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NEUTRINOS AND SOLAR MODELS*

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After summarizing principles of solar model construction and presenting an updated prediction for the neutrino counting rates, I focus this review on the question of reliability of current models. Methods and results of seismic sounding of the solar interior are presented in some detail. The results confirm the standard scenario of the solar evolution. This conclusion, combined with the evidences for neutrino oscillations, means the end of astrophysical aspect of the solar neutrino problem. The models of the Sun interior remain important for interpretation of the data from the neutrino detectors but the data cannot be used to contradict the models, not even to constrain them.

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1. Introduction

Motivation for measuring solar neutrino flux came from the *solar photon problem*, which has been realized in the early 1920's, after the first dating of the terrestrial rocks. The measured age of some 1.5 Gy implied that the gravitational and thermal energy content in the Sun accounted for only few percent of the photon energy emitted during the earth life time. Already in the first monograph on star internal structure sir Arthur Eddington [1] put forward the hypothesis that the energy is liberated in the building up helium nucleus from four protons as the most plausible solution. By the early 1940's, mostly thanks to works of Hans Bethe [2] and his collaborators, the main fusion reactions leading to formation of helium nuclei have been known. The whole physics essential for constructing models of the Sun and other main sequence stars become available. Measuring the solar neutrino flux was conceived [3] as a crucial test of the stellar nucleosynthesis theory.

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The theory did not pass the test. The *solar neutrino problem* was born to became one of the longest and most widely publicized challenges to the theory of stellar interior structure.

In 1969, shortly after the neutrino deficit has been established, Gribov and Pontecorvo [4] proposed the neutrino oscillations as an explanation of the discrepancy between the prediction and measurements. This time, however, the early good guess was pursuit by many. During subsequent years the main efforts in resolving the discrepancy focused on revising predictions from solar models. Bahcall in *Neutrino Astrophysics* [5] gives a comprehensive survey of the works done in this field up to 1989. He lists, in particular, various *nonstandard solar models* which yielded neutrino rates consistent with observation but were not constructed according to the standard stellar evolution theory.

Fortunately, the failure to explain the measured neutrino flux did not stop the development of the theory of stellar evolution. Soon after Bahcall's monograph has been published, the crucial pieces of evidence regarding nature of the solution to the solar neutrino problem became available. We now have a strong arguments that the stellar evolution theory, in its standard version, describes the Sun's interior with a very high precision.

This main message of this review is that the solar model is the most reliable ingredient in interpretation of the experimental data. In greater detail I will discuss this message in the last section. Before, in Section 2, I summarize methodology of solar model construction and predictions of neutrino fluxes for various detectors, which are now in operation. In Section 3, I explain principles of seismic sounding of the Sun's interior and present the results.

2. Neutrino fluxes from standard solar models with the standard electro-weak interaction theory

Calculations of the ν_e production rate in the Sun are based on two, largely independent, ingredients, the solar model and nuclear reaction cross-sections. The model yields thermodynamical parameters and chemical element abundances as functions of distance from the center of the Sun. A significant contribution to photon and neutrino fluxes arises only in the inner part of the model, called the core. The 99 percent of the photon flux is produced in the region extending to 25 percent of the solar radius and encompassing about 50 percent of the solar mass. Production of high energy neutrinos is confined to a still smaller central region of the Sun. Only some of the cross-sections are important for calculating models and the their uncertainty is only weakly reflected in the model. On the other hand, for calculating the neutrino fluxes, accurate the nuclear data are essential. Accordingly, the *astrophysical* and *nuclear physics* aspects of the problem may be regarded as separate.

2.1. Standard solar model

The principles of constructing the standard solar model (SSM) has not changed since 1963, when it was introduced [6] for the evaluation the expected neutrino capture rate for the chlorine detector. The model was calculated with the use of the most advanced stellar physics of the time and the solar data from observations.

The main assumptions behind the construction of SSM are those of the standard stellar evolution theory. These assumptions are

(a) hydrostatic equilibrium with only gravity and pressure forces included,
(b) mass conservation,
(c) complete mixing of chemical elements within zones unstable to convection and no mixing outside them.

