NEUTRINO MIXING AND MAXIMAL CP VIOLATION*

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We propose a phenomenological model of lepton mixing and CP violation based on the flavor democracy of charge leptons and the mass degeneracy of neutrinos. A nearly bi-maximal flavor mixing pattern, which is favored by current data on atmospheric and solar neutrino oscillations, emerges naturally from this model after explicit symmetry breaking. The rephasing-invariant strength of CP or T violation can be as large as one percent, leading to significant probability asymmetries between $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (or $\nu_e \rightarrow \nu_\mu$) transitions in the long-baseline neutrino experiments. The possible matter effects on CP- and T-violating asymmetries are also taken into account.

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Since its foundation in the 1960's the standard electroweak model, which unifies the weak and electromagnetic interactions, has passed all experimental tests. Neither significant evidence for the departures from the standard model nor convincing hints for the presence of new physics has been found thus far at HERA, LEP, SLC, Tevatron and other high-energy facilities. In spite of the impressive success of the standard model, many physicists believe that it does not represent the final theory, but serves merely as an effective theory originating from a more fundamental, yet unknown theoretical framework. For instance there is little understanding, within the standard model, about the intrinsic physics of the electroweak symmetry breaking, the hierarchy of charged fermion mass spectra, the vanishing or smallness of neutrino masses, and the origin of flavor mixing and CP viola-

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tion. Any attempt towards gaining an insight into such problems inevitably requires significant steps to go beyond the standard model.

The investigations of fermion mass generation and flavor mixing problems, which constitute an important part of today's particle physics, can be traced back to the early 1970's, soon after the establishment of the standard electroweak model. Since then many different models or approaches have been developed [1]. From the theoretical point of view, however, our understanding of the fermion mass spectrum remains quite unsatisfactory. Before a significant breakthrough takes place on the theoretical side, the phenomenological approaches will remain to play a crucial role in interpreting new experimental data on quark mixing, CP violation, and neutrino oscillations. They are expected to provide useful hints towards discovering the full dynamics of fermion mass generation and CP violation.

In this talk we shall concentrate on neutrino oscillations, the leptonic counterparts of the flavor mixing phenomena in hadronic physics, and study in particular the interesting prospects of finding CP violation in neutrino oscillations.

The recent observation of atmospheric and solar neutrino anomalies, in particular by the Super-Kamiokande experiment [2], has provided a strong indication that neutrinos are massive and lepton flavors are mixed. As there exist at least three different lepton families, the flavor mixing matrix may in general contain non-trivial complex phase terms. Hence CP or T violation is naturally expected in the lepton sector.

A violation of CP invariance in the quark sector can result in a variety of observable effects in hadronic weak decays. Similarly CP or T violation in the lepton sector can manifest itself in neutrino oscillations [3]. The best and probably the only way to observe CP- or T-violating effects in neutrino oscillations is to carry out the long-baseline appearance neutrino experiments (see Ref. [4] for a list of extensive references).

In the scheme of three lepton families, the 3×3 flavor mixing matrix V links the neutrino mass eigenstates (ν_1, ν_2, ν_3) to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} . \tag{1}$$

If neutrinos are massive Dirac fermions, V can be parametrized in terms of three rotation angles and one CP-violating phase. If neutrinos are Majorana fermions, however, two additional CP-violating phases are in general needed to fully parametrize V. The strength of CP violation in neutrino oscillations, no matter whether neutrinos are of the Dirac or Majorana type, depends only upon a universal parameter $\mathcal{J}$, which is defined through

$$\mathrm{Im}\left(V_{il}V_{jm}V_{im}^{*}V_{jl}^{*}\right) \; = \; \mathcal{J}\sum_{k,n}\epsilon_{ijk}\epsilon_{lmn} \; . \tag{2}$$

The asymmetry between the probabilities of two CP-conjugate neutrino transitions, due to the CPT invariance and the unitarity of V, is uniquely given as

$$\begin{aligned} \Delta_{\mathrm{CP}} \; &= \; P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \\ &= \; -16\mathcal{J}\sin F_{12}\sin F_{23}\sin F_{31} \end{aligned} \tag{3}$$

with $(\alpha,\beta) = (e,\mu)$, (μ,τ) or (τ,e), $F_{ij} = 1.27\Delta m_{ij}^2 L/E$ and $\Delta m_{ij}^2 = m_i^2 - m_j^2$, in which L is the distance between the neutrino source and the detector (in unit of km) and E denotes the neutrino beam energy (in unit of GeV). The T-violating asymmetry can be obtained in a similar way [1]:

$$\begin{aligned} \Delta_{\mathrm{T}} \; &= \; P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha) \\ &= \; -16\mathcal{J}\sin F_{12}\sin F_{23}\sin F_{31} \; . \end{aligned} \tag{4}$$

These formulas show clearly that CP or T violation is a feature of all three lepton families. The relationship $\Delta_{\mathrm{T}} = \Delta_{\mathrm{CP}}$ is a straightforward consequence of CPT invariance. The observation of Δ_{T} might basically be free from the matter effects of the earth, which is possible to fake the genuine CP asymmetry Δ_{CP} in any long-baseline neutrino experiment. The joint measurement of $\nu_\alpha \to \nu_\beta$ and $\nu_\beta \to \nu_\alpha$ transitions to determine Δ_{T} is, however, a challenging task in practice. Probably it could only be realized in a neutrino factory, whereby high-quality neutrino beams can be produced with high-intensity muon storage rings [4].

