} + {\rm O}(\lambda^3) \,. \tag{30}$$

For $\lambda \sim 0.2$, both the squared mass difference $(m_1^2 - m_2^2)/m_3^2$ and $\sin^2 2\theta_{\rm sun}$ are remarkably close to the values required by the vacuum oscillation solution to the solar neutrino problem. We have also checked that this property is reasonably stable against the perturbations induced by small terms (of order λ^5) replacing the zeros, coming from the diagonalization of the charged lepton sector and by the transformations that render the kinetic terms canonical. We find quite interesting that also the just-so solution, requiring an intriguingly small mass difference and a bimaximal mixing, can be reproduced, at least at the level of order of magnitudes, in the context of a "minimal" model of flavour compatible with supersymmetric SU(5). In this case the role played by supersymmetry is essential, a non-supersymmetric model with $\bar{\theta}$ alone not being distinguishable from the version with both θ and $\bar{\theta}$, as far as low-energy flavour properties are concerned.

4. Conclusions

If we start from three light neutrinos and the see-saw mechanism then a natural interpretation of the present data on neutrino oscillations is in terms of hierarchical light neutrino masses and asymmetric mass matrices (at least for d quarks and charged leptons). This has the advantage that no conspiracy is required between the Dirac and the Majorana sectors. There is also the peculiar possibility that large neutrino mixing is only produced by the see-saw mechanism starting from all nearly diagonal matrices. Although this possibility is certainly rather special, models of this sort can be constructed without an unrealistic amount of fine tuning. Both scenarios are well compatible with Abelian flavour symmetries and with grand unification ideas and the related phenomenology for quark and lepton masses.

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REFERENCES

[1] Y. Fukuda *et al.*, `hep-ex/9805006, hep-ex/9805021` and `hep-ex/9807003`.

[2] The data on solar neutrinos from Homestake, Gallex, Sage, Kamiokande and SuperKamiokande are summarized and the list of references given in J.N. Bahcall, P.I. Krastev and A.Yu Smirnov, `hep-ph/9807216`.

[3] C. Athanassopoulos *et al., Phys. Rev. Lett.* **77**, 3082 (1996); `nucl-ex/9706006, nucl-ex/9709006`.

[4] B. Armbruster *et al., Phys. Rev.* **C57**, 3414 (1998); G. Drexlin, talk at Wein'98.

[5] R. Barbieri, L.J. Hall, D. Smith, A. Strumia, N. Weiner, `hep/ph 9807235`.

[6] M. Apollonio *et al., Phys. Lett.* **B420**, 397 (1998).

[7] V. Barger, S. Pakvasa, T.J. Weiler, K. Whisnant, `hep-ph/9806387`; V. Barger, T.J. Weiler, K. Whisnant, `hep-ph/9807319`.

[8] S.M. Bilenky, C. Giunti, hep-ph/9802201; A.J. Baltz, A.S. Goldhaber, M. Goldhaber, hep-ph/9806540.

[9] L. Wolfenstein, *Phys. Rev.* **D17**, 2369 (1978); S.P. Mikheyev, A.Yu. Smirnov, *Sov. J. Nucl. Phys.* **42**, 913 (1986).

[10] M.C. Gonzalez-Garcia, H. Nunokawa, O.L.G. Peres, J.W.F. Valle, hep-ph/9807305; G.L. Fogli, E. Lisi, A. Marrone, G. Scioscia, hep-ph/9808205; S.M. Bilenkii, C. Giunti, W. Grimus, hep-ph/9809368.

[11] See for example L. K. Krauss, hep-ph/9807376.

[12] L. Baudis *et al., Phys. Lett.* **B407**, 219 (1997).

[13] F. Vissani, hep-ph/9708483; H. Georgi, S.L. Glashow, hep-ph/9808293.

[14] N. Haba, N. Okamura, M. Sugiura, hep-ph/9810471; J. Ellis, S. Lola, hep-ph/9904279; J.A. Casas, J.R. Espinosa, A. Ibarra, I. Navarro, hep-ph/9904395, hep-ph/9905381, hep-ph/9906281; N. Haba, N. Okamura, hep-ph/9906481; E. Ma, hep-ph/9907400.