In the solar case (a) yields an excellent approximation as confirmed by means of helioseismic sounding. However, in the early years of the solar neutrino problem, models with rapidly rotating and/or strongly magnetized core were contemplated. Departures from (b) due to the mass loss from the Sun at the current rate are totally negligible. The effects of much more intense mass loss in the young Sun are largely erased in the subsequent evolution. Questioning (c) used to be the most popular proposal for astrophysical solution of the neutrino problem. The results of helioseismic sounding, which I discuss in Section 3.3, rule out the mixing element in the core on the scale leading to a significant change in the neutrino flux prediction. However, the same results indicate that some mixing beneath the solar convective layer took place. Related to element mixing is the effect of gravitational settling of chemical elements heavier than hydrogen. This subtle effect, which is a relatively recent innovation in SSM [7], has a noticeable impact on the predicted neutrino counting rates.

Construction of stellar models requires microscopic physical data on nuclear cross-section, equation-of-state, photon absorption and scattering (opacity), and diffusion. The improvement in accuracy of these data continues. The most important recent revision concerned the opacity data [8]. It had some significance for the neutrino flux prediction. There has been a recent effort in assessing precision of the nuclear reaction data [9].

The observational data on the Sun used for constructing its model are mass, radius, total photon flux, the heavy elements to hydrogen abundance ratio in the atmosphere, and the age which is assumed equal to the age of the oldest meteorites. Of these parameters, the most uncertain are the element abundances and only this uncertainty really matters for the calculated neutrino fluxes. The atmospheric helium abundance cannot be determined accurately enough by means of spectroscopy. Hence it is an output rather than an input parameter of the SSM.

2.2. Calculated neutrino fluxes

In Table I, I quote after Bahcall *et al.* [10] the neutrino fluxes from individual reactions in the solar core as well as the total counting rates for the chlorine and gallium detectors. The 1σ ranges reflect uncertainties from all sources combined quadratically. The nuclear reaction data contribute about 80 percent of the uncertainties listed in the table. Only the remaining small part is attributed to SSM.

TABLE I

Calculated neutrino from SSM [10]

Reaction (or decay)	E_ν [MeV]	Flux [10^{10} cm^{-}2s^{-1}]	Cl [SNU]	Ga [SNU]
$^1\mathrm{H} + {}^1\mathrm{H}$	≤ 0.42	$5.94 \times (1.0^{+0.01}_{-0.01})$	0.0	69.6
$^1\mathrm{H} + {}^1\mathrm{H} + e^-$	1.42	$1.39 \times 10^{-2} \times (1.0^{+0.01}_{-0.01})$	0.2	2.8
$^7\mathrm{Be} + e^-$	0.86 or (10%) 0.38	$5.15 \times 10^{-4} \times (1.0^{+0.09}_{-0.09})$	5.9	12.4
$^8\mathrm{B}$	≤ 14.02	$4.80 \times 10^{-1} \times (1.0^{+0.19}_{-0.14})$	1.15	34.4
$^3\mathrm{He} + {}^1\mathrm{H}$	≤ 18.8	2.1×10^{-2}	0.0	0.0
$^{13}\mathrm{N}$	≤ 1.20	$6.05 \times 10^{-2} \times (1.0^{+0.19}_{-0.13})$	0.1	3.5
$^{15}\mathrm{O}$	≤ 1.73	$5.32 \times 10^{-1} \times (1.0^{+0.22}_{-0.15})$	0.4	6.0
$^{17}\mathrm{F}$	≤ 1.74	$6.33 \times 10^{-4} \times (1.0^{+0.12}_{-0.11})$	0.0	0.1
Total			$7.7^{+1.2}_{-1.0}$	129^{+8}_{-6}

This is not really a new situation. In fact, improvements in solar models played a relatively small role in the evolution of the calculated neutrino capture rates. In figures 2.1 and 10.1 of his monograph, Bahcall [5] plots as a function of time the capture rates for the Homestake detector from standard solar models calculated by him with various collaborators. The plot covers the 1963–1988 interval. The extension to 1998 is plotted in his recent paper [11]. Only in the first five years do we see large variations but they are not due to the changes in solar models. The factor five decrease between 1964 and 1968 is primarily due to a decrease of the $^3\mathrm{He} + {}^3\mathrm{He}$ cross-section, S_{33}. This reaction competes with $^3\mathrm{He} + {}^4\mathrm{He}$, which ultimately leads to the production of the neutrinos detectable at Homestake. A possibility that the cross-section for the former reaction could be reduced even further due to a hypothetical low-energy resonance has been considered as possible nuclear physics solution of the solar neutrino problem [12], but the measurements [13] put the end to this possibility.