Analyses of current experimental data [2,5] [2] yield $\Delta m_{\mathrm{sun}}^2 \ll \Delta m_{\mathrm{atm}}^2$ and $|V_{e3}|^2 \ll 1$, implying that the atmospheric and solar neutrino oscillations are approximately decoupled. A reasonable interpretation of those data follows from setting $\Delta m_{\mathrm{sun}}^2 = |\Delta m_{12}^2|$ and $\Delta m_{\mathrm{atm}}^2 = |\Delta m_{23}^2| \approx |\Delta m_{31}^2|$. In this approximation $F_{31} \approx -F_{23}$ holds. The CP- and T-violating asymmetries can then be simplified as

$$\Delta_{\mathrm{CP}} \; = \; \Delta_{\mathrm{T}} \; \approx \; 16\mathcal{J}\sin F_{12}\sin^2 F_{23} \; . \tag{5}$$

[1] Note that an asymmetry between the probabilities $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ and $P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$ signifies T violation too. This asymmetry and that defined in Eq. (4) have the same magnitude but opposite signs.

[2] Throughout this work we do not take the LSND evidence for neutrino oscillations [6], which was not confirmed by the KARMEN experiment [7], into account.

Note that $\Delta_{\rm CP}$ or $\Delta_{\rm T}$ depends linearly on the oscillating term $\sin F_{12}$, therefore the length of the baseline suitable for measuring CP and T asymmetries should satisfy the condition $|\Delta m^2_{12}| \sim E/L$. This requirement singles out the large-angle MSW solution, which has $\Delta m^2_{\rm sun} \sim 10^{-5}$ to 10^{-4} eV^2 and $\sin^2 2\theta_{\rm sun} \sim 0.65$ to 1 [8], among three possible solutions to the solar neutrino problem. The small-angle MSW solution is not favored; it does not give rise to a relatively large magnitude of $\mathcal{J}$, which determines the significance of practical CP- or T-violating signals. The long wave-length vacuum oscillation requires $\Delta m^2_{\rm sun} \sim 10^{-10}$ eV^2, too small to meet the realistic long-baseline prerequisite.

In this talk we extend our previous hypothesis of lepton flavor mixing [9], which arises naturally from the breaking of flavor democracy for charged leptons and that of mass degeneracy for neutrinos, to include CP violation [10]. It is found that the rephasing-invariant strength of CP or T violation can be as large as one percent. The flavor mixing pattern remains nearly bimaximal, thus both atmospheric and solar neutrino oscillations can well be interpreted. The consequences of the model on CP violation in the future long-baseline neutrino experiments will also be discussed by taking the matter effects into account.

The phenomenological constraints obtained from various neutrino oscillation experiments indicate that the mass differences in the neutrino sector are tiny compared to those in the charged lepton sector. One possible interpretation is that all three neutrinos are nearly degenerate in mass. In this case one might expect that the flavor mixing pattern of leptons differs qualitatively from that of quarks, where both up and down sectors exhibit a strong hierarchical structure in their mass spectra and the observed mixing angles are rather small. A number of authors have argued that the hierarchy of quark masses and the smallness of mixing angles are related to each other, by considering specific symmetry limits [11]. One particular way to proceed is to consider the limit of subnuclear democracy, in which the mass matrices of both the up- and down-type quarks are of rank one and have the structure

$$M_q \; = \; \frac{c_q}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{6}$$

with $q =$ u (up) or d (down) as well as $c_{\rm u} = m_t$ and $c_{\rm d} = m_b$. Small departures from the democratic limit lead to the flavor mixing and at the same time introduce the masses of the second and first families. Specific symmetry breaking schemes have been proposed in some literature in order to calculate the flavor mixing angles in terms of the quark mass eigenvalues (for a review, see Ref. [1]).

Since the charged leptons exhibit a similar hierarchical mass spectrum as the quarks, it would be natural to consider the limit of subnuclear democracy for the (e, μ, τ) system, *i.e.*, the mass matrix takes the form as Eq. (6). In the same limit three neutrinos are degenerate in mass. Then we have [9]

$$M_l^{(0)} = \frac{c_l}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} ,$$
$$M_\nu^{(0)} = c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{7}$$

where $c_l = m_\tau$ and $c_\nu = m_0$ measure the corresponding mass scales. If the three neutrinos are of the Majorana type, $M_\nu^{(0)}$ could take a more general form $M_\nu^{(0)} P_\nu$ with $P_\nu = \text{Diag}\{1, e^{i\phi_1}, e^{i\phi_2}\}$. As the Majorana phase matrix P_ν has no effect on the flavor mixing and CP-violating observables in neutrino oscillations, it will be neglected in the subsequent discussions. Clearly $M_\nu^{(0)}$ exhibits an S(3) symmetry, while $M_l^{(0)}$ an $\text{S}(3)_\text{L} \times \text{S}(3)_\text{R}$ symmetry.