[15] R. Barbieri, G.G. Ross, A. Strumia, hep-ph/9906470;

[16] A. Ioannisian, J.W.F. Valle, *Phys. Lett.* **B332**, 93 (1994), hep-ph/9402333; M. Fukugita, M. Tanimoto, T. Yanagida, *Phys. Rev.* **D57**, 4429 (1998), hep-ph/9709388; M. Tanimoto, hep-ph/9807283 and hep-ph/9807517; H. Fritzsch, Z. Xing, hep-ph/9808272; R.N. Mohapatra, S. Nussinov, hep-ph/9808301 and hep-ph/9809415; M. Fukugita, M. Tanimoto, T. Yanagida, hep-ph/9809554; Yue-Liang Wu, hep-ph/9810491; J.I. Silva-Marcos, hep-ph/9811381; C. Wetterich, hep-ph/9812426.

[17] G. Altarelli, F. Feruglio, *Phys. Lett.* **B439**, 112 (1998) [hep-ph/9807353]; *JHEP* **11**, 021 (1998) [hep-ph/9809596]; *Phys. Lett.* **B451**, 388 (1999) [hep-ph/9812475] ; hep-ph/9905536.

[18] A.Y. Smirnov, *Phys. Rev.* **D48**, 3264 (1993).

[19] Z.G. Berezhiani, A. Rossi, *Phys. Lett.* **B367**, 219 (1996).

[20] S.F. King, *Phys. Lett.* **B439**, 350 (1998), hep-ph/9806440; S.F. King, hep-ph/9904210; S. Davidson, S.F. King, *Phys. Lett.* **B445**, 191 (1998), hep-ph/9808333; E. Ma, D.P. Roy, *Phys. Rev.* **D59**, 097702 (1999).

[21] S. Lola, G.G. Ross, hep-ph/9902283; K. Babu, J. Pati, F. Wilczek, hep-ph/9912538.

[22] G. Altarelli, F. Feruglio, I. Masina, hep-ph/9907532.

[23] M. Jezabek, Y. Sumino, *Phys. Lett.* **B440**, 327 (1998), hep-ph/9807310; T. Blazek, S. Raby, K. Tobe, hep-ph/9903340; R. Barbieri, P. Creminelli, A. Romanino, hep-ph/9903460; M. Jezabek, Y. Sumino, hep-ph/9904382; B. Stech, hep-ph/9905440.

[24] G.G. Ross, *Grand Unified Theories*, Benjamin, 1985; R.N. Mohapatra, *Unification and Supersymmetry*, Springer-Verlag, 1986; C. Kounnas, A. Masiero, D.V. Nanopoulos, K.A. Olive, *Grand Unification with and without Supersymmetry and Cosmological Implications*, World Scientific, Singapore 1984.

[25] Z. Berezhiani, Z. Tavartkiladze, *Phys. Lett.* **B409**, 220 (1997); C.H. Albright, S.M. Barr, *Phys. Rev.* **D58**, 013002 (1998), hep-ph/9712488; C.H. Albright,

K.S. Babu, S.M. Barr, *Phys. Rev. Lett.* **81**, 1167 (1998), hep-ph/9802314; Z. Berezhiani, A. Rossi, hep-ph/9811447; K. Hagiwara, N. Okamura, hep-ph/9811495.

[26] H. Georgi, C. Jarlskog, *Phys. Lett.* **B86**, 297 (1979).

[27] J. Ellis, M.K. Gaillard, *Phys. Lett.* **B88**, 315 (1979).

[28] C. Froggatt, H.B. Nielsen, *Nucl. Phys.* **B147**, 277 (1979); J.K. Elwood, N. Irges, P. Ramond, hep-ph/9807228; P. Ramond, hep-ph/9808489, hep-ph/9809401; W. Buchmuller, T. Yanagida, hep-ph/9810308; P. Binetruy, S. Lavignac, S. Petcov, P. Ramond, *Nucl. Phys.* **B496**, 3 (1997), hep-ph/9610481; N. Irges, S. Lavignac, P. Ramond, *Phys. Rev.* **D58**, 5003 (1998), hep-ph/9802334; Y. Grossman, Y. Nir, Y. Shadmi, hep-ph/9808355; C.D. Froggart, M. Gibson, H.B. Nielsen, hep-ph/9811265; B. Stech, hep-ph/9905440.

[29] M. Leurer, Y. Nir, N. Seiberg, *Nucl. Phys.* **B420**, 468 (1994), hep-ph/9310320.

[30] M. Carena, S. Pokorski, C.E.M. Wagner, *Nucl. Phys.* **B406**, 59 (1993); P. Langacker, N. Polonsky, *Phys. Rev.* **D49**, 1451 (1194); N. Polonsky, *Phys. Rev.* **D54**, 4537 (1996).