Still, the values of S_{33} and S_{34} remain important contributors to the uncertainty of the calculated fluxes. The main contributor is the S_{17} value for the $^7\mathrm{Be} + {}^1\mathrm{H}$ reaction, which plays a negligible role in the solar energy production and hence in modeling the Sun. The remaining uncertainties in the S_{33} and S_{34} values are also of negligible consequences for the model. A 10% increase in S_{33} or a 5% decrease in S_{34}, which are equivalent, lead — in the center of the solar model — to a temperature lower by 0.005% and a hydrogen abundance higher by 0.08%. The latter increase is easy to understand. With the age and the photon flux constraints, the lower neutrino losses imply more economical use of the hydrogen fuel.

The only neutrino producing reaction with no uncertainties assigned is ${}^3\mathrm{He} + {}^1\mathrm{H}$. The recoil electron spectrum measured in the Superkamiokande experiment, showing the excess in the highest energy region above the prediction, indicates that the cross-section may be underestimated.

The calculated value of the Cl capture rate, given in Table I, is $\Phi_{\mathrm{Cl}} = 7.7^{+1.2}_{-1.0}$ SNU, which within the error agrees with the 1968 value. The very good agreement results in part from a cancellation of independent contributions. The value climbed up to nearly 10 SNU in a wake of improvements in modeling the Sun. New opacities increased Φ_{Cl} by about 1 SNU and the previously ignored effect of chemical element settling by nearly 2 SNU. These increases were, subsequently, nearly compensated by changes in the cross-section data and a decrease in the atmospheric heavy element content.

It seems highly unlikely that future improvements of SSM may lead to changes in the calculated neutrino fluxes beyond the ranges listed in Table I. To see large changes, one must depart from the standard stellar evolution theory. A number of departures have been suggested as astrophysical solutions to the neutrino problem. Their common part have always been a lowering temperature in the solar core, which results in lowering the high-energy-neutrino production rate. In order to keep the photon flux at the observed value, the hydrogen content in the core must be increased. The most plausible way of achieving it, as I already mentioned, is allowing some form of mixing within the core. The solar core is not unstable to convection. Other possibilities for generating a fluid motion within the core have been considered. They are reviewed by Bahcall [5]. None of them are fully satisfactory. Nuclear-reaction driven instability of certain oscillation modes [14] was perhaps the best proposal, but the resulting motion is very unlikely to cause material mixing. Nonetheless, bearing in mind that a minute microscopic velocities suffice to mix the material within the core during Sun's life-time, one cannot rule out such a possibility.

Consider gas circulation with a typical velocity v_c. Taking for the core radius $r_c \approx 2 \times 10^5$ km and for the solar age $\tau_\odot = 4.5$ Gy, we obtain $v_c \approx 4$ cm/s as the minimum circulation required for mixing. Alternatively, we may conjecture a turbulence characterized by a mean eddy velocity, v_T, and a mean free path , λ_T. Then, taking $v_T\lambda_T/3$ as an estimate of the eddy diffusivity, we get $v_T = 12 r_c/\lambda_T$ cm/s as the minimum velocity, which yields a number which is perhaps an order of magnitude higher than the minimum

v_c but by some six orders of magnitude lower than the local sound speed. This stringent upper limit for the fluid velocity is required to eliminate the possibility of a mixed solar core.

Microscopic diffusion velocity is by about an order of magnitude less than v_c and their net effect is a small enhancement of the abundance gradient resulting from hydrogen fusion.

3. Helioseismic probing of the Sun's interior

3.1. Solar oscillations

Discovery of oscillations in solar photosphere [15] and the showing that they manifest excitation of global oscillation modes [16] opened a new possibility of probing the Sun's interior by means of seismic sounding. The modes detected have frequencies in the 1–5 mHz interval and cover a wide range of spherical harmonic degrees, beginning with $\ell = 0$ (radial pulsation) up to $\ell \sim 10^4$. Modes of degrees higher than, say, $\ell = 200$ are not relevant for probing the interior. The detected modes belong to two distinct types: pressure modes (denoted p_n, where n is the radial order), which are trapped acoustic waves, and fundamental modes (denoted f or p_0), which are analogues of surface water waves.

Solar oscillations are excited as an acoustic noise by turbulent convection. The site of excitation is located near the surface, where turbulence is most vigorous. They are seen both in radial velocity and intensity fluctuations. The amplitudes of individual modes are very low: ~ 10 cm/s in radial velocity and $\sim 10^{-6}$ in the relative intensity fluctuations. Random fluctuations due to turbulent convection are by five orders of magnitude larger.