One can transform the charged lepton mass matrix from the democratic basis $M_l^{(0)}$ into the hierarchical basis

$$M_l^{(\text{H})} = c_l \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{8}$$

by means of an orthogonal transformation, *i.e.*, $M_l^{(\text{H})} = U M_l^{(0)} U^\text{T}$, where

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} . \tag{9}$$

We see $m_e = m_\mu = 0$ from $M_l^{(\text{H})}$ and $m_1 = m_2 = m_3 = m_0$ from $M_\nu^{(0)}$. Of course there is no flavor mixing in this symmetry limit.

A simple real diagonal breaking of the flavor democracy for $M_l^{(0)}$ and the mass degeneracy for $M_\nu^{(0)}$ may lead to instructive results for flavor mixing in neutrino oscillations [9, 12]. To accommodate CP violation, however, complex perturbative terms are required. Let us proceed with two different symmetry-breaking steps in close analogy to the symmetry breaking discussed previously for the quark mass matrices [13, 14].

First, small real perturbations to the (3,3) elements of $M_l^{(0)}$ and $M_\nu^{(0)}$ are respectively introduced:

$$
\begin{aligned}
\Delta M_l^{(1)} &= \frac{c_l}{3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_l \end{pmatrix} , \\
\Delta M_\nu^{(1)} &= c_\nu \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_\nu \end{pmatrix} .
\end{aligned}
\qquad (10)
$$

In this case the charged lepton mass matrix $M_l^{(1)} = M_l^{(0)} + \Delta M_l^{(1)}$ remains symmetric under an $S(2)_{\rm L} \times S(2)_{\rm R}$ transformation, and the neutrino mass matrix $M_\nu^{(1)} = M_\nu^{(0)} + \Delta M_\nu^{(0)}$ has an $S(2)$ symmetry. The muon becomes massive (*i.e.*, $m_\mu \approx 2|\varepsilon_l| m_\tau/9$), and the mass eigenvalue m_3 is no more degenerate with m_1 and m_2 (*i.e.*, $|m_3 - m_0| = m_0|\varepsilon_\nu|$). After the diagonalization of $M_l^{(1)}$ and $M_\nu^{(1)}$, one finds that the 2nd and 3rd lepton families have a definite flavor mixing angle θ. We obtain $\tan\theta \approx -\sqrt{2}$ if the small correction of $\mathcal{O}(m_\mu/m_\tau)$ is neglected. Then neutrino oscillations at the atmospheric scale may arise in $\nu_\mu \to \nu_\tau$ transitions with $\Delta m^2_{32} = \Delta m^2_{31} \approx 2m_0|\varepsilon_\nu|$. The corresponding mixing factor $\sin^2 2\theta \approx 8/9$ is in good agreement with current data.

The symmetry breaking given in Eq. (10) for the charged lepton mass matrix serves as a good illustrative example. One could consider a more general case, analogous to the one for quarks [13], to break the $\mathrm{S}(3)_{\rm L} \times \mathrm{S}(3)_{\rm R}$ symmetry of $M_l^{(0)}$ to an $\mathrm{S}(2)_{\rm L} \times \mathrm{S}(2)_{\rm R}$ symmetry. This would imply that $\Delta M_l^{(1)}$ takes the form

$$
\Delta M_l^{(1)} = \frac{c_l}{3} \begin{pmatrix} 0 & 0 & \varepsilon_l' \\ 0 & 0 & \varepsilon_l' \\ \varepsilon_l' & \varepsilon_l' & \varepsilon_l \end{pmatrix} , \qquad (11)
$$

where $|\varepsilon_l| \ll 1$ and $|\varepsilon_l'| \ll 1$. In this case the leading-order results obtained above, *i.e.*, $\tan\theta \approx -\sqrt{2}$ and $\sin^2 2\theta \approx 8/9$, remain unchanged.

At the next step we introduce a complex symmetry breaking perturbation, analogous to that for quark mass matrices discussed in Ref. [15], to the charged lepton mass matrix $M_l^{(1)}$:

$$
\Delta M_l^{(2)} = \frac{c_l}{3} \begin{pmatrix} 0 & -i\delta_l & i\delta \\ i\delta & 0 & -i\delta_l \\ -i\delta_l & i\delta_l & 0 \end{pmatrix} . \qquad (12)
$$

Transforming $M_l^{(2)} = M_l^{(1)} + \Delta M_l^{(2)}$ into the hierarchical basis, we obtain

$$M_l^{\rm H} \;=\; c_l \begin{pmatrix} 0 & -i\frac{1}{\sqrt{3}}\delta_l & 0 \\ i\frac{1}{\sqrt{3}}\delta_l & \frac{2}{9}\varepsilon_l & -\frac{\sqrt{2}}{9}\varepsilon_l \\ 0 & -\frac{\sqrt{2}}{9}\varepsilon_l & 1+\frac{1}{9}\varepsilon_l \end{pmatrix} . \tag{13}$$

Note that $M_l^{\rm H}$, just like a variety of realistic quark mass matrices [11], has texture zeros in the (1,1), (1,3) and (3,1) positions. The phases of its (1,2) and (2,1) elements are $\mp 90°$, which could lead to maximal CP violation if the neutrino mass matrix is essentially real. For the latter we consider a small perturbation, analogous to that in Eq. (10), to break the remaining mass degeneracy of $M_\nu^{(1)}$:

$$\Delta M_\nu^{(2)} \;=\; c_\nu \begin{pmatrix} -\delta_\nu & 0 & 0 \\ 0 & \delta_\nu & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{14}$$

From $\Delta M_l^{(2)}$ and $\Delta M_\nu^{(2)}$ we obtain $m_e \approx |\delta_l|^2 m_\tau^2/(27 m_\mu)$ and $m_{1,2} = m_0(1 \mp \delta_\nu)$, respectively. The simultaneous diagonalization of $M_l^{(2)} = M_l^{(1)} + \Delta M_l^{(2)}$ and $M_\nu^{(2)} = M_\nu^{(1)} + \Delta M_\nu^{(2)}$ leads to a full 3×3 flavor mixing matrix, which links neutrino mass eigenstates (ν_1, ν_2, ν_3) to neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ in the following manner [10]:

$$V \;=\; U \;+\; i\;\xi_V \sqrt{\frac{m_e}{m_\mu}} \;+\; \zeta_V \frac{m_\mu}{m_\tau} \; , \tag{15}$$

where U has been given in Eq. (9), and

$$\xi_V \;=\; \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \end{pmatrix} ,$$

$$\zeta_V \;=\; \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{-1}{\sqrt{12}} & \frac{-1}{\sqrt{12}} & \frac{1}{\sqrt{3}} \end{pmatrix} . \tag{16}$$

In comparison with the result of Ref. [9], the new feature of this lepton mixing scenario is that the term multiplying ξ_V becomes imaginary. Therefore CP or T violation has been incorporated.

The complex symmetry breaking perturbation given in Eq. (12) is certainly not the only one which can be considered for $M_l^{(1)}$. Below we list a number of other interesting possibilities, *i.e.*, the Hermitian perturbations

$$\Delta \tilde{M}_l^{(2)} = \frac{c_l}{3} \begin{pmatrix} 0 & -i\delta_l & 0 \\ i\delta_l & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ,$$
$$\Delta \hat{M}_l^{(2)} = \frac{c_l}{3} \begin{pmatrix} 0 & 0 & i\delta_l \\ 0 & 0 & -i\delta_l \\ -i\delta_l & i\delta_l & 0 \end{pmatrix} ; \quad (17)$$

and the non-hermitian perturbations

$$\Delta \boldsymbol{M}_l^{(2)} = \frac{c_l}{3} \begin{pmatrix} -i\delta_l & 0 & i\delta_l \\ 0 & i\delta & -i\delta_l \\ i\delta_l & -i\delta_l & 0 \end{pmatrix} ,$$
$$\Delta \tilde{\boldsymbol{M}}_l^{(2)} = \frac{c_l}{3} \begin{pmatrix} -i\delta_l & 0 & 0 \\ 0 & i\delta_l & 0 \\ 0 & 0 & 0 \end{pmatrix} ,$$
$$\Delta \hat{\boldsymbol{M}}_l^{(2)} = \frac{c_l}{3} \begin{pmatrix} 0 & 0 & i\delta_l \\ 0 & 0 & -i\delta_l \\ i\delta_l & -i\delta_l & 0 \end{pmatrix} . \quad (18)$$

The three hermitian and three non-hermitian perturbative mass matrices obey the following sum rules:

$$\Delta M_l^{(2)} = \Delta \tilde{M}_l^{(2)} + \Delta \hat{M}_l^{(2)} ,$$
$$\Delta \boldsymbol{M}_l^{(2)} = \Delta \tilde{\boldsymbol{M}}_l^{(2)} + \Delta \hat{\boldsymbol{M}}_l^{(2)} . \quad (19)$$

Let us remark that hermitian perturbations of the same forms as given in Eqs. (12) and (17) have been used to break the flavor democracy of quark mass matrices and to generate CP violation [13, 15]. The key point of this similarity between the charged lepton and quark mass matrices is that both of them have the strong mass hierarchy and might have the same dynamical origin or a symmetry relationship.