The observables of our interest here are mode frequencies. They are shown in Fig. 1. The accuracy of these data is impressive. The errors had to be multiplied by a factor of 10^3 to make the error bars visible. No other solar data are measured with such accuracy. What is shown in the figure are frequencies averaged over the $2\ell + 1$ components within multiplets. These averages yield a probe of the radial structure. The frequency dependence on the azimuthal order, m, is induced by rotation and asphericity. The latter arises primarily from the magnetic field and varies significantly in the activity cycle. It is always very small, which provides an empirical justification for the neglect of other forces beyond gravity and pressure in modeling the Sun.

The data plotted in Fig. 1 were obtained with two instruments (MDI [18] and GOLF[19]) on board of SOHO spacecraft, which began operation in April 1996. Except for a few month brake in 1998 when the contact with spacecraft was temporarily lost, the instruments continue operation to date. They follow seismic changes in the Sun through its current high activity phase. The first frequency data suitable for deep sounding of the interior became available by 1989 [19], in part prior to publication. Their accuracy was not as high as that of more recent data but in fact all most important inferences regarding the Sun were obtained from these early data.

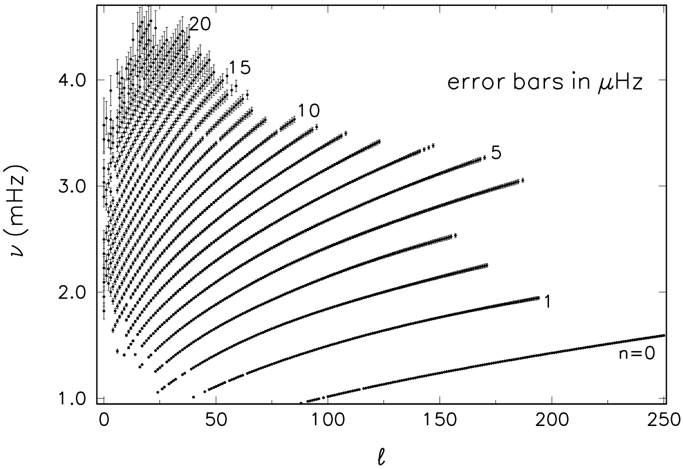

Fig. 1. Measured frequencies of solar oscillations are plotted against mode degree, ℓ. Numbers at selected branches indicate mode radial order, n. The f-mode branch is denoted $n = 0$. The error bars correspond to one standard deviations multiplied by 1000.

3.2. From p-mode frequencies to the internal structure

Having constructed solar model one may easily calculate frequencies of its oscillation in the adiabatic approximation, which consists in ignoring radiative losses and interaction with convection. Going beyond this approximation is difficult and results are unreliable.

In the upper panel of Fig. 2, I plot differences between measured frequencies of p-modes shown in Fig. 1 and frequencies of the same modes calculated in the adiabatic approximation for the SSM constructed by Sienkiewicz [20]. The differences are small but significant at 10–100σ level. At this stage, however, we do not know yet to what extent the differences are due to true differences between the Sun and its model and to what extent they are due to inadequacies of the adiabatic approximation.

Fortunately, the two contributions may be disentangled because we know that the approximation may be invalid only in a thin layer at the surface and there we may assume that radial dependence of mode eigenfunction is independent of ℓ. Making use of this property we may express the small differences in frequencies in the following form [21],