To be more specific we transform all the six charged lepton mass matrices

$$M_l^{(2)} = M_l^{(0)} + \Delta M_l^{(1)} + \Delta M_l^{(2)} ,$$
$$\tilde{M}_l^{(2)} = M_l^{(0)} + \Delta M_l^{(1)} + \Delta \tilde{M}_l^{(2)} ,$$
$$\hat{M}_l^{(2)} = M_l^{(0)} + \Delta M_l^{(1)} + \Delta \hat{M}_l^{(2)} \quad (20)$$

and

$$\boldsymbol{M}_l^{(2)} = M_l^{(0)} + \Delta M_l^{(1)} + \Delta \boldsymbol{M}_l^{(2)} ,$$

$$
\begin{aligned}
\tilde{\boldsymbol{M}}_l^{(2)} &= M_l^{(0)} + \Delta M_l^{(1)} + \Delta\tilde{\boldsymbol{M}}_l^{(2)} \;, \\
\hat{\boldsymbol{M}}_l^{(2)} &= M_l^{(0)} + \Delta M_l^{(1)} + \Delta\hat{\boldsymbol{M}}_l^{(2)}
\end{aligned} \tag{21}
$$

into their counterparts in the hierarchical basis and list the results in Table I. A common feature of these hierarchical mass matrices is that their (1,1) elements all vanish. For this reason the CP-violating effects, resulted from the hermitian perturbations $\Delta M_l^{(2)}$, $\Delta\tilde{M}_l^{(2)}$, $\Delta\hat{M}_l^{(2)}$ and the non-hermitian perturbations $\Delta\boldsymbol{M}_l^{(2)}$, $\Delta\tilde{\boldsymbol{M}}_l^{(2)}$, $\Delta\hat{\boldsymbol{M}}_l^{(2)}$ respectively, are approximately independent of other details of the flavor symmetry breaking and have the identical strength to a high degree of accuracy. Indeed it is easy to check that all the six charged lepton mass matrices in Eqs. (20) and (21), together with the neutrino mass matrix $M_\nu^{(2)} = M_\nu^{(0)} + \Delta M_\nu^{(1)} + \Delta M_\nu^{(2)}$, lead to the same flavor mixing pattern V as given in Eq. (15). Hence it is in practice difficult to distinguish one scenario from another. In our point of view, the similarity between $M_l^{(2)}$ and its quark counterpart [11, 15] could provide us a useful hint towards an underlying symmetry between quarks and charged leptons. One may also argue that the simplicity of $\tilde{\boldsymbol{M}}_l^{(2)}$ and its parallelism with $M_\nu^{(2)}$ might make it technically more natural to be derived from a yet unknown fundamental theory of lepton mixing and CP violation.

The flavor mixing matrix V can in general be parametrized in terms of three Euler angles and one CP-violating phase [3]. A suitable parametrization reads as follows [16]:

$$
\begin{aligned}
V &= \begin{pmatrix} c_l & s_l & 0 \\ -s_l & c_l & 0 \\ 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}
\begin{pmatrix} c_\nu & -s_\nu & 0 \\ s_\nu & c_\nu & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} s_l s_\nu c + c_l c_\nu e^{-i\phi} & s_l c_\nu c - c_l s_\nu e^{-i\phi} & s_l s \\ c_l s_\nu c - s_l c_\nu e^{-i\phi} & c_l c_\nu c + s_l s_\nu e^{-i\phi} & c_l s \\ -s_\nu s & -c_\nu s & c \end{pmatrix} ,
\end{aligned} \tag{22}
$$

in which $s_l \equiv \sin\theta_l$, $s_\nu \equiv \sin\theta_\nu$, $c \equiv \cos\theta$, etc. The three mixing angles can all be arranged to lie in the first quadrant, while the CP-violating phase may take values between 0 and 2π. It is straightforward to obtain $\mathcal{J} = s_l c_l s_\nu c_\nu s^2 c \sin\phi$. Numerically we find

$$
\theta_l \approx 4^\circ \;, \qquad \theta_\nu \approx 45^\circ \;, \qquad \theta \approx 52^\circ \;, \qquad \phi \approx 90^\circ \tag{23}
$$

[3] For neutrinos of the Majorana type, two additional CP-violating phases may enter. But they are irrelevant to neutrino oscillations and can be neglected for our present purpose.

TABLE I

Counterparts of six charged lepton mass matrices in the hierarchical basis

$M_l^{(2)}/c_l$	$\boldsymbol{M}_l^{(2)}/c_l$
$\begin{pmatrix} 0 & -i\frac{1}{\sqrt{3}}\delta_l & 0 \\ i\frac{1}{\sqrt{3}}\delta_l & \frac{2}{9}\varepsilon_l & -\frac{\sqrt{2}}{9}\varepsilon_l \\ 0 & -\frac{\sqrt{2}}{9}\varepsilon_l & 1+\frac{1}{9}\varepsilon_l \end{pmatrix}$	$\begin{pmatrix} 0 & -i\frac{1}{\sqrt{3}}\delta_l & 0 \\ -i\frac{1}{\sqrt{3}}\delta_l & \frac{2}{9}\varepsilon_l & -\frac{\sqrt{2}}{9}\varepsilon_l \\ 0 & -\frac{\sqrt{2}}{9}\varepsilon_l & 1+\frac{1}{9}\varepsilon_l \end{pmatrix}$
$\tilde{M}_l^{(2)}/c_l$	$\tilde{\boldsymbol{M}}_l^{(2)}/c_l$
$\begin{pmatrix} 0 & -i\frac{\sqrt{3}}{9}\delta_l & -i\frac{\sqrt{6}}{9}\delta_l \\ i\frac{\sqrt{3}}{9}\delta_l & \frac{2}{9}\varepsilon_l & -\frac{\sqrt{2}}{9}\varepsilon_l \\ i\frac{\sqrt{6}}{9}\delta_l & -\frac{\sqrt{2}}{9}\varepsilon_l & 1+\frac{1}{9}\varepsilon_l \end{pmatrix}$	$\begin{pmatrix} 0 & -i\frac{\sqrt{3}}{9}\delta_l & -i\frac{\sqrt{6}}{9}\delta_l \\ -i\frac{\sqrt{3}}{9}\delta_l & \frac{2}{9}\varepsilon_l & -\frac{\sqrt{2}}{9}\varepsilon_l \\ -i\frac{\sqrt{6}}{9}\delta_l & -\frac{\sqrt{2}}{9}\varepsilon_l & 1+\frac{1}{9}\varepsilon_l \end{pmatrix}$
$\hat{M}_l^{(2)}/c_l$	$\hat{\boldsymbol{M}}_l^{(2)}/c_l$
$\begin{pmatrix} 0 & -i\frac{2\sqrt{3}}{9}\delta_l & i\frac{\sqrt{6}}{9}\delta_l \\ i\frac{2\sqrt{3}}{9}\delta_l & \frac{2}{9}\varepsilon_l & -\frac{\sqrt{2}}{9}\varepsilon_l \\ -i\frac{\sqrt{6}}{9}\delta_l & -\frac{\sqrt{2}}{9}\varepsilon_l & 1+\frac{1}{9}\varepsilon_l \end{pmatrix}$	$\begin{pmatrix} 0 & -i\frac{2\sqrt{3}}{9}\delta_l & i\frac{\sqrt{6}}{9}\delta_l \\ -i\frac{2\sqrt{3}}{9}\delta_l & \frac{2}{9}\varepsilon_l & -\frac{\sqrt{2}}{9}\varepsilon_l \\ i\frac{\sqrt{6}}{9}\delta_l & -\frac{\sqrt{2}}{9}\varepsilon_l & 1+\frac{1}{9}\varepsilon_l \end{pmatrix}$

from Eq. (15). The smallness of θ_l is a natural consequence of the mass hierarchy in the charged lepton sector, just as the smallness of $\theta_{\rm u}$ in quark mixing [11]. On the other hand, both θ_ν and θ are too large to be comparable with the corresponding quark mixing angles (*i.e.*, $\theta_{\rm d}$ and θ as defined in Ref. [11]), reflecting the qualitative difference between quark and lepton flavor mixing phenomena. It is worth emphasizing that the leptonic CP-violating phase ϕ takes a special value ($\approx 90^\circ$) in our model. The same possibility is also favored for the quark mixing phenomenon in a variety of realistic mass matrices [17]. Therefore maximal leptonic CP violation, in the sense that the magnitude of $\mathcal{J}$ is maximal for the fixed values of three flavor mixing angles, appears naturally as in the quark sector.

Some consequences of this lepton mixing scenario can be drawn as follows:

(1) The mixing pattern in Eq. (15), after neglecting small corrections from the charged lepton masses, is quite similar to that of the pseudoscalar mesons π^0, η and η' in QCD in the limit of the chiral ${\rm SU(3)_L} \times {\rm SU(3)_R}$

symmetry [14, 18]:

$$\begin{aligned} \pi^0 &= \frac{1}{\sqrt{2}}\left(|\bar{u}u\rangle - |\bar{d}d\rangle\right) , \\ \eta &= \frac{1}{\sqrt{6}}\left(|\bar{u}u\rangle + |\bar{d}d\rangle - 2|\bar{s}s\rangle\right) , \\ \eta' &= \frac{1}{\sqrt{3}}\left(|\bar{u}u\rangle + |\bar{d}d\rangle + |\bar{s}s\rangle\right) . \end{aligned} \tag{24}$$

Some preliminary theoretical attempts towards deriving the flavor mixing matrix $V \approx U$ have been reviewed in Ref. [1].

(2) The V_{e3} element, of magnitude

$$|V_{e3}| = \frac{2}{\sqrt{6}}\sqrt{\frac{m_e}{m_\mu}} , \tag{25}$$

is naturally suppressed in the symmetry breaking scheme outlined above. A similar feature appears in the 3×3 quark flavor mixing matrix, *i.e.*, $|V_{ub}|$ is the smallest among the nine quark mixing elements. Indeed the smallness of V_{e3} provides a necessary condition for the decoupling of solar and atmospheric neutrino oscillations, even though neutrino masses are nearly degenerate. The effect of small but nonvanishing V_{e3} will manifest itself in long-baseline $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\tau$ transitions, as already shown in Ref. [9].