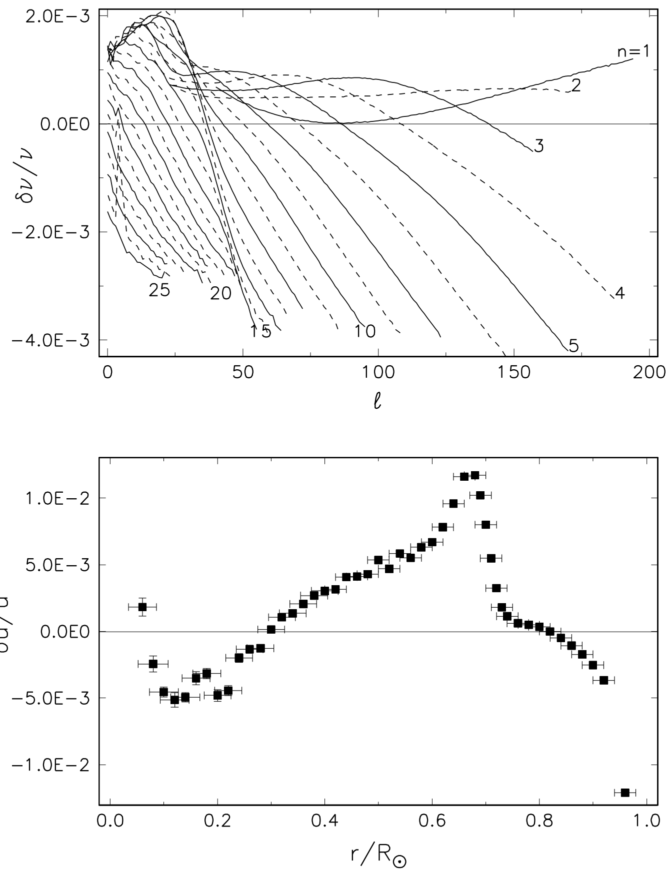

Fig. 2. (top) The relative differences in frequencies between the Sun and its model. (bottom) The inferred relative differences in $u(r)$. Vertical error bars directly reflect 1σ errors of the frequencies. Horizontal bars represent the full width at half-maximum of the Gaussian-like averaging kernels.

$$\left(\frac{\delta\nu}{\nu}\right)_i = \int_r^{R_\odot} \mathcal{K}_{u,i}\frac{\delta u}{u}dr + \mathcal{J}_i\delta Y_e + \frac{F_{\mathrm{surf}(\nu)}}{I_i}, \tag{1}$$

where $i \equiv (\ell, n)$ is a mode identifier; $R_\odot$ is solar radius; $u(r)$ is a selected structural parameter, which we choose to be $u = P/\rho$ (pressure to density ratio); Y_e is the mass fractional abundance of helium in the outer part of the Sun which is chemically homogeneous; F_{surf} describes the near surface inadequacies of the model; kernels $\mathcal{K}_{u,i}$ and the $\mathcal{J}_i$ and I_i (mode inertia) coefficients are determined numerically from mode eigenfunctions in the reference solar model. To obtain Eq. (1) we make use only of the hydrostatic equilibrium condition and a thermodynamical expression for the adiabatic exponent $\gamma = (\partial \ln P / \partial \ln \rho)_{\rm ad}$.

The differences δP and $\delta\rho$ may be expressed in terms of δu with simple integral relations, which follow from the linearized hydrostatic equilibrium condition. The term involving δY_e arises from the derivative of γ with respect of the helium abundance. It departs significantly from the ideal gas value 5/3 only in the outer layers, where hydrogen an helium are in the state of partial ionization. This is why Y refers to the outer part of the Sun which is convective and hence mixed. This implies that we cannot determine temperature in the solar core without additional constraints. These are thermal balance, equilibrium abundance as well as the laws for opacity, nuclear energy generation and diffusion coefficients. Without these constraints we know only the T/μ ratio in the core, where μ is the mean molecular weight. Thus, the interpretation of δu in the core cannot be not unique.

The lower panel of Fig. 2 shows the result of inversion of the frequency difference displayed in the upper panel. An optimal averaging method [22] has been applied to the set of Eq. (1) for all 1945 p-modes in the data. The simultaneously inferred difference in the helium abundance is $\delta Y_e = 0.006$, with a tiny formal error. The main source of uncertainty, which is difficult to asses, is the $\gamma(P, \rho, Y)$ dependence. These results are from [20] but there are essentially equivalent results published recently [23, 24], which were obtained with the use of different solar models, with somewhat different methods, and with the use of the same or other contemporaneous data sets. Ironically, somewhat smaller differences were found [25] for a model [26] calculated with an earlier release of the opacity data from the same source [8].

3.3. Discussion

The differences in the structural parameters between the Sun and its model are indeed quite small. Within chemically homogeneous envelope, $r \geq 0.72 R_{\odot}$, where $\delta T/T \approx \delta u/u + 0.75\delta Y_e$, nearly all the difference arises from inadequacies in the treatment of the convective energy transport. Cooler outer envelope implies that the transport is more efficient than assumed (note that the surface temperature is constrained by measurements). A more advanced treatment of convection than the one adopted in the solar model points in the same direction.

The differences in the non-convective interior may be explained by suitable changes in the opacity coefficient which are within the uncertainty of its calculation [27]. The spike in $\delta u/u$ just below the convective zone may mean that we missed an opacity source in the model. Alternatively, it may mean that there is mixing of chemical elements extending into the non-convective interior [25]. The latter interpretation is supported by the small positive value of δY_e. Larger mass in the outer mixed zone implies smaller decrease of the relative helium abundance due to the gravitational settling. All such changes are far too small to influence significantly the calculated neutrino fluxes [24].