(3) The flavor mixing between the 1st and 2nd lepton families and that between the 2nd and 3rd lepton families are nearly maximal. This property, together with the natural smallness of $|V_{e3}|$, allows a satisfactory interpretation of the observed large mixing in atmospheric and solar neutrino oscillations. We obtain [4]

$$\begin{aligned} \sin^2 2\theta_{\rm sun} &= 1 - \frac{4}{3}\frac{m_e}{m_\mu} , \\ \sin^2 2\theta_{\rm atm} &= \frac{8}{9} + \frac{8}{9}\frac{m_\mu}{m_\tau} \end{aligned} \tag{26}$$

to a high degree of accuracy. Explicitly $\sin^2 2\theta_{\rm sun} \approx 0.99$ and $\sin^2 2\theta_{\rm atm} \approx 0.94$, favored by current data [2]. It is obvious that the model is fully consistent with the vacuum oscillation solution to the solar neutrino problem and might also be able to incorporate the large-angle MSW solution [5].

[4] In calculating $\sin^2 2\theta_{\rm sun}$ we have taken the $\mathcal{O}(m_e/m_\mu)$ correction to the expression of V into account.

[5] A slightly different symmetry-breaking pattern of the neutrino mass matrix [19], which involves four free parameters, allows the magnitude of $\sin^2 2\theta_{\rm sun}$ to be smaller and also consistent with the large-angle MSW solution.

(4) It is worth remarking that our lepton mixing pattern has no conflict with current constraints on the neutrinoless double beta decay [20], if neutrinos are of the Majorana type. In the presence of CP violation, the effective mass term of the $(\beta\beta)_{0\nu}$ decay can be written as

$$\langle M \rangle_{(\beta\beta)_{0\nu}} \; = \; \sum_{i=1}^{3} \left(m_i \, \tilde{V}_{ei}^2 \right) \; , \tag{27}$$

where $\tilde{V} = V P_\nu$ and $P_\nu = {\rm Diag}\{1, e^{i\phi_1}, e^{i\phi_2}\}$ is the Majorana phase matrix. If the unknown phases are taken to be $\phi_1 = \phi_2 = 90^\circ$ for example, then one arrives at

$$\left| \langle M \rangle_{(\beta\beta)_{0\nu}} \right| \; = \; \frac{2}{\sqrt{3}} \sqrt{\frac{m_e}{m_\mu}} \; m_i \; , \tag{28}$$

in which $m_i \sim 1 - 2$ eV (for $i = 1, 2, 3$) as required by the near degeneracy of three neutrinos in our model to accommodate the hot dark matter of the universe. Obviously $|\langle M \rangle_{(\beta\beta)_{0\nu}}| \approx 0.08 m_i \leq 0.2$ eV, the latest bound of the $(\beta\beta)_{0\nu}$ decay [20].

(5) The rephasing-invariant strength of CP violation in this scheme is given as [10]

$$\mathcal{J} \; = \; \frac{1}{3\sqrt{3}} \sqrt{\frac{m_e}{m_\mu}} \left(1 + \frac{1}{2} \frac{m_\mu}{m_\tau} \right) \; . \tag{29}$$

Explicitly we have $\mathcal{J} \approx 1.4\%$. The large magnitude of $\mathcal{J}$ for lepton mixing turns out to be very non-trivial, as the same quantity for quark mixing is only of order 10^{-5} [11,17]. If the mixing pattern under discussion were in no conflict with the large-angle MSW solution to the solar neutrino problem, then the CP- and T-violating signals $\Delta_{\rm CP} = \Delta_{\rm T} \propto -16\mathcal{J} \approx -0.2$ could be significant enough to be measured from the asymmetry between $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ or that between $P(\nu_\mu \rightarrow \nu_e)$ and $P(\nu_e \rightarrow \nu_\mu)$ in the long-baseline neutrino experiments. In the leading-order approximation we arrive at

$$\begin{aligned} \mathcal{A} \; &= \; \frac{P(\nu_\mu \rightarrow \nu_e) \; - \; P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}{P(\nu_\mu \rightarrow \nu_e) \; + \; P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)} \\ &= \; \frac{P(\nu_\mu \rightarrow \nu_e) \; - \; P(\nu_e \rightarrow \nu_\mu)}{P(\nu_\mu \rightarrow \nu_e) \; + \; P(\nu_e \rightarrow \nu_\mu)} \\ &= \; \frac{-\dfrac{8}{\sqrt{3}} \sqrt{\dfrac{m_e}{m_\mu}}}{\dfrac{16}{3} \dfrac{m_e}{m_\mu} + \left(\dfrac{\sin F_{12}}{\sin F_{23}} \right)^2} \; \sin F_{12} \; . \end{aligned} \tag{30}$$

The asymmetry $\mathcal{A}$ depends linearly on the oscillating term $\sin F_{12}$, which is associated essentially with the solar neutrino anomaly.

Note that $\mathcal{A}$ signals CP or T violation solely in vacuum. For most of the proposed long-baseline neutrino experiments the earth-induced matter effects on neutrino oscillations are non-negligible and should be carefully handled. In matter the effective Hamiltonians of neutrinos and antineutrinos can be written as [21]

$$
\begin{aligned}
\mathcal{H}_\nu &= \frac{1}{2E}\left[V\begin{pmatrix} m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix}V^\dagger + A\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right] \;, \\
\mathcal{H}_{\bar\nu} &= \frac{1}{2E}\left[V^*\begin{pmatrix} m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix}V^{\rm T} - A\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right] \;, \qquad (31)
\end{aligned}
$$

where $\pm A$ describes the charged-current contribution to the $\nu_e e^-$ or $\bar\nu_e e^+$ forward scattering. $A = 2\sqrt{2}G_{\rm F}N_e E$, in which N_e is the background density of electrons and E stands for the neutrino beam energy. The neutral-current contributions are universal for ν_e, ν_μ and ν_τ neutrinos, leading only to an overall unobservable phase and have been neglected. With the help of $\mathcal{H}_\nu$ and $\mathcal{H}_{\bar\nu}$ one can calculate the effective neutrino mass eigenvalues and the effective flavor mixing matrix in matter. The exact analytical results have been given in Ref. [22].

For simplicity we only present the numerical results of the matter-corrected CP- and T-violating asymmetries in the assumption of the baseline length $L = 732$ km, *i.e.*, a neutrino source at Fermilab pointing toward the Soudan mine in Minnesota or that at CERN toward the Gran Sasso underground laboratory in Italy. The inputs include the flavor mixing and CP-violating parameters obtained in Eq. (15) as well as the typical neutrino mass-squared differences $\Delta m^2_{21} = 5\cdot 10^{-5}$ eV^2 and $\Delta m^2_{32} = 3\cdot 10^{-3}$ eV^2. Assuming a constant earth density profile, one has $A \approx 2.28\cdot 10^{-4}$ eV$^2 E/[\mathrm{GeV}]$ [23]. The behaviors of the CP and T asymmetries changing with the beam energy E in the range 3 GeV $\leq E \leq$ 20 GeV are shown in Figs. 1 and 2, respectively. Clearly the vacuum asymmetry $\mathcal{A}$ can be of $\mathcal{O}(0.1)$. The matter-induced correction to the T-violating asymmetry is negligibly small for the experimental conditions under consideration. In contrast, the matter effect on the CP-violating asymmetry cannot be neglected, although it is unable to fake the genuine CP-violating signal.

If the upcoming data appeared to rule out the consistency between our model and the large-angle MSW solution to the solar neutrino problem, then it would be quite difficult to test the model itself from its prediction for large CP and T asymmetries in any realistic long-baseline experiment.

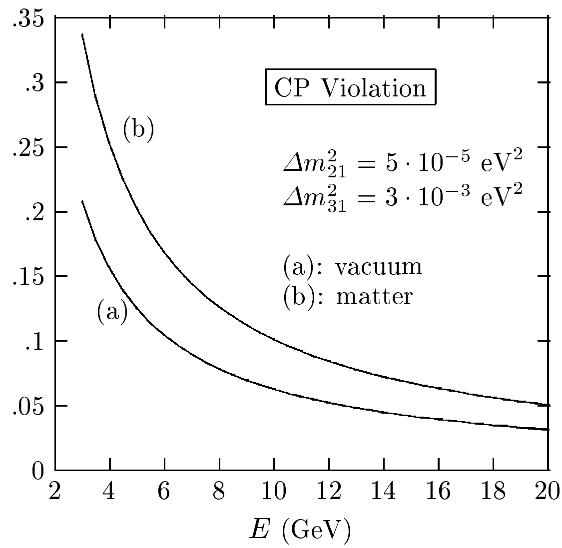

Fig. 1. Illustrative plot for the CP-violating asymmetries between $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions, in vacuum and in matter, changing with the neutrino beam energy E.

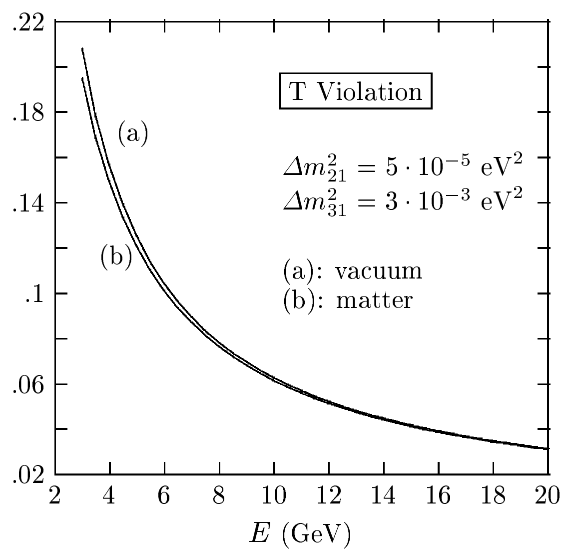

Fig. 2. Illustrative plot for the T-violating asymmetries between $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$ transitions, in vacuum and in matter, changing with the neutrino beam energy E.

In summary, we have extended our previous model of the nearly bi-maximal lepton flavor mixing to incorporate large CP violation. The new model remains favored by current data on atmospheric and solar neutrino oscillations, and it predicts significant CP- and T-violating effects in the long-baseline neutrino experiments. We expect that more data from the Super-Kamiokande and other neutrino experiments could soon provide stringent tests of the existing lepton mixing models and give useful hints towards the symmetry or dynamics of lepton mass generation.

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