When the frequency inversion based on Eq. (1) was first applied more than 10 years ago [21], the Sun-model differences were much larger. They pointed to a need for augmented opacity in the model. After considering plausible profiles for helium distribution in the core the conclusion was made that the results of helioseismic inversion exacerbate the solar neutrino problem and that its solution must be found in the particle physics. Except of regions very close to the center ($r \leq 0.05 R_{\odot}$), the decrease in the differences is mostly due to the improvements in solar models, consisting in the use of more advanced opacities and taking into account gravitational element settling. The two improvements have a qualitatively similar effect both on u and on the calculated neutrino fluxes. The remaining small differences in u suggest a small increase in the fluxes.

The bottom line conclusion from helioseismic sounding is a support for the standard model of the Sun. The only addition suggested — some element mixing just beyond the convective envelope — is not unexpected and in fact helps to explain the observed deficit of lithium in solar atmosphere relative to meteorites [28]. Such a mixing has virtually no effect on the structure of deeper layers, where the slow gravitational settling goes undisturbed for billions of years. The solar interior appears to be an unusually quiet place and indeed as simple as the simplest theory predicts.

4. The end of the solar neutrino problem

The evidence that the solar neutrino problem cannot be solved by changing model of the Sun's interior was available already in 1990s. It is not only the results of helioseismic sounding that effectively ruled out such a solution but, more importantly, a comparison of neutrino flux measurements from the Chlorine and Kamiokande detectors. Such a comparison, as Bahcall end Bethe [29] first noted, rules out both the astrophysical and the nuclear physics solutions leaving neutrino mixing as the only possibility. This conclusion got a further support with results from the Gallium detectors. The current status of the solar neutrino problem is summarized in Table II. The entries are based on numbers quoted after Bahcall *et al.* [10], where the references are given to original sources of the experimental data.

The largest deficit in the case of Homestake, implying virtually an absence of the Be- and CNO-neutrinos, which have intermediate energies, cannot be explained without conversion of the ν_e to a non-detectable fla-

TABLE II

Deficit of the neutrino counting rates in the three types of detectors

Detector	Threshold ν_e [MeV]	Flux ratio measured/predicted
GALLEX & SAGE	0.233 MeV	0.56 ± 0.05
Homestake	0.814 MeV	0.33 ± 0.03
Kamiokande & SuperK.	~ 6 MeV	0.48 ± 0.02

vor. A lower temperature in the core implies mainly the reduction of the B-neutrino production rate. A lower value of the $S_{3,4}/S_{3,3}$ ratio implies a similar reduction factor for the Be- and B-neutrinos. Neutrino flavor mixing remains as the only possibility. Its case has been strengthen by an independent evidence from Kamiokande data on the atmospheric neutrinos [30].

The Sun is a powerful source of low energy neutrinos which cannot be replaced with atmospheric or man-made ones. However, in a sense the role of astrophysics in interpretation of the solar experiment neutrino measurements came to the end. I believe that models we have now will remain for a long time sufficiently accurate for calculation of the neutrino production rate as well as the MSW effect. There is an uncertainty in calculated fluxes due to the data on nuclear reaction, in particular that on the $^3\mathrm{He} + {}^1\mathrm{H}$ fusion. Possible revisions, however, will have no impact for solar models and the assignment is for nuclear physicists not for astrophysicists. Perhaps their only role is just to remind that we have a reliable model of the Sun. Interpretation of the solar neutrino experiments without solar model [30] seems to me an irrational exercise. Why should we disregard the best understood and confirmed part of physics?

The goal of undertaking solar neutrino measurements was to test the hypothesis concerning the source of the solar energy. It has been emphasized that the goal has been accomplished by the fact that solar neutrinos from the pp chain have been measured [31]. I do not disagree. Indeed, the fact that the high energy neutrinos have been detected with the Kamiokande detectors proves that the solar photon flux is predominantly produced in the pp chain. Neutrinos from the CNO cycle are not detectable with this detector. It is also true, however, that it would be difficult to find an astronomer who had any doubts on this matter when the Kamiokande became available. Helioseismic sounding was unquestionably much more important as an empirical test of the solar interior models. In contrast, the benefit for whole science from undertaking the solar neutrino measurements exceeded expectation. Astrophysicists may be proud of the fact that the solar model they provided was an essential contribution to the discovery that neutrinos have masses and may change their flavors.

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